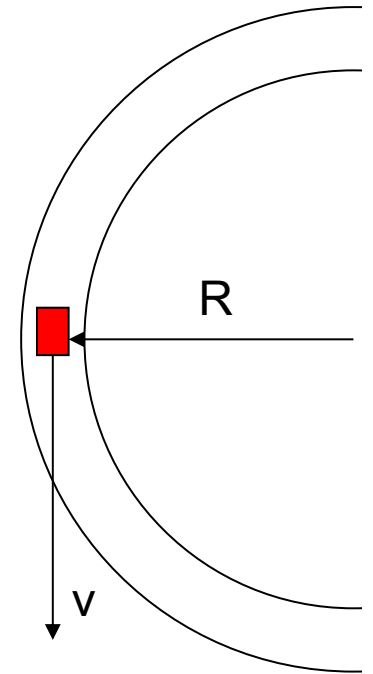
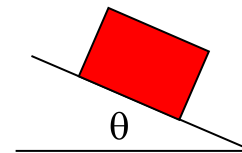
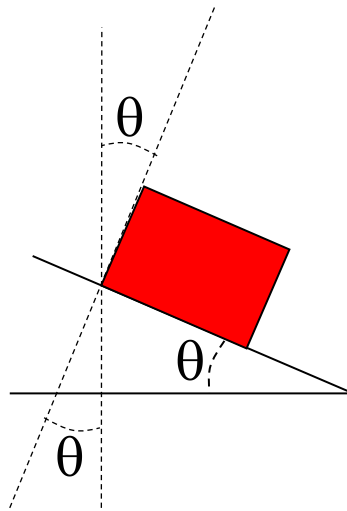


# Banked curve - no friction

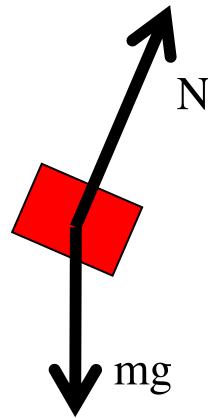
At what velocity must a car have to take a banked curve of radius  $R$  and angle  $\theta$  if the bank's surface is FRICTIONLESS?

As viewed from head-on

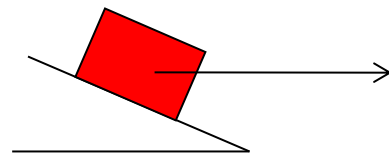


# Banked curve - no friction

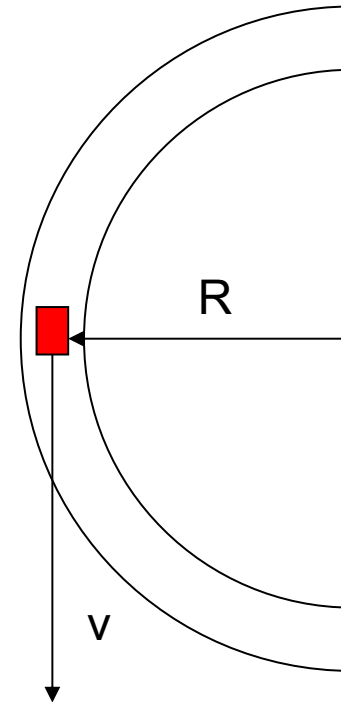
As always, start with a free body diagram (in this case, viewed from head-on):



It's CRITICAL that you get the right direction for the centripetal axis (see below). In this case, that axis is in the horizontal (think about where the center of the arc is located). With that, the axes look like:

