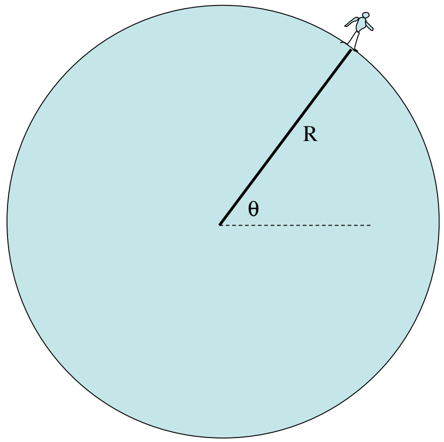
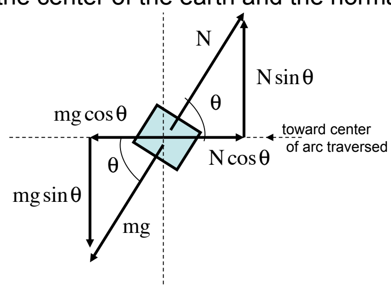
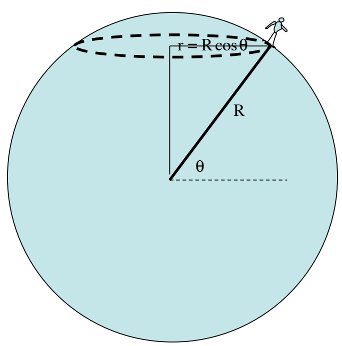
WHAT IS THE WEIGHT OF A MAN AT ANY LATITUDE

What you are about to see is a lesson in the adage, “If you get the free body diagram wrong, you’re screwed.” Follow along!



Draw what appears to be a reasonable f.b.d. with gravity oriented toward the center of the earth and the normal directed outward, we get:





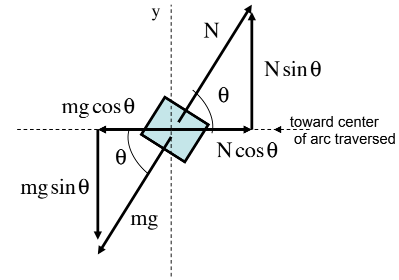
$\Sigma F_{\text{centripetal direction}}$:

$$N\cos\theta - mg\cos\theta = -ma_c$$

$$= -m\frac{v^2}{(R\cos\theta)}$$

$$\Rightarrow N = mg - m\frac{v^2}{R\cos\theta^2}$$

The problem becomes apparent when you sum the forces in the “y” direction. Doing that yields:



ΣF_y :

$$N\sin\theta - mg\sin\theta = ma_y \;(= 0)$$

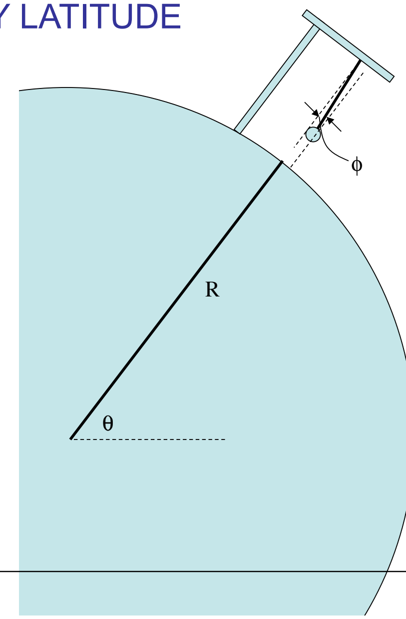
$$\Rightarrow N = mg$$

But this makes no sense in light of the previous derived relationship for “N,” and from common sense (the weight should be a function of the angle). So what’s wrong?

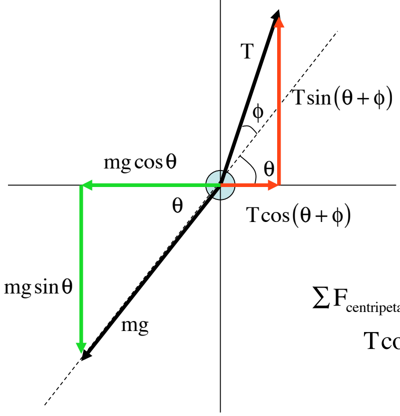
To answer that, we need to look at a related problem.

