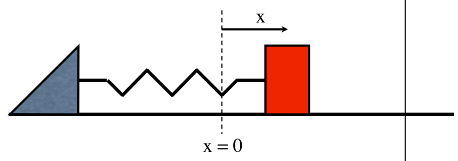
# SPRINGS



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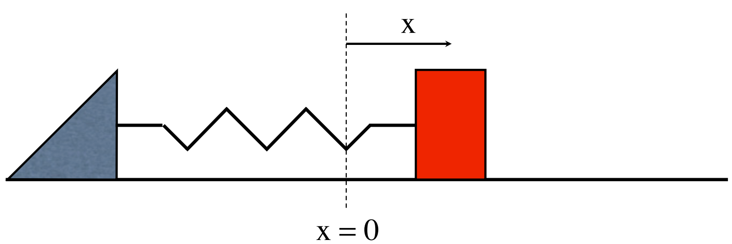
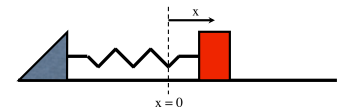



Attach a mass to a spring and push or pull it. If the spring is displaced a distance "x" from its equilibrium position, the spring will exert a force on the mass. The magnitude of that force will be:

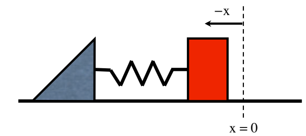
$$F_{spring} = -kx$$



1.) What does the "k" term tell you in $F_{spring} = -kx$ and what is it called?




2.) What does the negative sign do in the expression? Does your explanation make sense for the second set-up shown to the right?




3.) If the mass in the first sketch was held in that position, would it have energy as a consequence of its contact with the spring? If so, how much?



The spring, assumed to be "ideal" (this means it doesn't lose energy to friction as it oscillates back and forth), generates a "conservative force" that acts on the mass. That means the spring has a potential energy function associated with it.

The whole idea behind a potential energy function is to find a function that, when evaluated at two points in the function's force field, will have a difference (actually, it's minus the difference, but we're quibbling here) equal the work done by the field as a body moves between those two points. We did this for gravity by comparing the solution of "mg" dotted into "d" to $W_{gravity} = -\Delta U_{gravity} = -\left(U_{2,gravity} - U_{1,gravity}\right)$

The problem with doing this with a spring is that spring forces are not constant (the more you pull a spring, the more force it applies back on you) as was gravity near the earth's surface. That means we need to use Calculus to get the job done.

The approach is simple. We determine how much work is involved in moving the spring a differential (read this "very small") displacement "dx," then sum up all such values between our start and finish position. So let's assume the spring starts at distance $x_1$ from its equilibrium position, and moves to distance $x_2$ (note that it doesn't matter which of these is larger). Using an integral sign to do the summation, this reads:

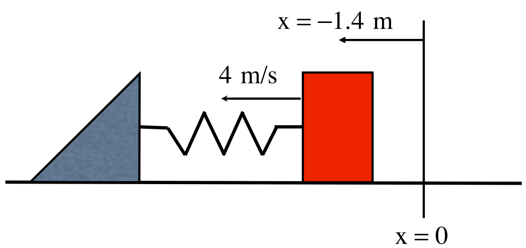
$$
\begin{aligned}
W_{spring} &= \int \vec{F}_{spring} \bullet d\vec{x} \\
&= \int \left(-kx\hat{i}\right) \bullet \left(dx\hat{i}\right) \\
&= -\int_{x_1}^{x_2} (kx)\,dx \\
&= -\left(\frac{1}{2}kx^2\right)_{x_1}^{x_2} \\
&= -\left(\frac{1}{2}k{x_2}^2 - \frac{1}{2}k{x_1}^2\right)
\end{aligned}
$$

From observation, it would appear that the potential energy function for a spring is

$$U_{spring} = \frac{1}{2}kx^2$$

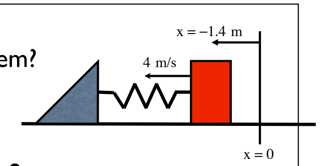


Problem: What you are seeing below is a snapshot of a 600 gram mass attached to a spring whose spring constant "k" is 2 nt/m. At t=0 seconds, the mass is moving to the left at x = -1.4 meters with a speed of 4 m/s.






a.) What is the total mechanical energy in the system?



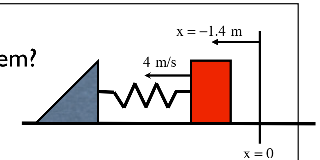

b.) Determine the mass's speed how when at x = -.2 meters.

c.) Determine its maximum deflection.

d.) At what coordinate will its potential and kinetic energy be equal?



a.) What is the total mechanical energy in the system?




$$
\begin{aligned}
E_1 &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\
&= \frac{1}{2}(.6\text{ kg})(4\text{ m/s})^2 + \frac{1}{2}(2\text{ nt/m})(-1.4\text{ m})^2 \\
&= 6.76\text{ J}
\end{aligned}
$$

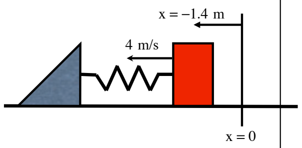
b.) Determine the mass's speed how when at x = -.2 meters.

If we know the total energy in the system, we can use that information and the total energy expression to determine the velocity at x = -.2 meters.

$$(E_{total}) = 6.76\text{ J} = \frac{1}{2}(.6\text{ kg})v^2 + \frac{1}{2}(2\text{ nt/m})(-.2\text{ m})^2$$

$$\Rightarrow \quad v = 4.73\text{ m/s}$$

## c.) Determine its maximum deflection.




At maximum deflection, the body isn't moving so the kinetic energy is zero, and all the energy in the system is potential. Sooo . . .

$$(E_{total} =)\ 6.76\ \text{J} = \cancelto{0}{KE} + U$$

$$\Rightarrow \quad 6.76\ \text{J} = \frac{1}{2}(2\ \text{nt/m})x_{max}{}^{2}$$

$$\Rightarrow \quad x_{max} = 2.6\ \text{m}$$

## d.) At what coordinate will its potential and kinetic energy be equal?

This will occur when half the **6.76** joules is wrapped up in potential energy, or:

$$3.38\ \text{J} = \frac{1}{2}(2\ \text{nt/m})x^{2}$$

$$\Rightarrow \quad x_{max} = 1.84\ \text{m}$$