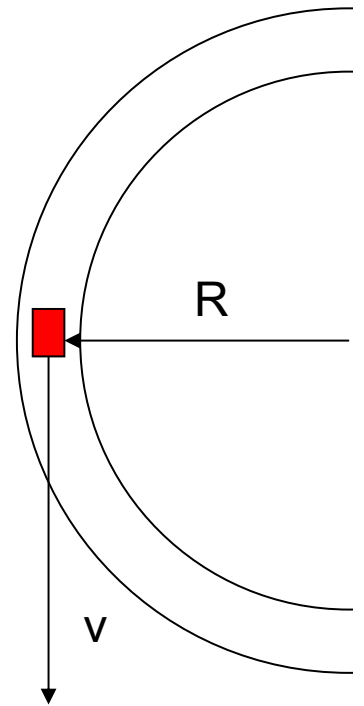
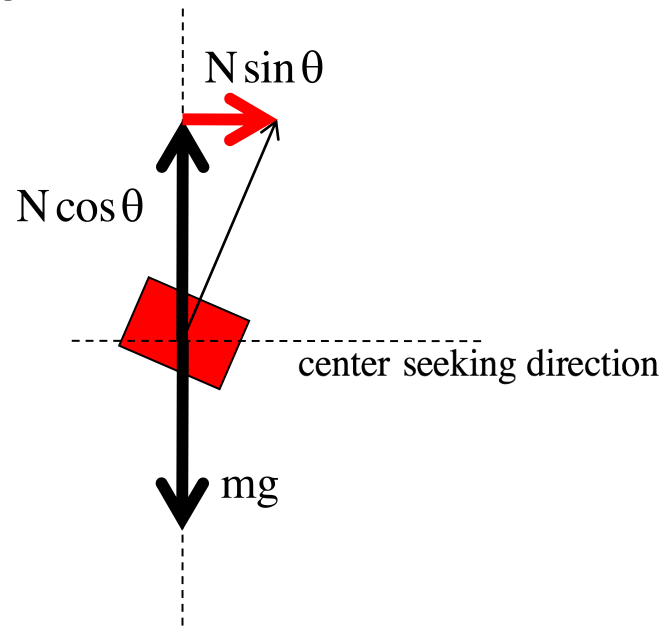


Center of the arc  
is this way!!!



# Banked curve - no friction

With the axes, we get:



N.S.L. yields:

$$\sum F_y$$

$$N \cos \theta - mg = ma_y$$

$$\Rightarrow N = \frac{mg}{\cos \theta}$$

and

$$\sum F_{cs}$$

$$N \sin \theta = ma_{c.s.}$$

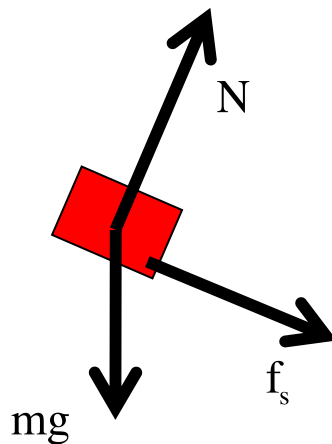
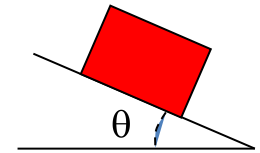
$$\Rightarrow \left( \frac{mg}{\cos \theta} \right) = m \left( \frac{v^2}{R} \right)$$

$$\Rightarrow v = (Rg \tan \theta)^{1/2}$$

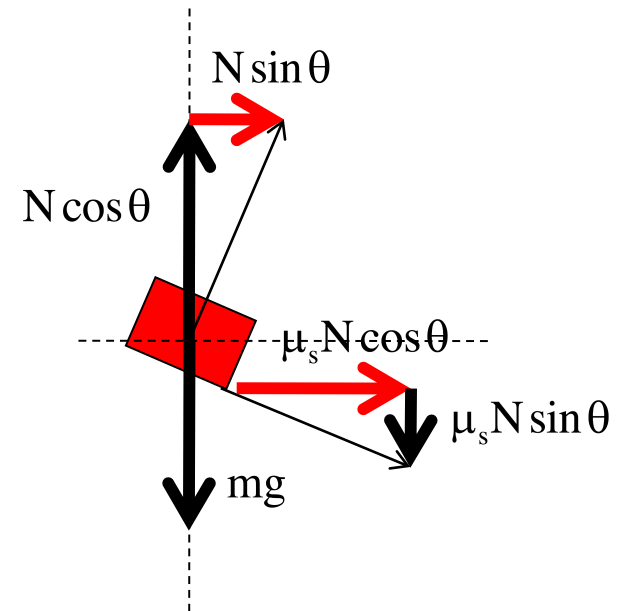
# Banked curve - WITH friction

What would happen if there was friction? It would depend upon whether the car was traveling slowly or fast.

