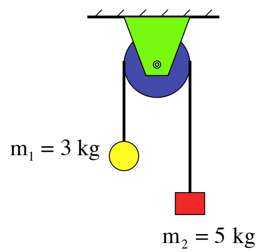
## Problem 4.38

This device is called an Atwood Machine.
For the masses shown, what is:




a.) the tension in the strings?

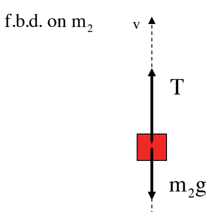
b.) the acceleration of the system?

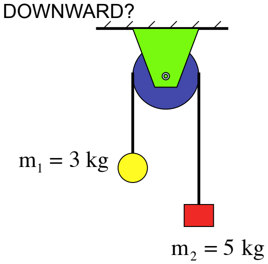
c.) the distance each initially stationary mass will travel after 1 second?



a.) the tension in the strings, assuming the 3 kg mass accelerates DOWNWARD?

For the right mass:



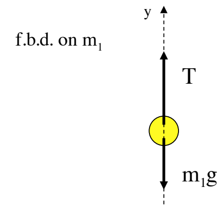

We will assume that the right mass is accelerating upward (it isn't, but I want you to see what happens in Part b if you assume the wrong direction for the acceleration). As I always call upwards positive, the sign in front of the acceleration term in N.S.L will be positive!

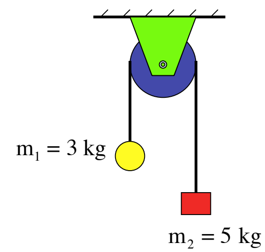
$$\sum F_y :$$
$$T - m_2 g = m_2 a$$
$$\Rightarrow \quad a = \frac{T - m_2 g}{m_2}$$

Notice that I've solved for "a." Why? We have two unknowns, "a" and "T." We want to determine "T." The way to do this is to solve for "a," then substitute it into the relationship we will get when we do the left mass. Watch and see.



For the left mass:




If the right mass is assumed to be accelerating upward, the left mass must be accelerating downward. As I always call upwards positive, the sign in front of the acceleration term in N.S.L will be NEGATIVE!

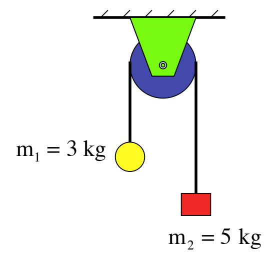
$$\sum F_y :$$
$$T - m_1 g = -m_1 a$$
$$\Rightarrow \quad a = \frac{-T + m_1 g}{m_1}$$

Notice that with the exception of the subscripts and the sign in front of the acceleration term, the f.b.d. and the N.S.L. expression for both masses looks the same. This is OK. Those small variations will make all the difference!



Using the two expressions shown below:

$$a = \frac{T - m_2 g}{m_2} \quad \text{and} \quad a = \frac{-T + m_1 g}{m_1}$$



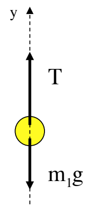

we can solve simultaneously for "T."

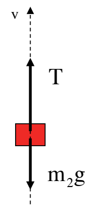
$$\frac{-T + m_1 g}{m_1} = \frac{T - m_2 g}{m_2}$$
$$\Rightarrow \quad -m_2 T + m_2 m_1 g = m_1 T - m_1 m_2 g$$
$$\Rightarrow \quad (-m_2 - m_1) T = -m_2 m_1 g - m_1 m_2 g$$
$$\Rightarrow \quad T = \frac{m_2 m_1 g + m_1 m_2 g}{(m_2 + m_1)}$$

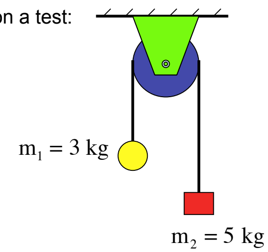
Crazy, eh?

b.) the acceleration of the system, assuming the 3 kg mass accelerates DOWNWARD?

Starting from scratch and showing you how you might do this on a test:




$\sum F_y$ :

$$T - m_1g = -m_1a$$
$$\Rightarrow \quad T = m_1g - m_1a$$

$\sum F_V$ :

$$T - m_2g = m_2a$$
$$\Rightarrow \quad T = m_2g + m_2a$$

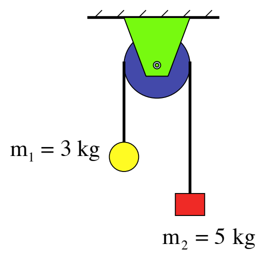
Combining yields:

$$m_1g - m_1a = m_2g + m_2a$$
$$\Rightarrow \quad a = \frac{m_1g - m_2g}{m_1 + m_2}$$



Solving with numbers yields:

$$a = \frac{m_1g - m_2g}{m_1 + m_2}$$
$$= \frac{(3\text{ kg})(9.8\text{ m/s}^2) - (5\text{ kg})(9.8\text{ m/s}^2)}{(3\text{ kg} + 5\text{ kg})}$$
$$= -2.45\text{ m/s}^2$$

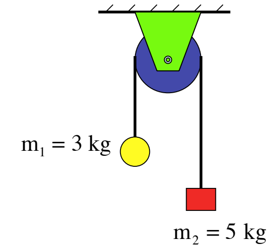



So what does the negative sign mean?

At the beginning of the problem, I stated that I was going to assume the “wrong direction” for the acceleration of the block mass (I made it upward in the positive direction). I said then that I was doing that because I wanted you to see what happens when you assume what turns out to be the wrong direction for the acceleration of the system. Well, you are seeing it. When you put in the numbers, a negative sign shows up in the solution. That sign does NOT mean the acceleration is in the negative direction (in fact, the “a” you calculate is supposed to be a magnitude, which is to say, always positive). It is simply signaling to you that you’ve got the acceleration’s direction wrong. No big deal! The magnitude is still OK. All you have to do to make things right is to make the statement, “And the direction of acceleration is opposite the assumed direction.“



c.) the distance each initially stationary mass will travel after 1 second?



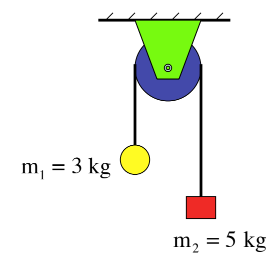

This is a simple kinematics problem.

$$\Delta y = v_o t + \frac{1}{2}at^2$$
$$= 0 + .5\left(2.45\text{m/s}^2\right)\left(1\text{ sec}\right)^2$$
$$= 1.225\text{ meters}$$



b.) The QUICK AND DIRTY APPROACH for the acceleration of the system, assuming the 3 kg mass accelerates DOWNWARD.

Looking at the sketch, you know the forces that are motivating the system are gravity on the two masses (the tension forces are internal, which is to say that they cancel one another out). One motivates the ball mass to accelerate upward, the other downward, so one must be deemed positive and one negative. As we have been instructed to assume that the ball is accelerating downward, we will take the gravitational force acting on it to be positive with the other being negative. Summing those forces and putting them equal to the acceleration times the TOTAL mass accelerating in the system, we get:




$\sum F_{external}$ :

$$m_1g - m_2g = (m_1 + m_2)a$$
$$\Rightarrow \quad a = \frac{m_1g - m_2g}{m_1 + m_2}$$

This is exactly what we got on Slide #5 using the long, formal approach. Pretty nifty, eh?