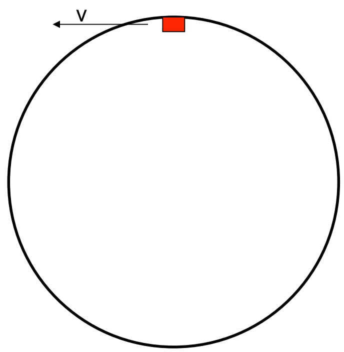
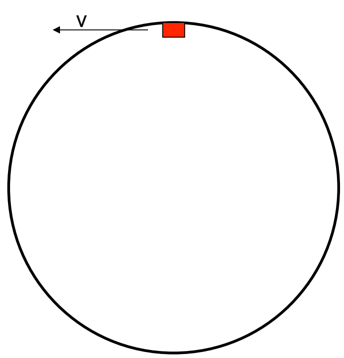
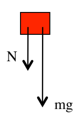
Supplementary Problem 5.38

What is the minimum velocity the cart must have to stay in its circular motion at the top of the loop?



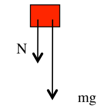
As always, start with an accurate free body diagram. In the most general case, which we should always start with, there is a normal force and gravity working. Being sure we get the correct direction for the normal force (how are normal directions defined?--think about it), we can draw:





HERE'S THE TWIST: If the body is moving really fast as it passes through the top of the arc, gravity isn't enough to pull it out of straight line motion. To provide the extra push needed, the loop provides a normal force (those are the two forces shown on the free body diagram). But if the block just barely free falls through the top of the arc, what does the normal force do? It goes to zero.

The appropriate way to do a problem like this, then, is to use Newton's Second Law as usual including ALL the forces acting in the general case, THEN strike out the normal force as equaling zero and solve what's left. That is:



$\sum F_{c.s.}$:

$$-\cancel{N}^{\,0} - mg = -ma_c$$

$$\Rightarrow \quad mg = m\left(\frac{v^2}{r}\right)$$

$$\Rightarrow \quad v = (gr)^{1/2}$$