If the car was moving extremely fast, the car would slide up and over the top if it broke loose suggesting that the static frictional force would be DOWN the embankment. Its free body diagram would look like:

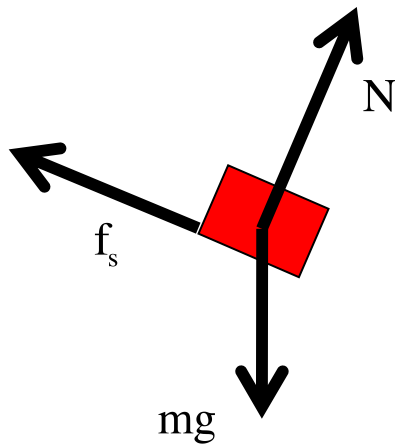
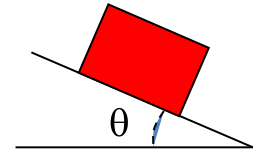


The center seeking direction would still be to the right in the horizontal, so the static frictional force would have to be broken into its components (as shown) before using N.S.L.

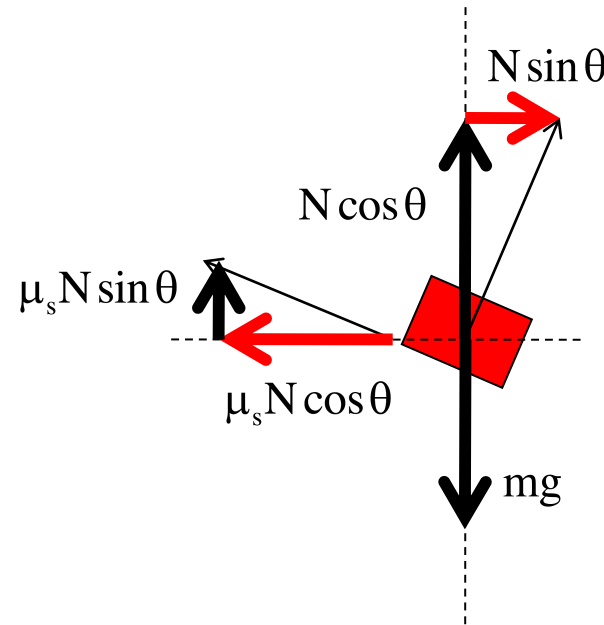


# Banked curve - WITH friction

If the car was moving extremely slow, the car would tend to slide down the embankment if it broke loose suggesting that the static frictional force would be UP the embankment in that case. Its free body diagram would look like:



And the components would look like:

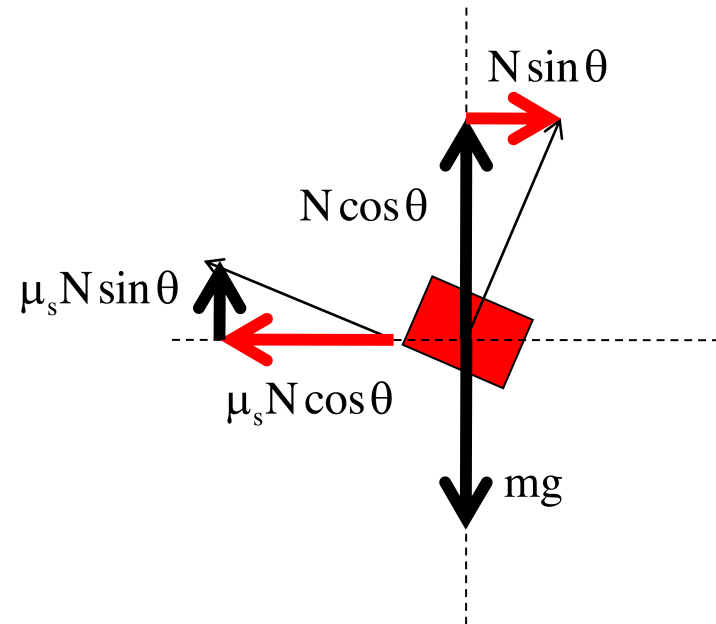


# Banked curve - WITH friction

In the case of going too slow, the math would look like:

$$\begin{aligned}\sum F_y \\ \mu_s N \sin \theta + N \cos \theta - mg &= ma_y \\ \Rightarrow N &= \frac{mg}{(\mu_s \sin \theta + \cos \theta)}\end{aligned}$$

