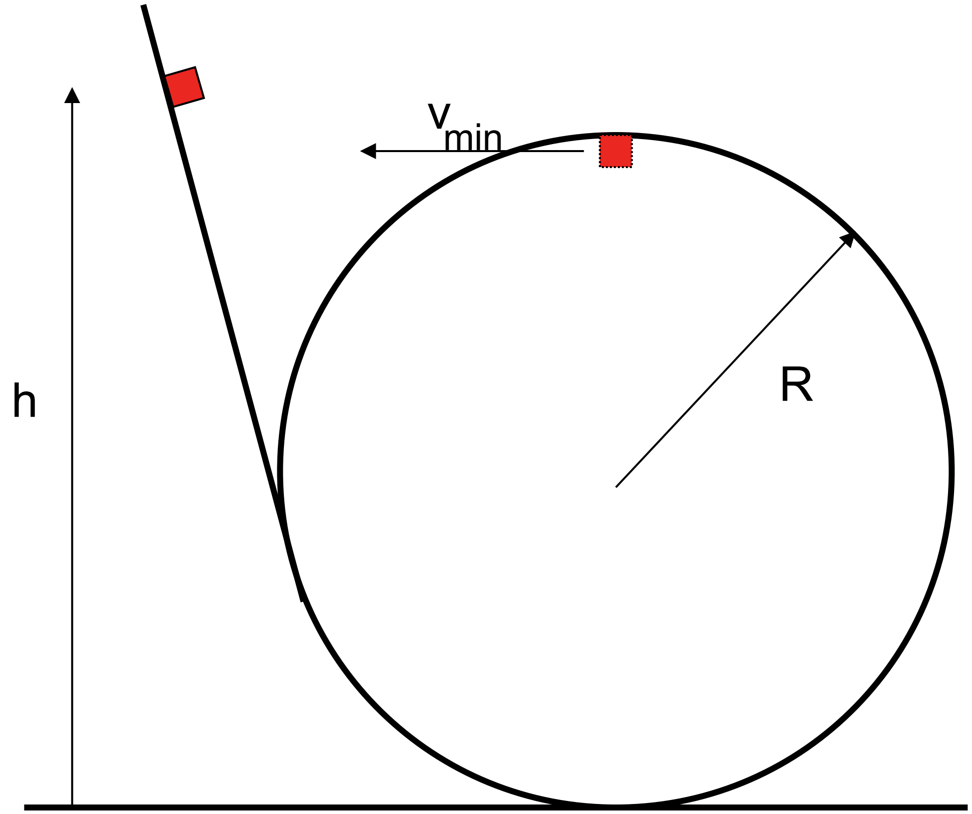
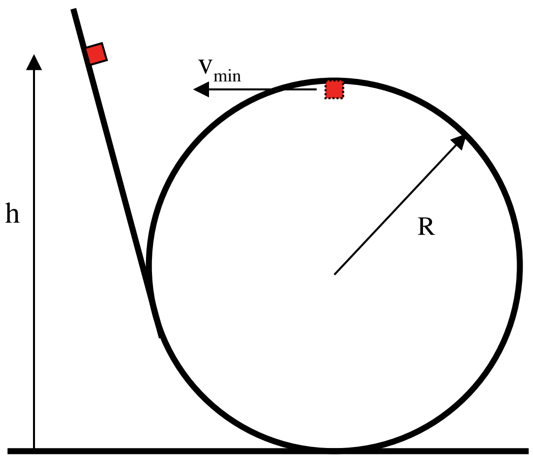
Loop the Loop

At what vertical height "h" above the ground must a block of mass "m" that is initially at rest start if it is to just make it through the top of the frictionless arc shown in the sketch? (Assume frictionless throughout the entire motion.)



The block of mass “m” starts from rest a minimum vertical height “h” units above the ground so that it just barely freefalls through the top of the arc in a frictionless loop-the-loop. Determine “h” for this to happen?



