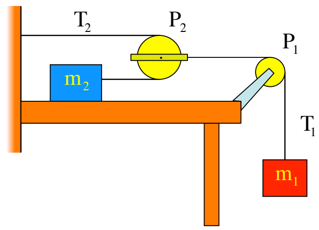
Problem 4.68

The masses m_1 and m_2 are connected to one another as shown in the sketch.

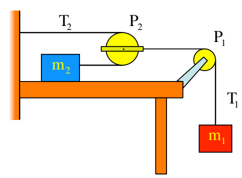
a.) How are the accelerations of the masses related (1:1, 2:1, 3:1, 2:3, what?)

b.) Derive expressions for the tensions denoted.

c.) Derive expressions for the mass's accelerations.



Problem 4.68



The masses m_1 and m_2 are connected to one another as shown in the sketch.

a.) How are the accelerations of the masses related (1:1, 2:1, 3:1, 2:3, what?)

Notice that the hanging mass will be taking up twice as much string (it will travel twice as far in a given amount of time) as will the mass on the table. As such, it's acceleration will be twice as much as that of the mass on the table. If we denote the acceleration of the mass on the tabletop as "a," we can write:

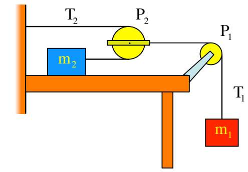
$$a_1 = a \quad and \quad a_2 = 2a$$

b.) Derive expressions for the tensions denoted.

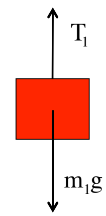
This is a Newton's Second Law problem.

b.) Derive expressions for the tensions denoted.

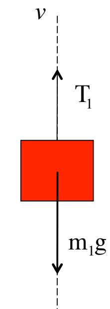
We will start with the hanging mass:



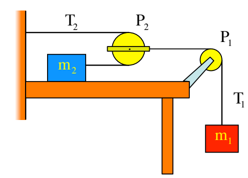
f.b.d. on m_1:



The *line of the acceleration* is in the vertical, so I will put an axis along that line.



We will start with the hanging mass.
Remembering that $a_1 = a \quad and \quad a_2 = 2a$

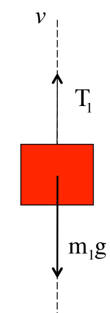


$\underline{\sum F_v}$:

$$T_1 - m_1 g = -m_1 a_1$$

$$\Rightarrow \quad T_1 - m_1 g = -m_1 a$$

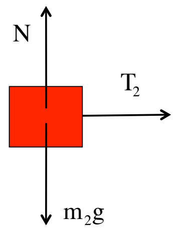
$$\Rightarrow \quad a = \frac{-T_1 + m_1 g}{m_1}$$

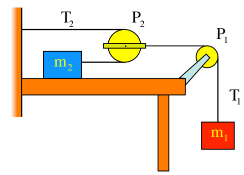


We need another equation (two unknowns, *a* and *T*).

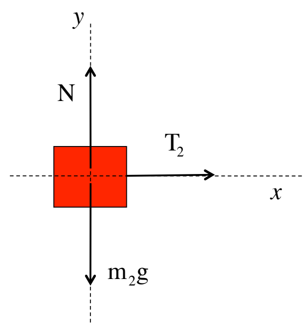
As for the mass on the table:

f.b.d. on m_2:

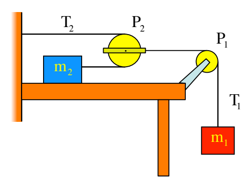




The *line of the acceleration* is in the horizontal, so I will put an axis along that line and one perpendicular to that line.



As for the mass on the table:



$\sum F_x$:

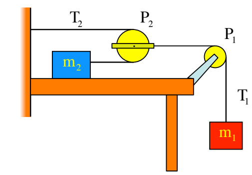
$$T_2 = m_2(2a)$$

$$\Rightarrow \quad a = \frac{T_2}{2m_2}$$

This problem has a twist that is somewhat mundane but might have messed you up. Pulley 2 has tension T_2 pulling on *top* AND on the *bottom*, which is to say that the force exerted on that pulley to the left is equal to $2T_2$. As the pulley has little mass associated with it, we can assume that the tension to the right (T_1) is equal to the net tension to the left, or:

$$2T_2 = T_1$$

Combining the acceleration and tension relationships we got from the f.b.d.'s, we can write:



$$a = \frac{T_2}{2m_2}$$

$$T_1 - m_1 g = -m_1 a$$

$$\Rightarrow \quad T_1 - m_1 g = -m_1\left(\frac{T_2}{2m_2}\right)$$

Using the fact that $2T_2 = T_1$, we can write:

$$T_1 - m_1 g = -m_1\left(\frac{T_2}{2m_2}\right)$$

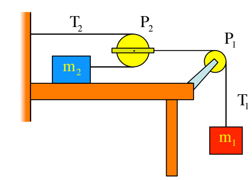
$$\Rightarrow \quad 2T_2 - m_1 g = -m_1\left(\frac{T_2}{2m_2}\right)$$

$$\Rightarrow \quad 2T_2 + m_1\left(\frac{T_2}{2m_2}\right) = m_1 g$$

$$\Rightarrow \quad T_2\left[2 + \frac{m_1}{2m_2}\right] = m_1 g$$

$$\Rightarrow \quad T_2\left[\frac{4m_2 + m_1}{2m_2}\right] = m_1 g$$

$$\Rightarrow \quad T_2 = \frac{2m_2 m_1 g}{4m_2 + m_1}$$



Again, using the fact that $2T_2 = T_1$, we can write:

$$T_1 = 2T_2$$

$$\Rightarrow \quad T_1 = \frac{4m_2 m_1 g}{4m_2 + m_1}$$

c.) Derive expressions for the mass's accelerations.

All of the physics is wrapped up in using N.S.L., so because there is nothing left to do but some math, I'm going to let this section ride.