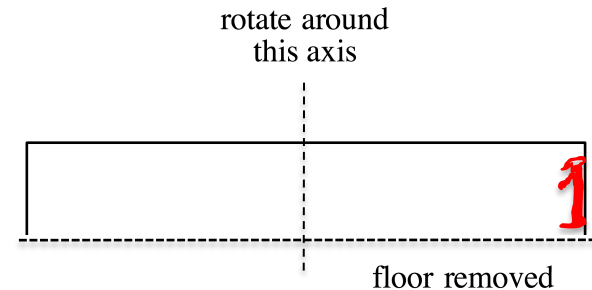
$$\begin{aligned}\sum F_{cs} \\ -N \cos \theta + N \sin \theta &= ma_{cs} \\ \Rightarrow -\left(\frac{mg}{\mu_s \sin \theta + \cos \theta}\right) \cos \theta + \left(\frac{mg}{\mu_s \sin \theta + \cos \theta}\right) \sin \theta &= m \left(\frac{v^2}{R}\right) \\ \Rightarrow v &= \sqrt{Rg \left(\frac{-\cos \theta + \sin \theta}{\mu_s \sin \theta + \cos \theta}\right)}\end{aligned}$$



Crazy, huh? (you do NOT need to know how to do all this math – it's to show you how it WOULD be done). You DO need to know how to draw the FBDs.

# Carnival ride – take 1

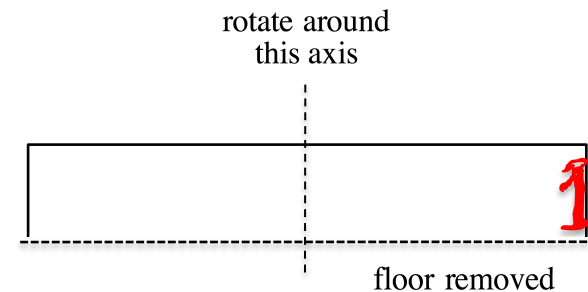
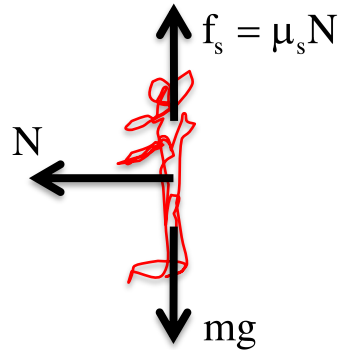
A rider stands against the wall of a huge cylinder of radius “R” that is constrained to rotate about its central axis. Once up to speed, the rider finds himself pinned against the wall as the floor drops out from under him.



If the coefficient of static friction between the rider and the wall is  $\mu_s$  what is the minimum speed the cylinder can rotate and keep the man from falling through to the ground?

# Carnival ride – take 1

As always, we must start with a f.b.d. on the man as he stands in the “snapshot” we are looking at.



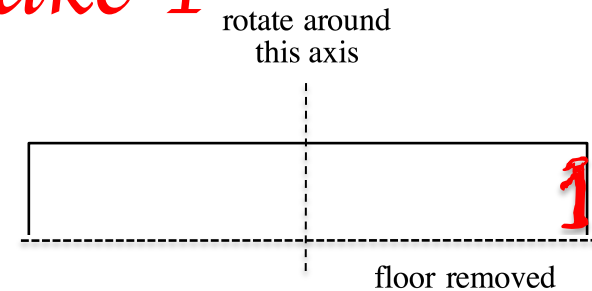
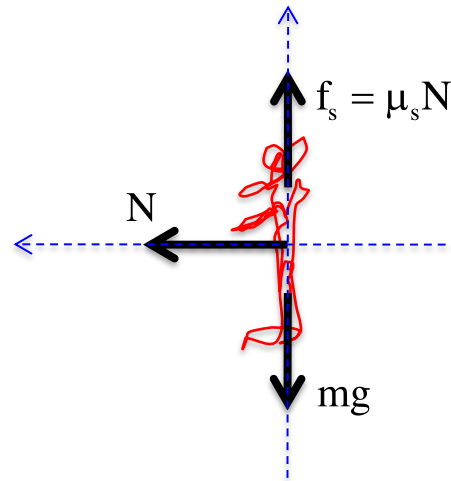
Note 1: Remember, the normal force acts away from the support that provides it, and,

Note 2: The static frictional force is UPWARD as the body would move DOWNWARD if it broke loose. Remember, it's the static frictional force that KEEPS the body from breaking loose!



