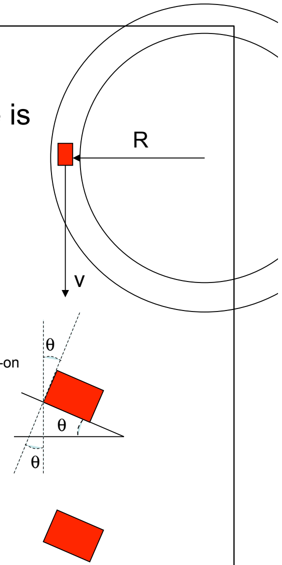
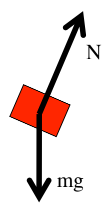
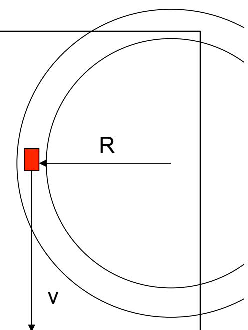
At what velocity must a car have to take a banked curve of radius R if the bank's surface is FRICTIONLESS?



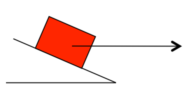
1.)

As always, start with a free body diagram (in this case, viewed from head-on):





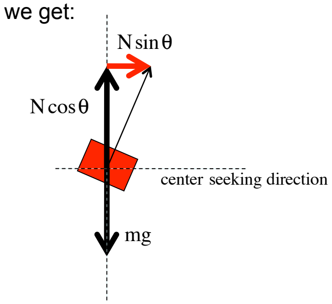
It's CRITICAL that you get the right direction for the centripetal axis (see below). In this case, that axis is in the horizontal (think about where the center of the arc is located). With that, the axes look like:

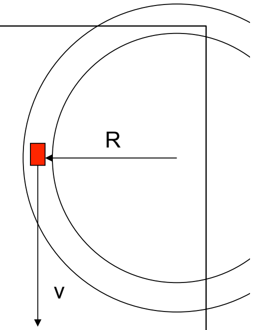


Center of the arc is this way!!!

2.)

With the axes, we get:





N.S.L. yields:

$\sum F_y$

$N\cos\theta - mg = m\cancelto{0}{a_y}$

$\Rightarrow \quad N = \dfrac{mg}{\cos\theta}$

and

$\sum F_{c.s.}$

$N\sin\theta = ma_{c.s.}$

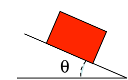
$\Rightarrow \quad \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = m\left(\dfrac{v^2}{R}\right)$

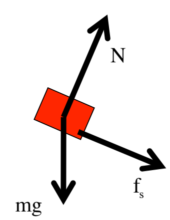
$\Rightarrow \quad v = \sqrt{Rg\tan\theta}$

3.)

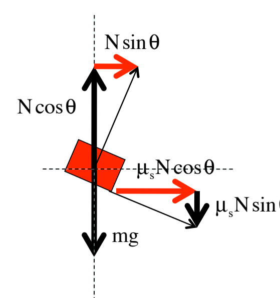
What would happen if there was friction? It would depend upon whether the car was traveling slowly of fast.

If the car was moving extremely fast, the car would slide up and over the top if it broke loose suggesting that the static frictional force would be DOWN the embankment. It's free body diagram would look like:



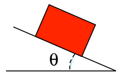


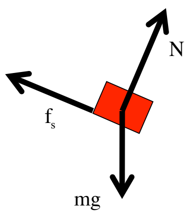
The center seeking direction would still be to the right in the horizontal, so the static frictional force would have to be broken into its components (as shown) before using N.S.L.



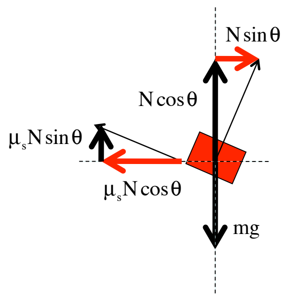
4.)

If the car was moving extremely slow, the car would tend to slide down the embankment if it broke loose suggesting that the static frictional force would be UP the embankment in that case. It's free body diagram would look like:

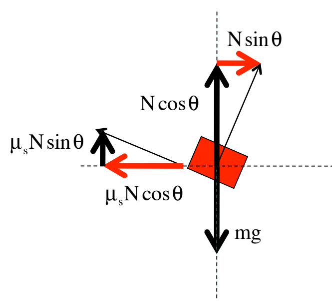




And the components would look like:



In the case of going too slow, the math would look like:



$$\sum F_y$$

$$\mu_s N\sin\theta + N\cos\theta - mg = \cancelto{0}{ma_y}$$

$$\Rightarrow \quad N = \frac{mg}{(\mu_s \sin\theta + \cos\theta)}$$

$$\sum F_{c.s.}$$

$$-N\cos\theta + N\sin\theta = ma_{c.s.}$$

$$\Rightarrow \quad -\left(\frac{mg}{\mu_s \sin\theta + \cos\theta}\right)\cos\theta + \left(\frac{mg}{\mu_s \sin\theta + \cos\theta}\right)\sin\theta = m\left(\frac{v^2}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{Rg\left(\frac{-\cos\theta + \sin\theta}{\mu_s \sin\theta + \cos\theta}\right)}$$

Crazy, huh?