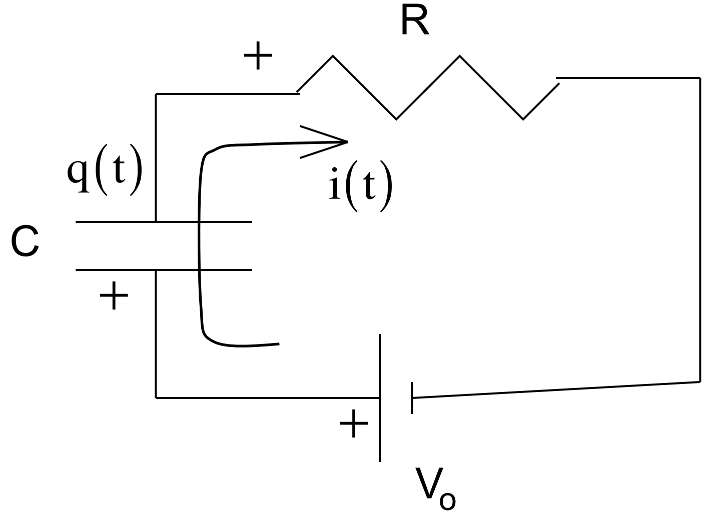
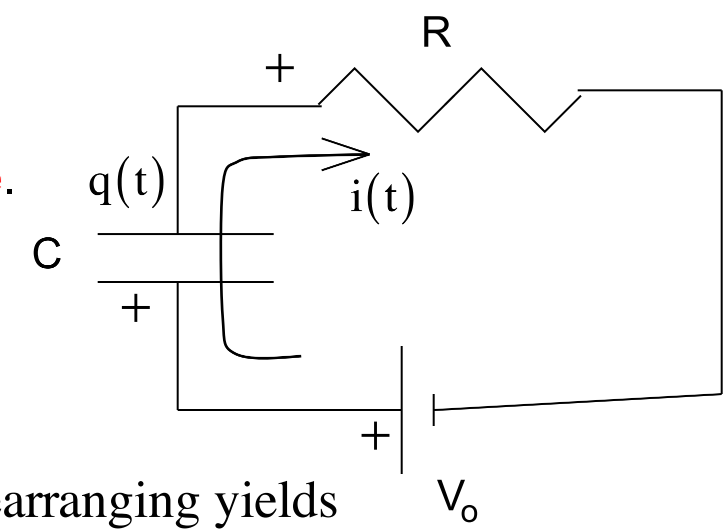
TIME CONSTANT FOR CAPACITORS

The question arises: If you have a capacitor either charging or discharging through a resistor, how fast will it act? Will the action happen fast or slow? The idea of the *RC time constant* gives us the information needed to answer that question.



TIME CONSTANT FOR CAPACITORS

The question arises: If you have a capacitor either charging or discharging through a resistor, how fast will it act? Will the action happen fast or slow? The idea of the *RC time constant* gives us the information needed to answer that question.



Let the switch be thrown at t = 0 across an initially uncharged capacitor and watch it charge. This process begins with the use of Kirchoff's Laws on an RC circuit. Summing the voltage changes around the circuit, we can write:

$$+V_o - \frac{q(t)}{C} - i(t)R = 0;$$ Dividing by R and rearranging yields

$$i(t) \quad + \frac{1}{RC}q(t) \quad = \frac{V_o}{R},$$ and with $i(t) = \frac{d(q(t))}{dt}$ we can write:

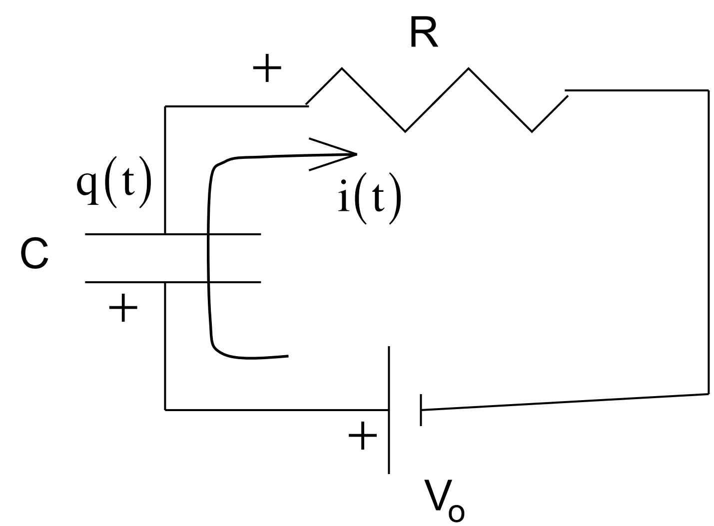
$$\frac{d(q(t))}{dt} + \left(\frac{1}{RC}\right)q(t) = \frac{V_o}{R}$$