# Carnival ride - take 1

The center seeking direction is toward the center of the arc, so our axis becomes:



With the f.b.d., we can write:

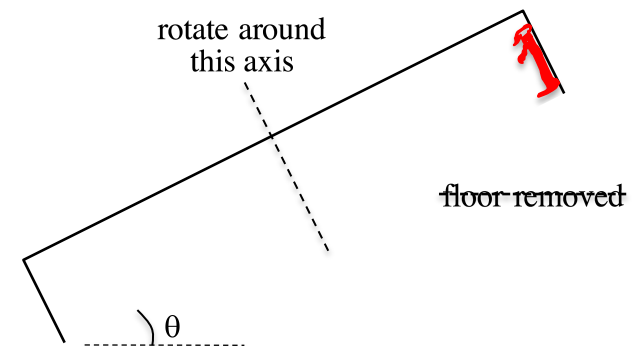
$$\begin{aligned} \sum F_y : \\ \mu_s N - mg &= ma_y^0 \\ \Rightarrow N &= \frac{mg}{\mu_s} \end{aligned}$$

and

$$\begin{aligned} \sum F_{c.s.} : \\ N &= ma_{c.s.} \\ \Rightarrow \left( \frac{mg}{\mu_s} \right) &= m \left( \frac{v^2}{R} \right) \\ \Rightarrow v &= \left( \frac{Rg}{\mu_s} \right)^{1/2} \end{aligned}$$

# Carnival ride...take 2!

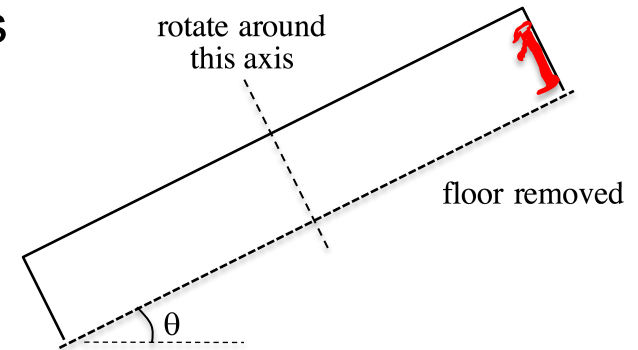
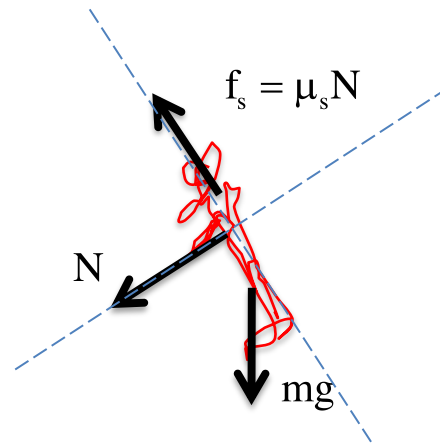
A rider stands against the wall of a huge cylinder of radius “R” that is constrained to rotate about its central axis. Once up to speed, the cylinder tilts upward to an angle  $\theta$  while the rider finds himself pinned against the wall as the floor drops out from under him.



If the coefficient of static friction between the rider and the wall is  $\mu_s$ , what is the minimum speed the cylinder can rotate and keep the man from falling through to the ground?

# Carnival ride – take 2

Clearly, we want to observe the man when he is most vulnerable to falling. That is shown to the right. The f.b.d. that goes with that position is:

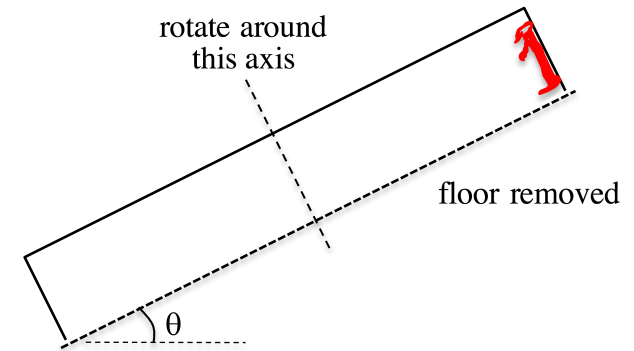
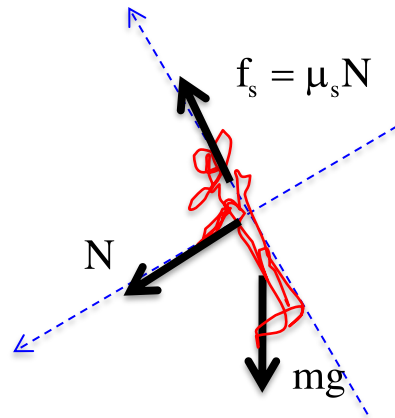


