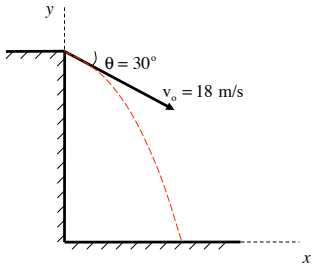
Problem 3.23

A student throws a rock 30 degrees below the horizontal moving with velocity magnitude 18 m/s. The cliff is 50 meters high.



a.) What are the stone's initial coordinates?

b.) What are the components of the initial velocity?

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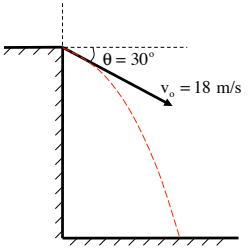

c.) Write out the velocity as a function of time for both the y and x direction.

d.) Write out the position as a function of time for both the y and x direction.

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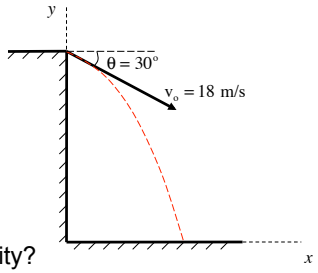
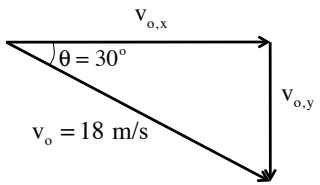


a.) What are the stone's initial coordinates?

(0, 50 m)

b.) What are the components of the initial velocity?

$$v_{o,x} = v_o \cos\theta$$
$$= (18\ \text{m/s})\cos 30^\circ$$
$$= 15.6\ \text{m/s}$$



$$v_{o,y} = -v_o \sin\theta$$
$$= -(18\ \text{m/s})\sin 30^\circ$$
$$= -9\ \text{m/s}$$

Note I had to manually put the negative sign in for the y-component of the velocity.

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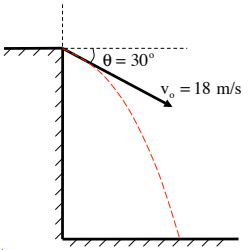

c.) Write out the velocity as a function of time for both the y and x direction.

$$v_{2,x} = v_{o,x} + \cancel{a_x}^{0}\Delta t = 15.6\ \text{m/s} \qquad v_{2,y} = v_{o,y} + a_y\Delta t = (-9\ \text{m/s}) + (-9.8\ \text{m/s}^2)\Delta t$$

d.) Write out the position as a function of time for both the y and x direction.

$$x_2 = x_o + v_{o,x}\Delta t + \cancel{\left(\frac{1}{2}\right)a_x}^{0}(\Delta t)^2 = (15.6\ \text{m/s})\Delta t$$

$$y_2 = y_o + v_{o,y}\Delta t + \left(\frac{1}{2}\right)a_y(\Delta t)^2 = (50\ \text{m}) + (-9\ \text{m/s})\Delta t + \left(\frac{1}{2}\right)(-9.8\ \text{m/s}^2)(\Delta t)^2$$

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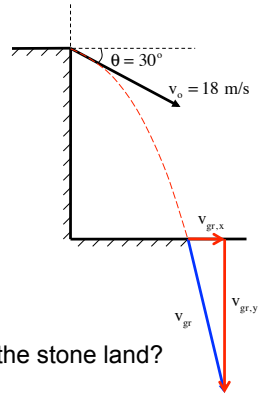

e.) How long does it take for the stone to reach the ground below?

f.) With what speed and angle of impact does the stone land?

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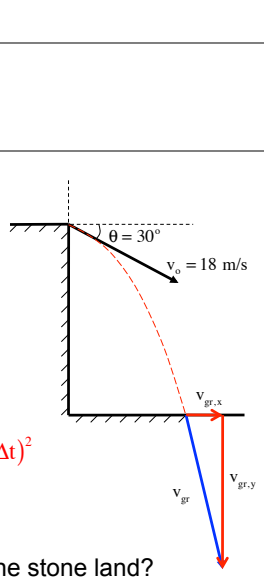

e.) How long does it take for the stone to reach the ground below?

$$y_2 = y_o + v_{o,y}\Delta t + \left(\frac{1}{2}\right)a_y(\Delta t)^2$$

$$\Rightarrow \quad 0 = (50\text{ m}) + (-9\text{ m/s})\Delta t + \left(\frac{1}{2}\right)(-9.8\text{ m/s}^2)(\Delta t)^2$$

$$\Rightarrow \quad t = 2.4\text{ seconds}$$

f.) With what speed and angle of impact does the stone land?

For the x-direction:

$$v_{2,x} = v_{o,x} + a_x\cancel{(\Delta t)}^{0}$$
$$= (15.6\text{ m/s})$$

This shouldn't be surprising as the velocity in the x-direction DOESN'T CHANGE unless there's a force in the x-direction, which there isn't in this case!

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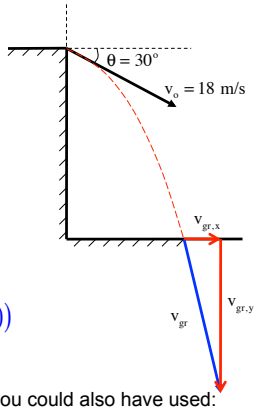


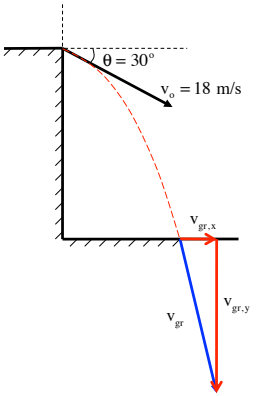
f. (con't.) With what speed and angle of impact does the stone land?

For the y-direction:

$$(v_{gr,y})^2 = (v_{o,y})^2 + 2a_y(y_{gr} - y_o)$$
$$= (-9\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(0 - (50\text{ m}))$$
$$\Rightarrow \quad v_{gr,y} = -32.6\text{ m/s}$$

Note that the negative sign was inserted manually. Also, you could also have used:

$$v_{gr,y} = v_{1,y} + a_y\Delta t$$
$$= (-9\text{ m/s}) + (-9.8\text{ m/s}^2)(2.4\text{ s})$$
$$\Rightarrow \quad v_{gr} = -32.6\text{ m/s}$$



f. (con't.) With what speed and angle of impact does the stone land?

To get the net velocity, the Pythagorean relationship holds:

$$v_{gr} = \left((v_{gr,x})^2 + (v_{gr,y})^2\right)^{1/2}$$
$$= \left((15.6\text{ m/s})^2 + (-32.6\text{ m/s})^2\right)^{1/2}$$
$$\Rightarrow \quad v_{gr} = 36.1\text{ m/s}$$

at an angle of:

$$\theta = \tan\left(\frac{v_{gr,y}}{v_{gr,x}}\right)$$
$$= \tan\left(\frac{-32.6\text{ m/s}}{15.6\text{ m/s}}\right)$$
$$\Rightarrow \quad \theta = -64.4^\circ$$

so:

$$\vec{v}_{gr} = (36.1\text{ m/s})\angle -64.4^\circ$$