The solution to this first-order differential equation is:

$$q(t) = q_{max}\left(1 - e^{-t/RC}\right)$$

and

$$
\begin{aligned}
i(t) = \frac{dq}{dt} &= \left(-\frac{1}{RC}\right) q_{max}\left(-e^{-t/RC}\right) \\
&= \left(\frac{1}{R}\right)\left(\frac{q_{max}}{C}\right)\left(e^{-t/RC}\right) \\
&= \frac{V_o}{R} e^{-t/RC} \\
&= i_o e^{-t/RC}
\end{aligned}
$$



Don't believe me? Check it at the extremes.

The charge on the cap at t=0 is zero, as calculated. The charge on the cap at time infinity is the maximum charge, as predicted. The current at t=0 is the maximum current (V/R). The current at infinity is zero. The functions all check out.

Let's look at the charging function.

$$q(t) = q_{max}\left(1 - e^{-t/RC}\right)$$

At some point, some bright soul asked the question, "How much charge has accumulated after an amount of time *RC*?" Doing the math yields:

$$\begin{aligned} q(t) &= q_{max}\left(1 - e^{-t/RC}\right) \\ &= q_{max}\left(1 - e^{-RC/RC}\right) \\ &= q_{max}\left(1 - e^{-1}\right) \\ &= q_{max}\left(1 - \frac{1}{e^{1}}\right) \\ &= q_{max}\left(1 - \frac{1}{2.7}\right) \\ &= q_{max}\left(1 - .37\right) \\ &= .63\, q_{max} \end{aligned}$$

Evidently, after a time equal to RC, the capacitor will be at 63% of its maximum charge.

The number "RC" is given a special name. It is called the RC circuit's *time constant*. The symbol used to denote the time constant is τ.

By the same reasoning, two time constants of time yields a charge:

$$
\begin{aligned}
q(t) &= q_{max}\left(1 - e^{-t/RC}\right) \\
&= q_{max}\left(1 - e^{-2RC/RC}\right) \\
&= .87\ q_{max}
\end{aligned}
$$

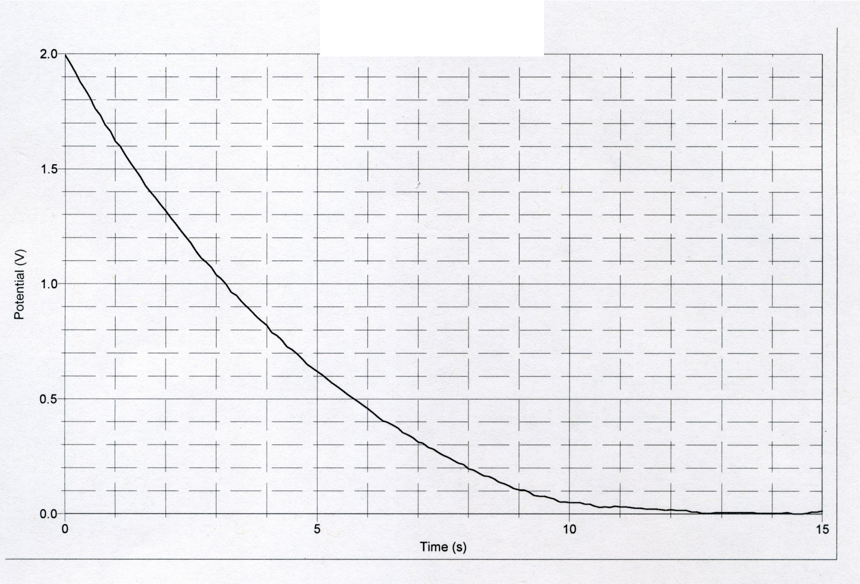
And there is symmetry to this. After time τ , a DISCHARGING capacitor will dump 63% of its charge. After time 2τ, the cap will dump 87%.

