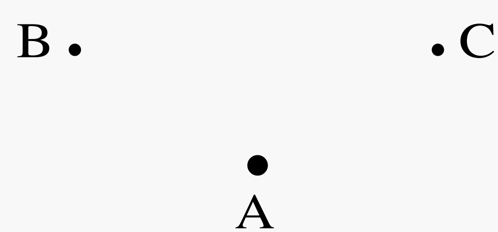
BELL'S SPACE-SHIP PARADOX

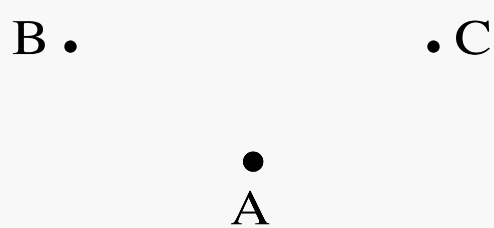
NOTE: The information in this PowerPoint was gleaned from a paper written by Francisco J Flores of the Department of Philosophy at Cal Poly State University, San Luis Obispo, in 2005. The paper was titled: "Bell's Spaceships: A Useful Relativistic Paradox." A copy of the article can be found on the Web at:

http://www.google.com/search?client=safari&rls=en&q=Francisco+J+Flores,+how+to+teach+special+relativity&ie=UTF-8&oe=UTF-8



The set-up: Three space ships in flat space and initially at rest with respect to each other are arranged as shown to the right. Specifically, ships B and C, which are assumed to be identical, are each the same distance from A. At some instant, both B and C receive a light signal whereupon they gently begin to accelerate. Their acceleration process is identical. A string just long enough to reach between ships B and C is tied to the ships. The paradox: as they begin to accelerate, will the string break?

From ship A's frame of reference, which we will refer to as the unprimed frame of reference, draw the world lines for ships B and C. Denote when the light leaves ship A as *event a*, and denote when the light arrives at ships B and C as *event b* and *event c*.



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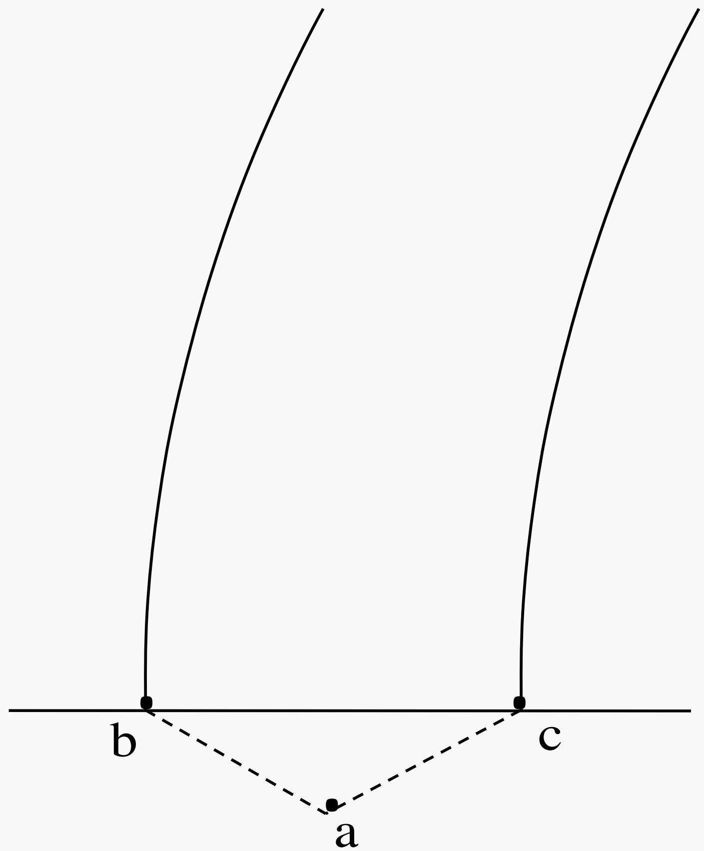
This problem is technically a General Relativity problem as it has acceleration in it, but it can be analyzed from a Special Relativity perspective because the acceleration is "gentle." The set-up is as follows:

Event a denotes the light signal being sent by ship A.

Event b and Event c denote the signal arrivals at ships B and C respectively.

As acceleration changes velocity, each ship's world line begins in the vertical along the time axis (zero velocity) and moves slowly toward the photon "light-line" at forty-five degrees.