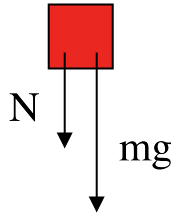
Starting with *conservation of energy*, we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$0 + (mgh) + 0 = \frac{1}{2} m v_{top}{}^2 + (mg(2R))$$

$$\Rightarrow v_{top}{}^2 = 2(gh) - 2(g(2R))$$

Using the fact that the body is moving centripetally, we can write:



$$\sum F_c :$$

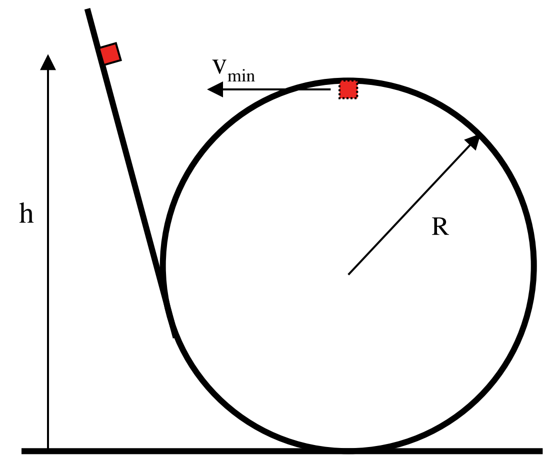
$$-N - mg = -m\frac{v^2}{R}$$

$$\Rightarrow v^2 = gR$$

Equating the two velocity terms yields:

$$(v_{top}{}^2 =)\ gR = 2(gh) - 2(g(2R))$$

$$\Rightarrow \quad h = \frac{5}{2}R$$



What would change if the block had an initial velocity up the incline?

In the Conservation of Energy relationship, there would have been an initial kinetic energy.

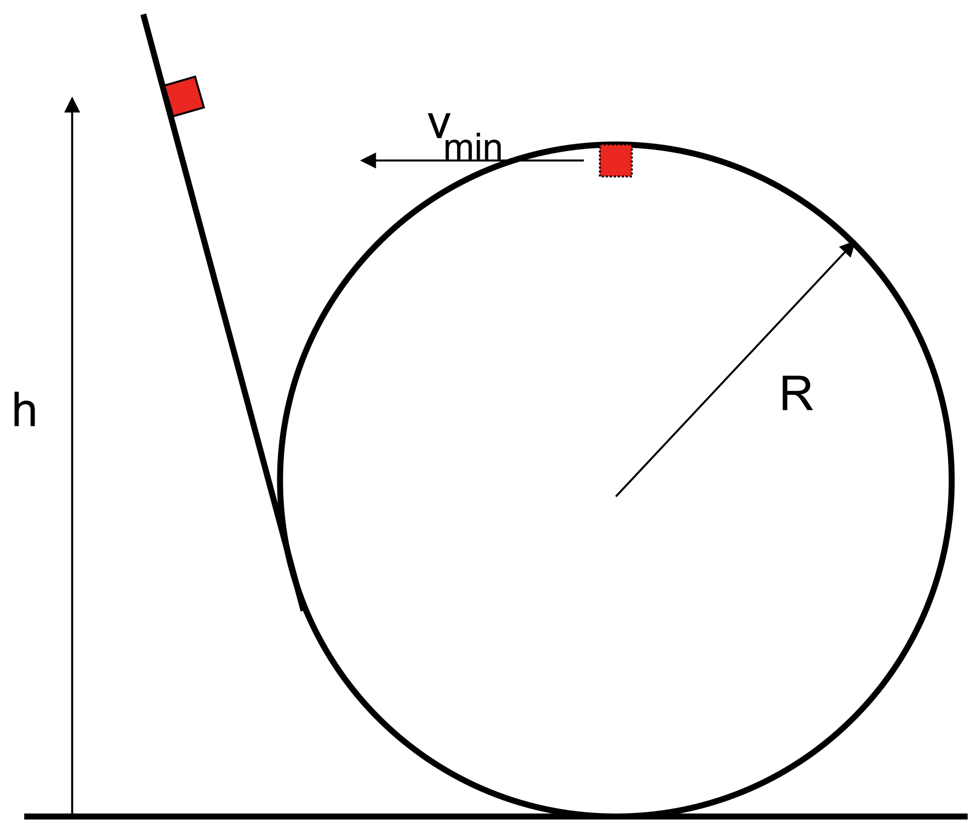
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\left(\frac{1}{2}mv_1^2\right) + (mgh) + \quad 0 \quad = \frac{1}{2}mv_{top}^2 + (mg(2R))$$