Note 1: As always, the normal force acts away from the support that provides it, and

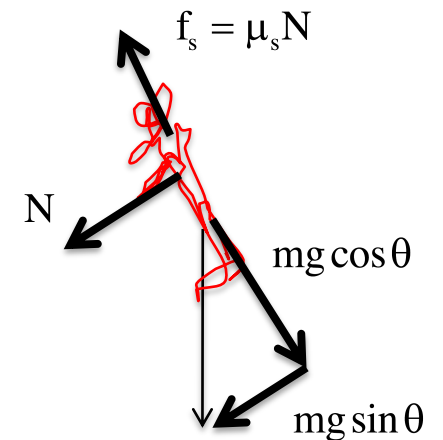
Note 2: The static frictional force is parallel to the surface providing it.

# Carnival ride - take 2

The center seeking direction is toward the center of the arc, so our axis becomes:

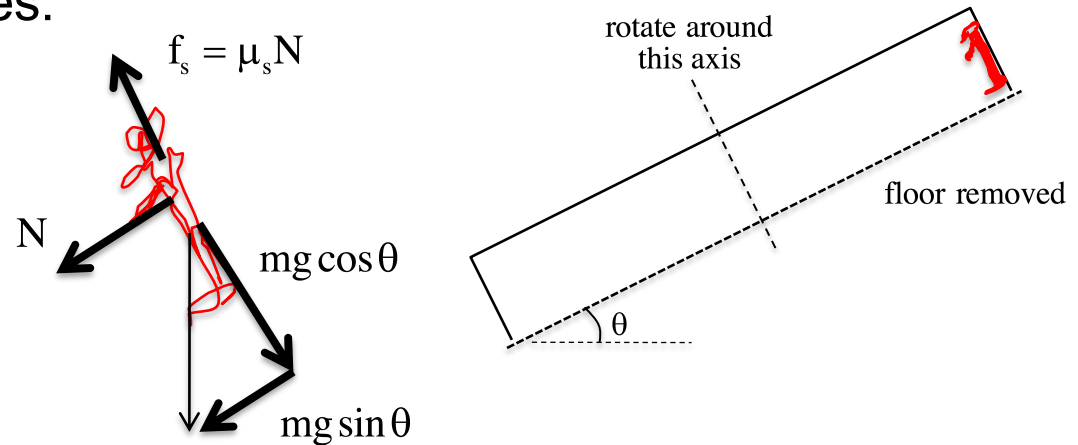


We need to break " $mg$ " into its component parts along our axis, or:



# Carnival ride - take 2

The center seeking direction is toward the center of the arc, so our axis becomes:



$$\sum F_y :$$

$$\mu_s N - mg \cos \theta = ma_y^0$$

$$\Rightarrow N = \frac{mg \cos \theta}{\mu_s}$$

and

$$\sum F_{c.s.} :$$

$$N + mg \sin \theta = ma_{c.s.}$$

$$\Rightarrow \left( \frac{mg \cos \theta}{\mu_s} \right) + mg \sin \theta = m \left( \frac{v^2}{R} \right)$$

$$\Rightarrow v = \left( \left( \frac{Rg \cos \theta}{\mu_s} \right) + Rg \sin \theta \right)^{1/2}$$