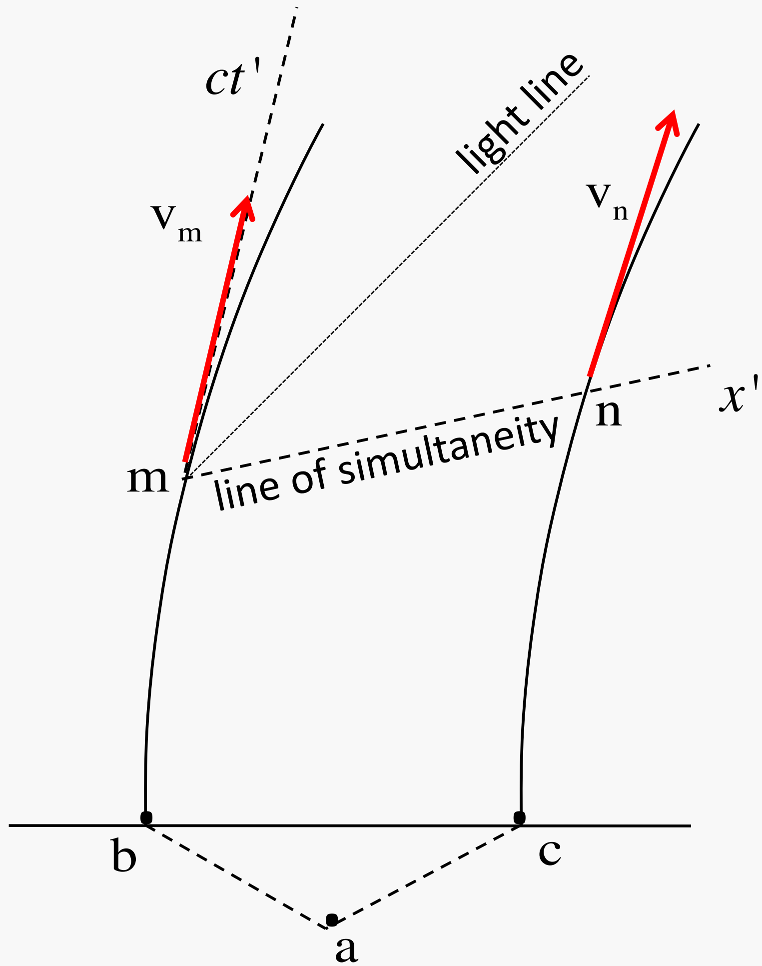


Ship B moves along its world line. At event m (see sketch) it's unprimed velocity vector is tangent to its world line and is denoted as v_m .

If we place a primed axis at event m, note that the vector v_m will be along the time (ct') axis at x'=0 (the ship never moves from it's initial coordinate in its own coordinate system).

Remembering that lines of simultaneity in the accelerated frame are parallel to the x' axis, we can follow the line of simultaneity from event m to C's world line whereupon we find event n (again, look at sketch).

Looking at a tangent to C's world line at event n (this is also the direction of v_n), we find that its slope is different from v_m. In other words, at that point in time, FROM SHIP B'S FRAME OF REFERENCE, the two ships are moving at different velocities . . . hence the string will break.

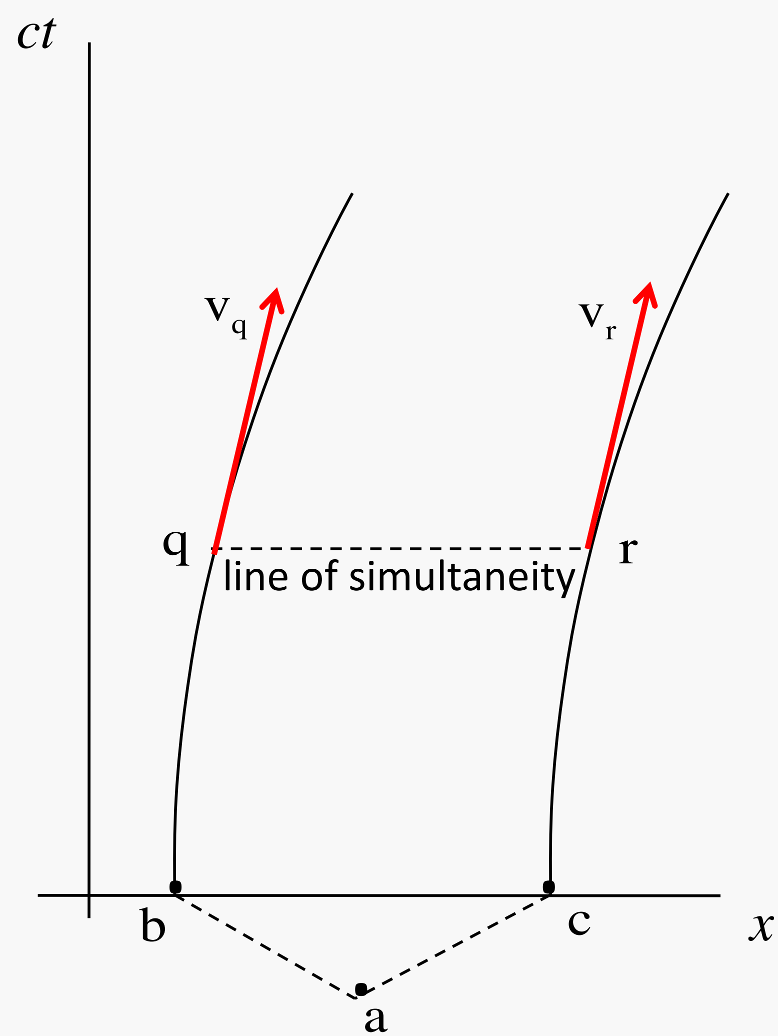


FROM SHIP A's FRAME OF REFERENCE?