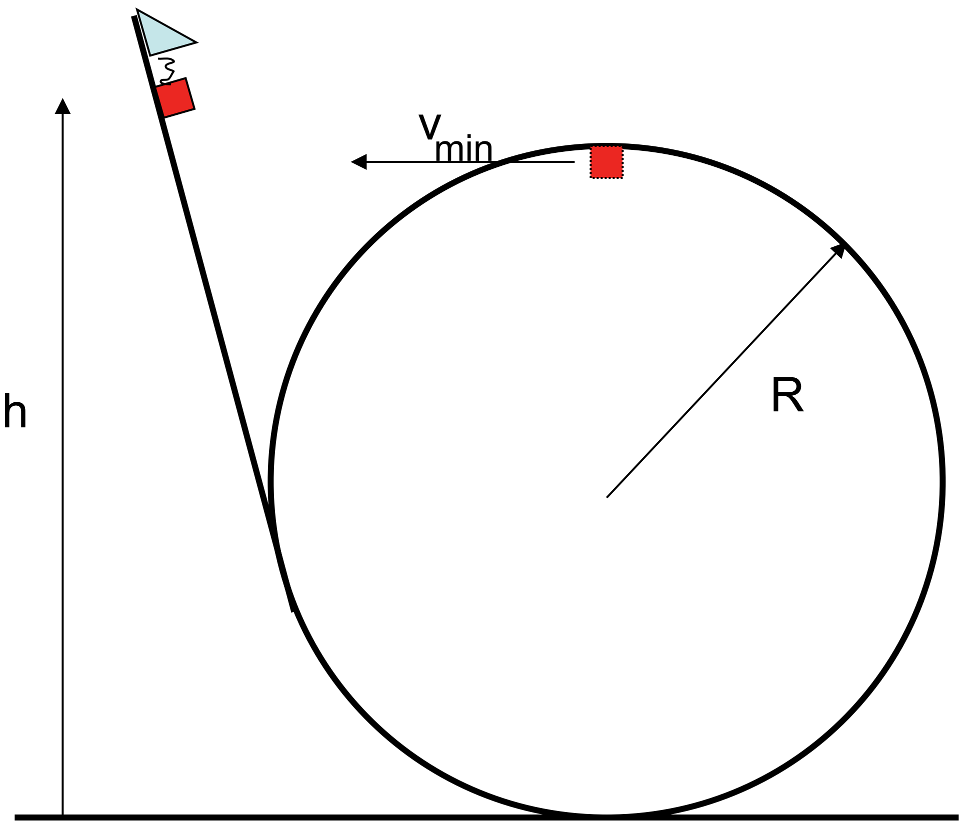
$$\Rightarrow \quad v_{top}^2 = v_1^2 + 2gh - 2(g(2R))$$

What would change if the block had an initial velocity down the incline?

Assuming no friction in the system, there would be no difference between the *conservation of energy* relationship with the velocity moving down the incline and a *conservation of energy relationship* with the velocity of same magnitude moving up the incline. The minimum velocity at the *top* would still be such that the normal force would go to zero, and you'd have to use both the *conservation of energy* and *centripetal force* and *Newton's Second* Law solved together to disembowel the problem.



