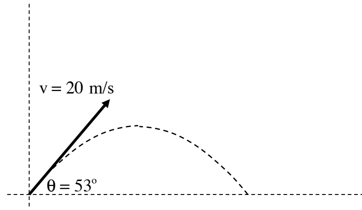
Problem 13.27

A place kicker kicks a ball at the 36 meter line (weird, but applicable). The ball leaves the ground as shown in the sketch.



a.) Does the ball clear the cross bar?

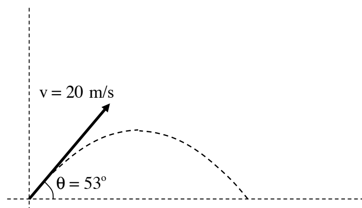
b.) As the ball approach the cross bar, is it rising or dropping?

c.) For the sake of amusement, what is the velocity of the ball at the top of its flight?

d.) If the ball makes it, what is its velocity as it crosses the cross bar?

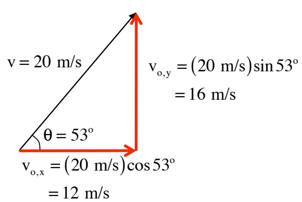
e.) What is the ball's velocity when it hits the ground?

A place kicker kicks a ball at the 36 meter line (weird, but applicable). The ball leaves the ground as shown in the sketch.



a.) Does the ball clear the cross bar?

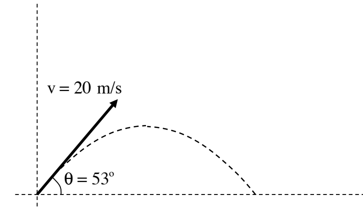
Approach: Determine the elapsed time for the ball to travel 36 horizontal meters. Determine the y-coordinate associated with that time. If that value is greater than 3.05 meters (the height of the bar), the ball makes it!



x-dir	y-dir
$x_1 = 0$	$y_1 = 0$
$x_2 = 36\text{ m}$	$y_2 = ?$
$v_{1,x} = 12\text{ m/s}$	$v_{1,y} = 16\text{ m/s}$
$v_{2,x} = 12\text{ m/s}$	$v_{2,y} = ?$
$a_x = 0$	$a_x = -9.8\text{ m/s}^2$

Horizontal motion over 36 m:

$$x_2 = x_1 + v_{1,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$
$$(36\text{ m}) = 0 + 12t + 0$$
$$\Rightarrow t = 3\text{ sec}$$



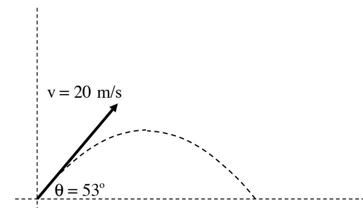
Vertical motion over that time:

$$y_2 = y_1 + v_{1,y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$y_2 = 0 + (16\text{ m/s})(3\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(3\text{ s})^2$$
$$\Rightarrow y_2 = 3.9\text{ m}$$

x-dir	y-dir
$x_1 = 0$	$y_1 = 0$
$x_2 = 36\text{ m}$	$y_2 = ?$
$v_{1,x} = 12\text{ m/s}$	$v_{1,y} = 16\text{ m/s}$
$v_{2,x} = 12\text{ m/s}$	$v_{2,y} = ?$
$a_x = 0$	$a_x = -9.8\text{ m/s}^2$

The y-component when the x-component is 36 meters is greater than the cross bar's height of 3.05 meters, so the kick clears the cross bar.

A place kicker kicks a ball at the 36 meter line (weird, but applicable). The ball leaves the ground as shown in the sketch.



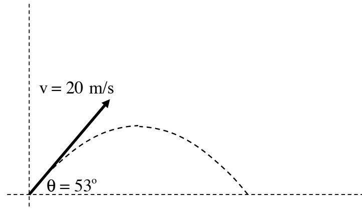
b.) As the ball approach the cross bar, is it rising or dropping?

Interesting question: If the ball is rising, it's y-coordinate will be larger a few instances after the time when the x-coordinate is 36 meters. As that time was 3 seconds, let's determine the y-coordinate at t = 3.01 seconds.

$$y_2 = y_1 + v_{1,y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$y_2 = 0 + (16\text{ m/s})(3.01\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(3.01\text{ s})^2$$
$$\Rightarrow y_2 = 3.77\text{ m}$$

This y-coordinate is lower than the 3.9 meters at t = 3 seconds, so the ball appears to be falling.

c.) For the sake of amusement, what is the velocity of the ball at the top of its flight?



At the top, the y-component of the velocity will always be zero. Assuming there is no acceleration in the x-direction (i.e., no jet pack attached to the ball), the velocity in the x-direction won't change throughout the motion, which means the net velocity at the top will be:

$$\vec{v}_{top} = (12 \text{ m/s})\hat{i} + 0\hat{j}$$

d.) What is the ball's velocity as it crosses the cross bar?

The x-component is EASY. It's the same as the initial velocity in the x-direction (12 m/s). For the y component (negative because it's moving downward):

$$v_{bar,y}^2 = v_{1,y}^2 + 2 a_y (y_{bar} - y_1)$$

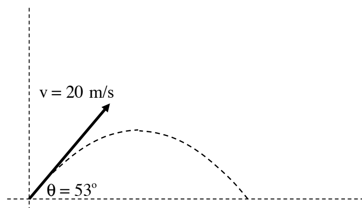
$$v_{bar,y}^2 = (16 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)[(3.9 \text{ m}) - 0]$$

$$\Rightarrow v_{bar,y} = -13.4 \text{ m/s}$$

d.) What is the ball's velocity as it crosses the cross bar?

The net velocity as it cross the bar is:

$$\vec{v} = (12 \text{ m/s})\hat{i} - (13.4 \text{ m/s})\hat{j}$$



e.) What is the ball's velocity when it hits the ground?

Because there is no air friction, the velocity should be 20 m/s when it gets back down to the ground (the same velocity magnitude it left the ground with) at an angle of 53 degrees downward.