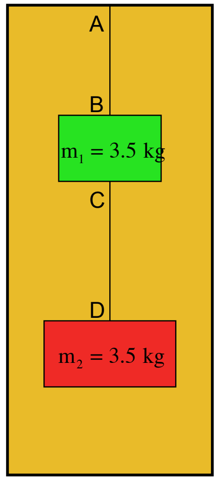
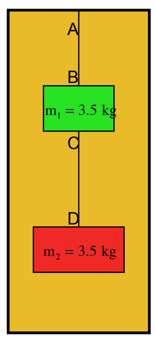
Problem 4.21

Two masses are attached to one another by rope as shown in the sketch.

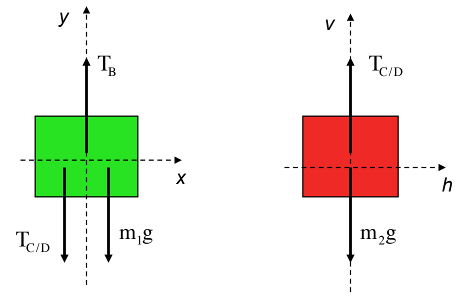
a.) What is the tension in the two lines if the acceleration is 1.6 m/s/s upward?

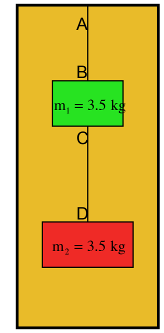
b.) If the strings can withstand a tension of 85 newtons, what maximum acceleration can the system handle?





Step 2: Identify direction of acceleration and line of axes.





Step 3: If there are any off-axis forces, break them into their component parts. (There are none in this problem.)

Step 4: Sum the forces on ONE body in ONE direction and put equal to "ma," where "a" is acceleration in that direction.

Noting that there are forces only in the vertical v-direction of the bottom mass, we can sum the forces on that mass yielding:

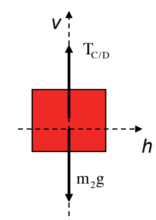
$$\sum F_v :$$

$$T_{C/D} - m_2 g = m_2 a$$

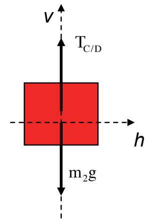
$$\Rightarrow \quad T_{C/D} = m_2 g + m_2 a$$

$$\Rightarrow \quad T_{C/D} = (3.5\ \text{kg})(9.8\ \text{m/s}^2) + (3.5\ \text{kg})(1.6\ \text{m/s}^2)$$

$$\Rightarrow \quad T_{C/D} = 39.9\ \text{nts}$$



Notice the difference if the acceleration had been DOWN. In that case, we would have had to unembed the negative sign from the acceleration term so the “a” stood for the MAGNITUDE of the acceleration and the math would have looked like:



$$\sum F_v :$$

$$T_{C/D} - m_2 g = -m_2 a$$

$$\Rightarrow \quad T_{C/D} = m_2 g - m_2 a$$

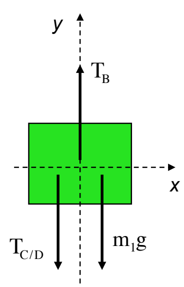
$$\Rightarrow \quad T_{C/D} = (3.5 \text{ kg})(9.8 \text{ m/s}^2) - (3.5 \text{ kg})(1.6 \text{ m/s}^2)$$

$$\Rightarrow \quad T_{C/D} = 28.7 \text{ nts}$$

Back to the original problem: Noting that there are no forces in the horizontal (i.e., x-direction), we can sum the forces on the top mass in the y-direction yielding:

$$\underline{\sum F_y :}$$

$$T_B - T_{C/D} - m_1 g = m_1 a$$

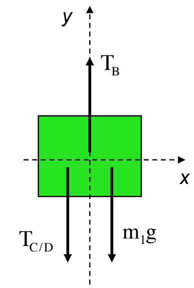


Note that I haven’t subscripted the “a” term as the only acceleration happening is in the “y” direction so there’s no reason to delineate.

Note what would have happened if the acceleration had been DOWN. Again, the acceleration term would have to have its negative sign unembedded yielding:

$$\underline{\sum F_y :}$$

$$T_B - T_{C/D} - m_1 g = -m_1 a$$



In any case, going back to the original problem:

Doing everything algebraically first with our two relationships as shown below:

$$T_{C/D} = m_2 g + m_2 a \qquad\qquad T_B - T_{C/D} - m_1 g = m_1 a$$

We can substitute the first equation into the second yielding:

$$T_B - \quad T_{C/D} \quad - m_1 g = m_1 a$$

$$T_B - (m_2 g + m_2 a) - m_1 g = m_1 a$$

$$\Rightarrow \quad T_B = (m_1 + m_2)(a + g)$$

$$\Rightarrow \quad T_B = (3.5\text{kg} + 3.5\text{kg})(1.6 \text{ m/s}^2 + 9.8 \text{ m/s}^2)$$

$$\Rightarrow \quad T_B = 79.8 \text{ nts}$$

b.) For the final thrill, let's assume the maximum tension possible in either rope is 85 newtons. What is the maximum upward acceleration the elevator can execute and not break the rope?

The rope that is most vulnerable is the top one. Using the derived expression from a few pages back, we can write:

$$T_B = (m_1 + m_2)(a + g)$$

$$\Rightarrow \quad 85 \text{ nt} = (3.5\text{kg} + 3.5\text{kg})(a + 9.8 \text{ m/s}^2)$$

$$\Rightarrow \quad a = 2.34 \text{ m/s}^2$$