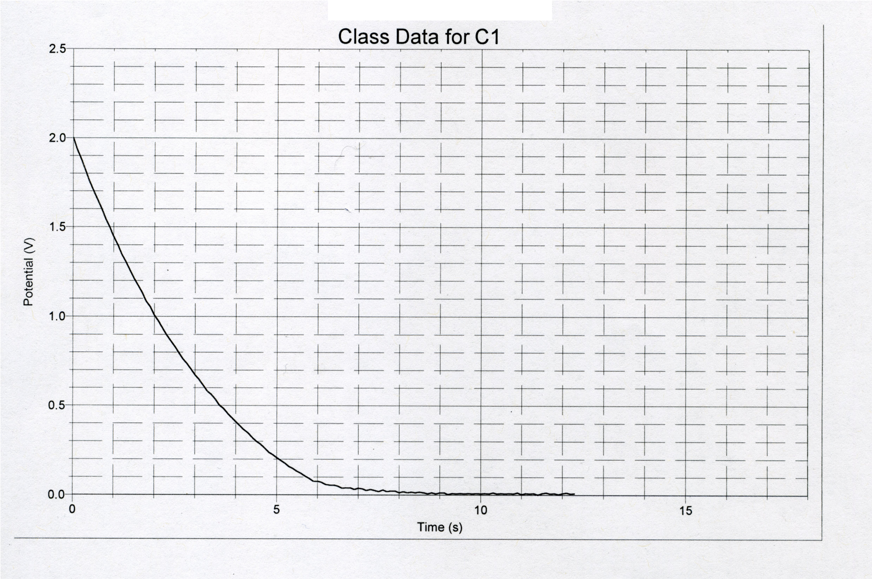
A similar calculation can be done for the amount of current in the circuit as time proceeds. The bottom line is that after time τ, there will be 37% of the maximum current still left in the circuit, and after time 2τ there will be approximately 13% left. (Remember, as time proceeds the current in a capacitor circuit will go to zero.)

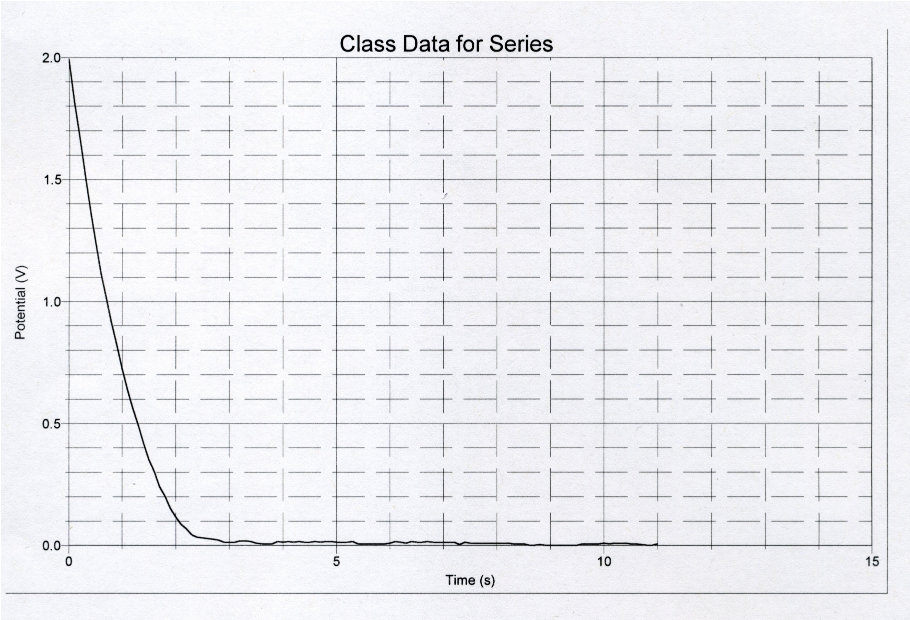
As for CURRENT, with a current function of $i(t) = i_o e^{-t/RC}$, after one time constant the current will be at .37 of the maximum current no matter whether the cap is charging or discharging, and after two time constant the number is .13 of the maximum current.

So check out the graph. What is the time constant for this RC circuit discharge?

These are discharge graphs for three different capacitances (same resistor in each case). Which cap must have been largest?



