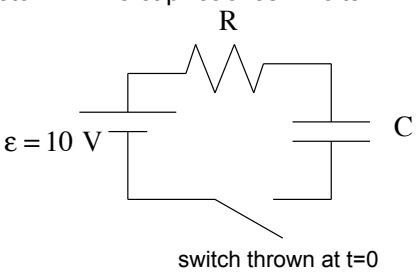
Problem 18.36

A 10 microfarad cap is charged by a 10 volts through a resistor R. The cap reaches 4 volts in 3 seconds after charging begins. What's R?



This isn't a great problem, but it won't hurt you to think a little about the time characteristics of charge build-up on a capacitor.

From its definition:

$$C = \frac{q(t)}{V_c}$$

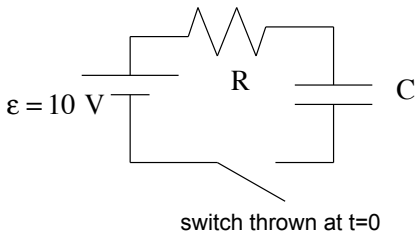
$$\Rightarrow \quad V_c = \frac{q(t)}{C}$$

$$= \frac{1}{C}\left(Q_{max}\left(1 - e^{-t/RC}\right)\right) = \frac{Q_{max}}{C}\left(1 - e^{-t/RC}\right)$$

$$= \varepsilon\left(1 - e^{-t/RC}\right)$$

where V_C is the voltage across the capacitor plates at time "t."

Solving this for R yields:



$$V_c = \varepsilon\left(1 - e^{-t/RC}\right)$$

$$\Rightarrow \quad \frac{V_c}{\varepsilon} - 1 = -e^{-t/RC}$$

$$\Rightarrow \quad \ln\left(1 - \frac{V_c}{\varepsilon}\right) = \ln\left(e^{-t/RC}\right)$$

$$\Rightarrow \quad \ln\left(1 - \frac{V_c}{\varepsilon}\right) = \frac{-t}{RC}$$

$$\Rightarrow \quad R = \frac{-t/C}{\ln\left(1 - \frac{V_c}{\varepsilon}\right)}$$

$$\Rightarrow \quad R = \frac{-(3\ \text{sec})/(10\text{x}10^{-6}\ \text{f})}{\ln\left(1 - \frac{(4\ \text{V})}{(10\ \text{V})}\right)}$$

$$\Rightarrow \quad R = 5.87\text{x}10^{5}\ \Omega$$