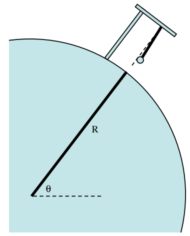
WHAT IS THE STRING TENSION OF A HANGING BOB AT ANY LATITUDE

What’s important to note is that the bob will not hang directly down toward the center of the earth. It will swing out a bit southward.



f.b.d. on bob taking the tension's deviation from the normal into account:





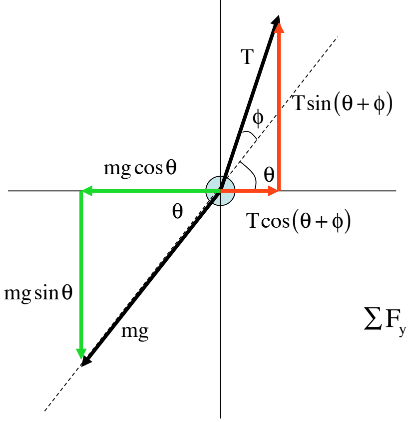
$\sum F_{\text{centripetal direction}}$:

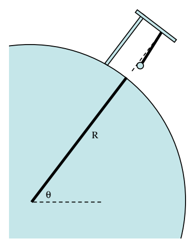
$$T\cos(\theta+\phi) - mg\cos\theta = -ma_c$$

$$= -m\frac{v^2}{(R\cos\theta)}$$

$$\Rightarrow\ T = mg - m\frac{v^2}{(R\cos\theta)(\cos(\theta+\phi))}$$

f.b.d. on bob



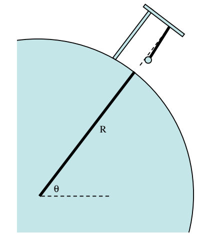


$\sum F_y$:

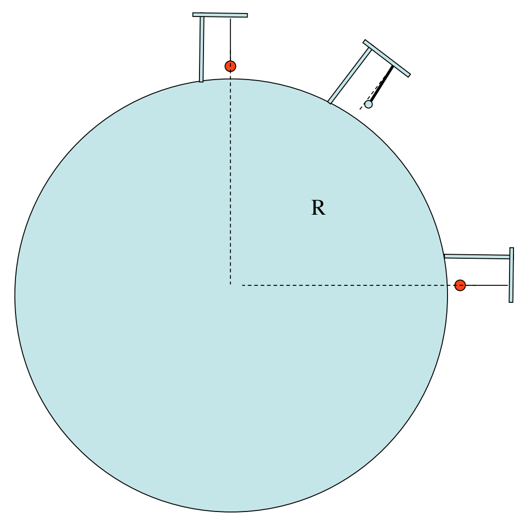
$$T\sin(\theta+\phi) - mg\sin\theta = ma_y$$

$$\Rightarrow\ T = \frac{mg\sin\theta}{\sin(\theta+\phi)}$$

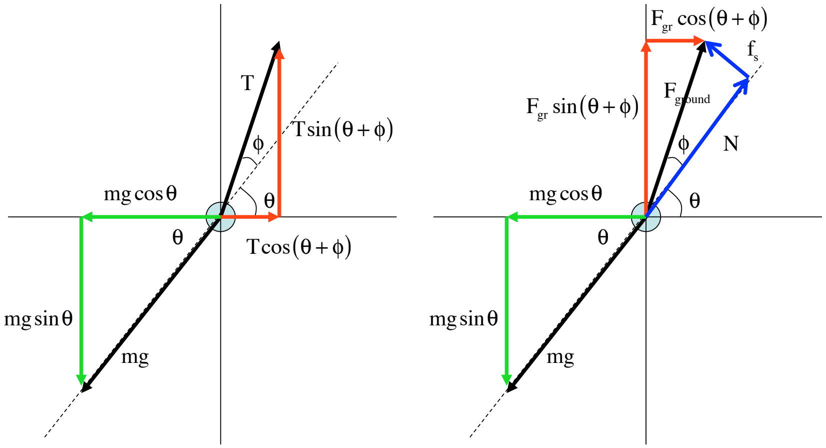
If we think of the bob as really a swing seat with our man sitting on it, then the tension in the line is really acting like the normal force in our "man at any latitude" problem. The only difficulty is that the normal force will ALWAYS be perpendicular to the surface providing it. It can't be off-angle with a line toward the center of the earth (in this case). So where is the skewing force coming from? Evidently, there must be a very small static frictional force acting at the individual's feet. If we include that, we end up with a free body diagram that allows for a net outward force (from the earth's surface) that is NOT along the line of gravity. And with that, our difficulty evaporates.



Minor note: One other thing that is interesting to notice is that if we did the pendulum problem with the structure located at the pole, we'd get tension and gravitational forces lining up, just as we'd expect of a normal and gravitational force in our original problem.



Apparently, when look at someone standing on a latitude other than zero or ninety, there must be an additional force—presumably one of friction, that makes the two situations comparable. See sketch below!



With friction, we find the same situation.