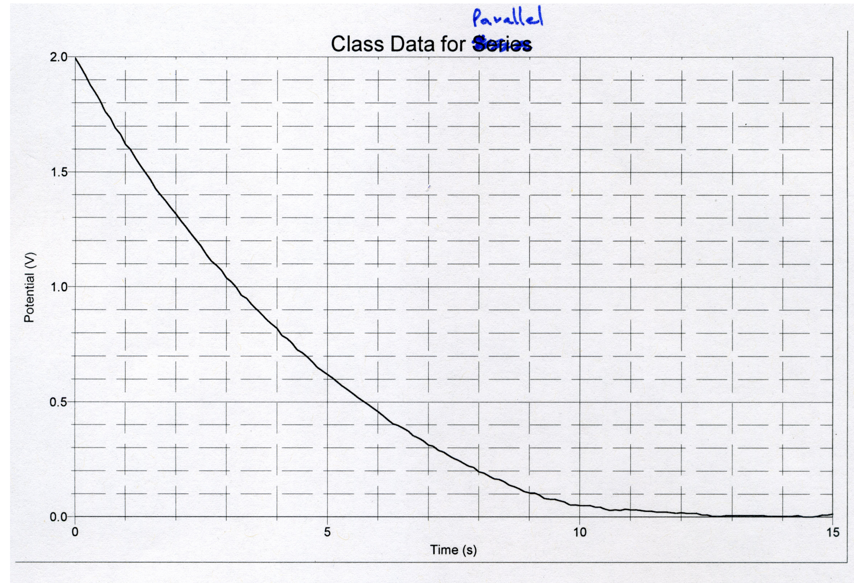


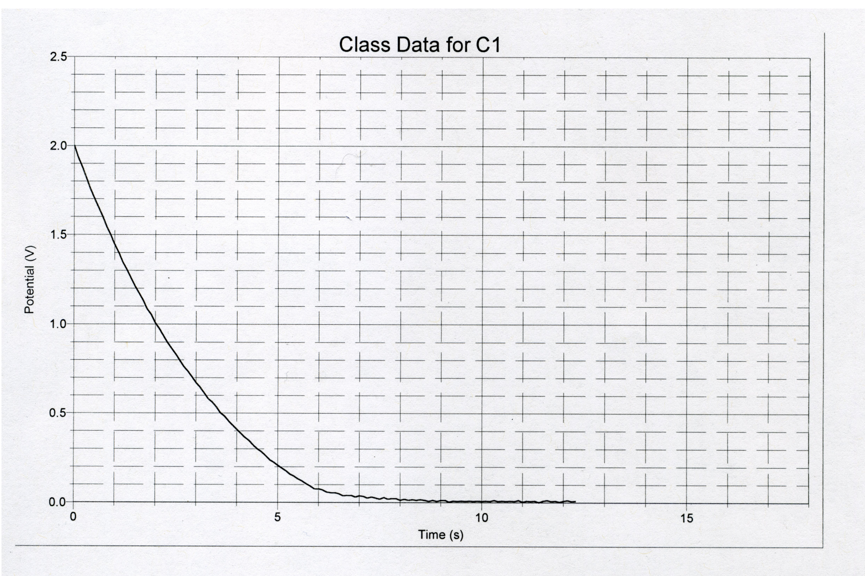


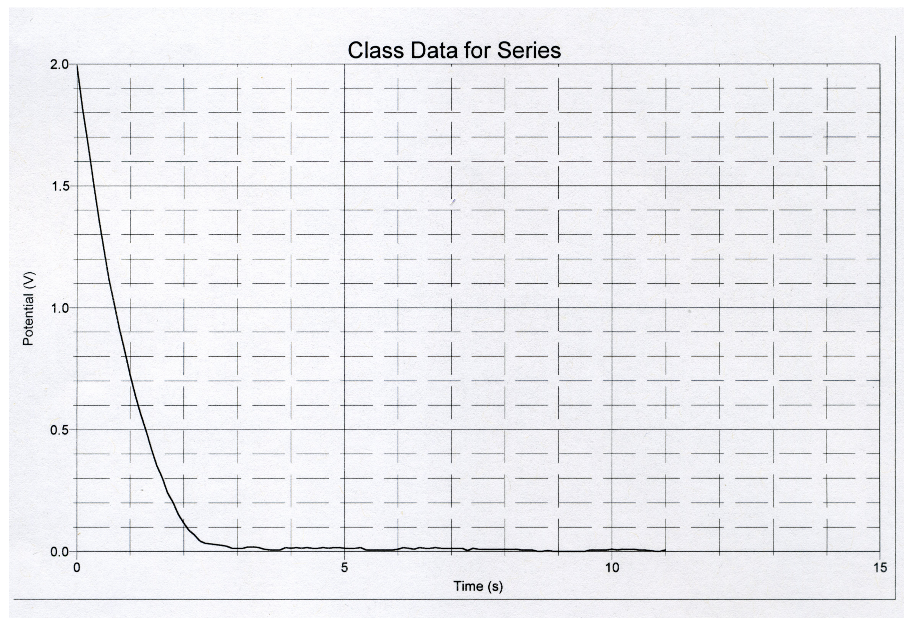
The resistor and capacitor values for one of the graphs is given. With which graph do they go?

$C_{series} = 29.2\ \mu F$

$R = 40.5\ k\Omega$







The resistor and cap values are shown. Determine the time constant for each situation.

$C_1 = 91.4\ \mu F$

$C_2 = 42.8\ \mu F$

$C_{series} = 29.2\ \mu F$

$C_{parallel} = 134.1\ \mu F$

$R = 40.5\ k\Omega$