**Mini-lab:** An airplane circles with some unknown *velocity*  $v$  (see demo at front of room).

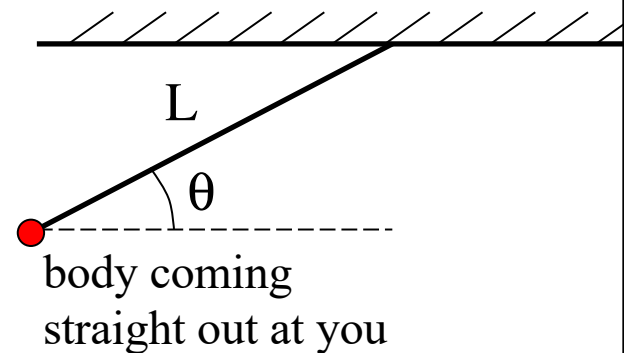
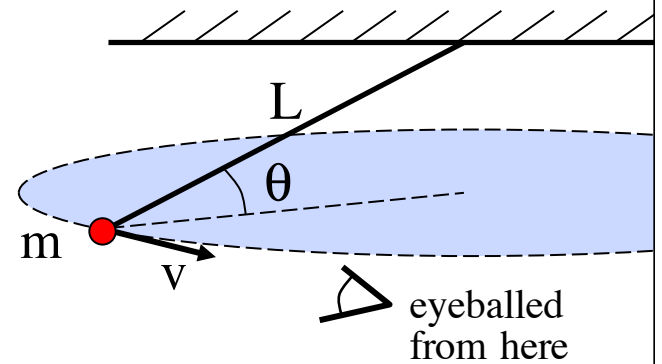
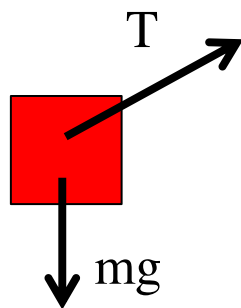
**Derive** an expression for  $v$ , then check to see if that value matches up with the actual velocity of the plane. Determine also the motion's **period**  $T$ .

**From what** perspective will you view the system?

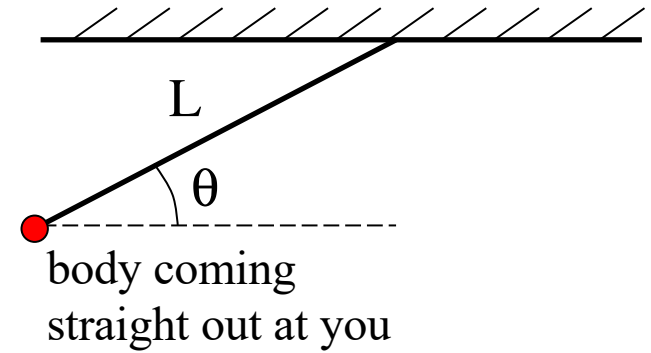
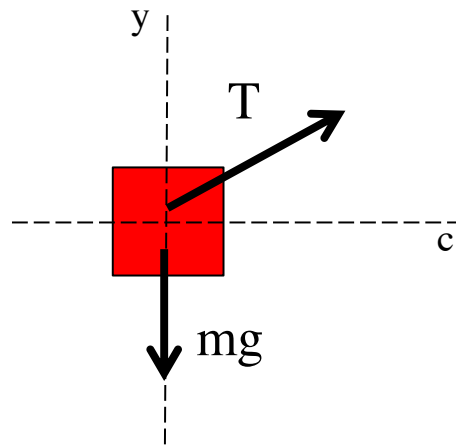
Look at it from head-on.

**Hint for f.b.d.:** Think of viewing the system as though the body was at an extreme—that is, with the velocity coming straight at you and the mass and string in the plane of the page.)

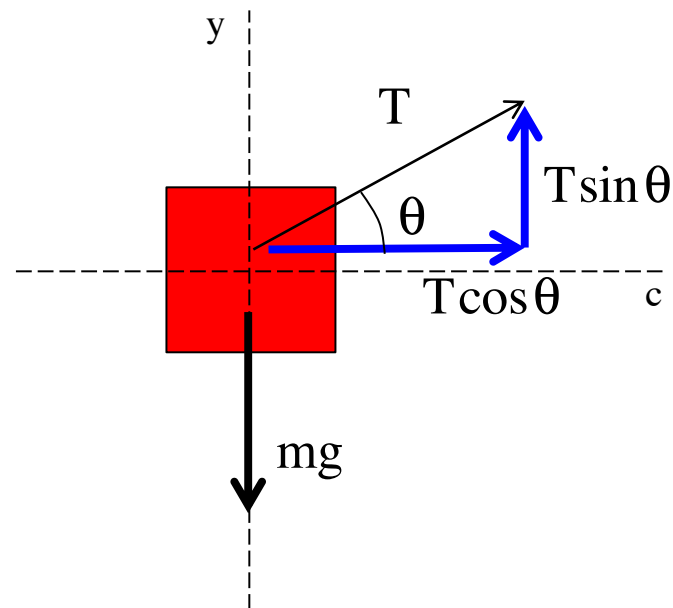
f.b.d. (looking at the body from head-on)



So *how* will you orient your axes?



Breaking off-axis forces into components:



etc.