What would change if the block was initially shoved a distance "L" meters up against an ideal spring whose spring constant was "k," if the block was then released from rest.



Ah, the old "add a spring" twist. In that case, there would simply be initial spring potential energy to be added into the *conservation of energy* relationship at the start. (Note that there would be no spring potential energy once the spring had sprung after the beginning of the run, and that I can simplify by multiplying everything by "2/m.") Doing so yields:

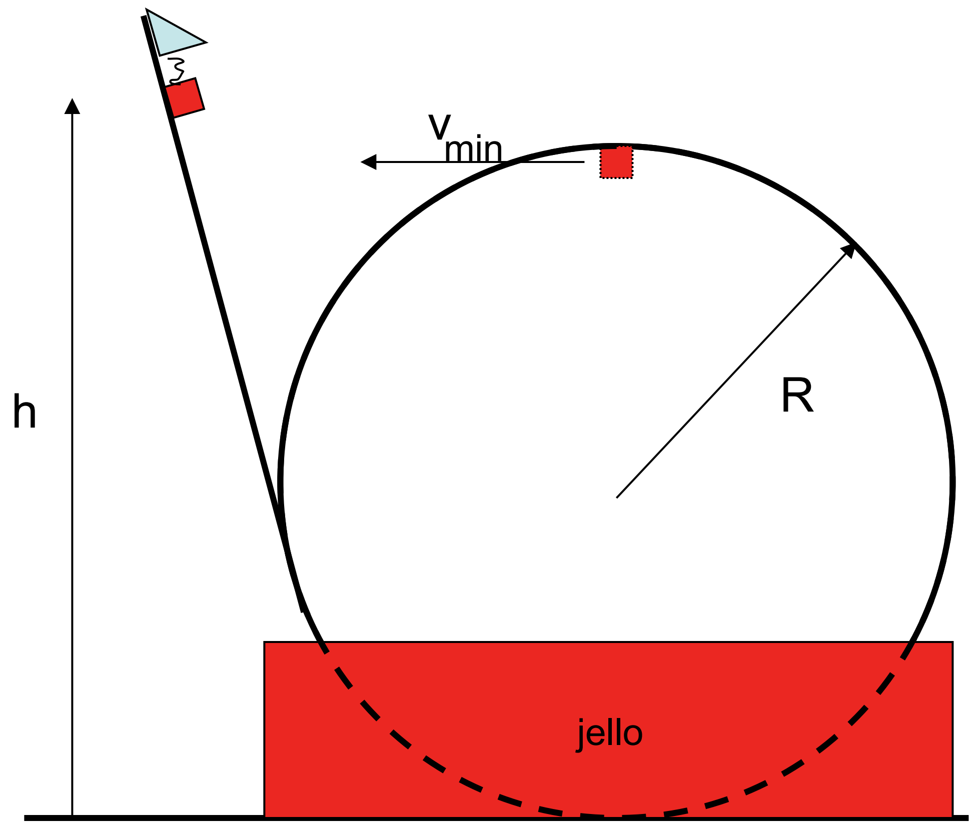
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$0 + \left(\frac{1}{2}kL^2 + mgh\right) + 0 = \frac{1}{2}mv_{top}^2 + \left(mg(2R)\right)$$

$$\Rightarrow \quad v_{top}^2 = \frac{kL^2}{m} + 2gh - 2\left(g(2R)\right)$$

How would things change if the spring stayed but the block had to slosh through liquid jello at the bottom of the arc, losing 4000 joules of energy in the process?

In that case, you'd have to include the energy lost to the jello in the "extraneous work" part of *conservation of energy*.



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$0 + \left(\frac{1}{2}kL^2 + mgh\right) + (\text{-}4000\ J) = \frac{1}{2}mv_{top}^2 + \left(mg(2R)\right)$$

$$\Rightarrow v_{top}^2 = \frac{kL^2}{m} + 2gh - \frac{2(4000\ J)}{m} - 2\left(g(2R)\right)$$