An unprimed axis is added to our space-time diagram.

A line of simultaneity is drawn between event q and event r (see sketch).

The ship velocities are the same at that point in time, which suggests that the rope will not break, but that's not the case.



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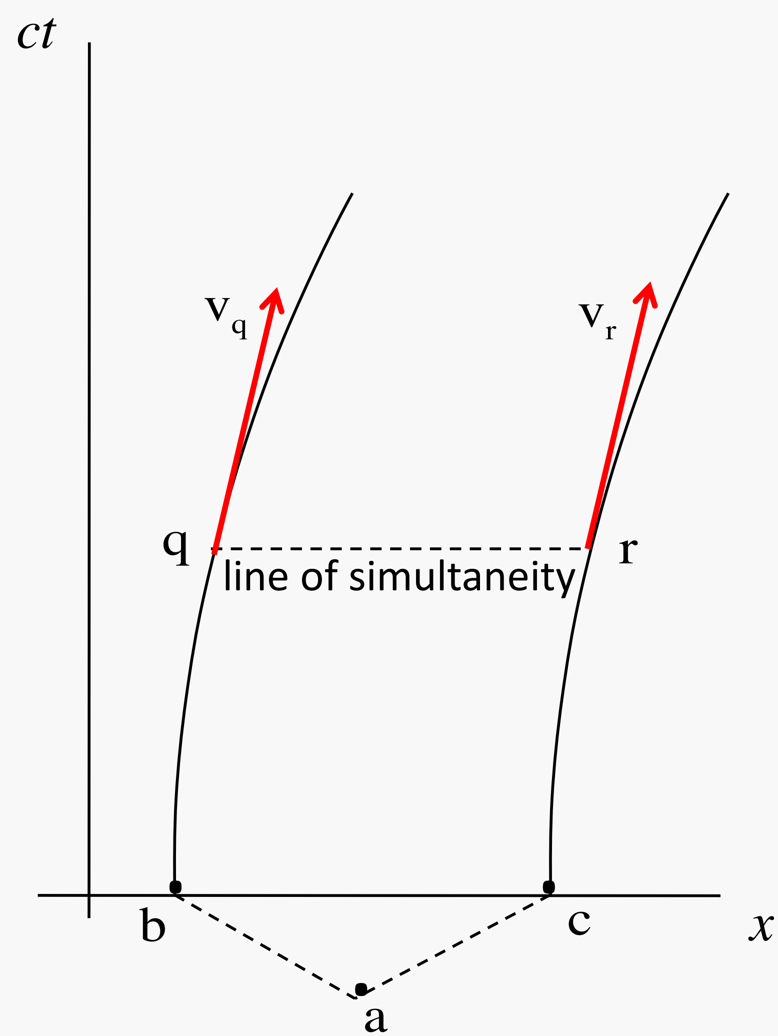


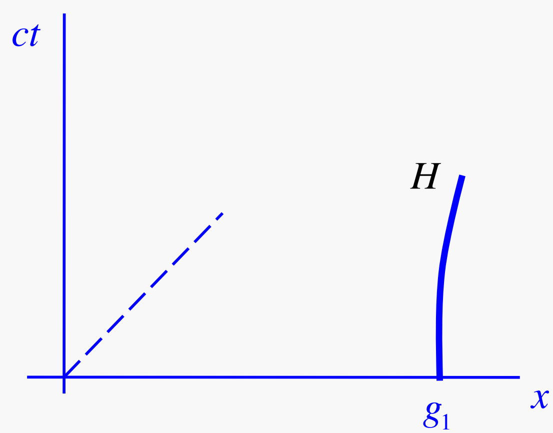
What needs to be noticed is that as the world line changes, the relativistic factor, which governs the length of the string, get bigger and bigger as the acceleration takes the ships to higher and higher velocities. That means, as bizarre as this is going to sound, that for the string to keep from breaking, the ships world lines would have to be getting closer and closer to one another. As that is not happening, the string will break.

That last argument was a bit of a hand-waving gem. A little more mathematically satisfying approach follows:

We know that relativity requires Minkowskian geometry, and we know that in that geometry the length *s* of the string must satisfy the *interval* expression:

$$\Delta x^2 - (\Delta ct)^2 = \Delta s^2$$

This curve is a hyperbola and graphs as *H* shown below (the graph is called a *calibration curve*).



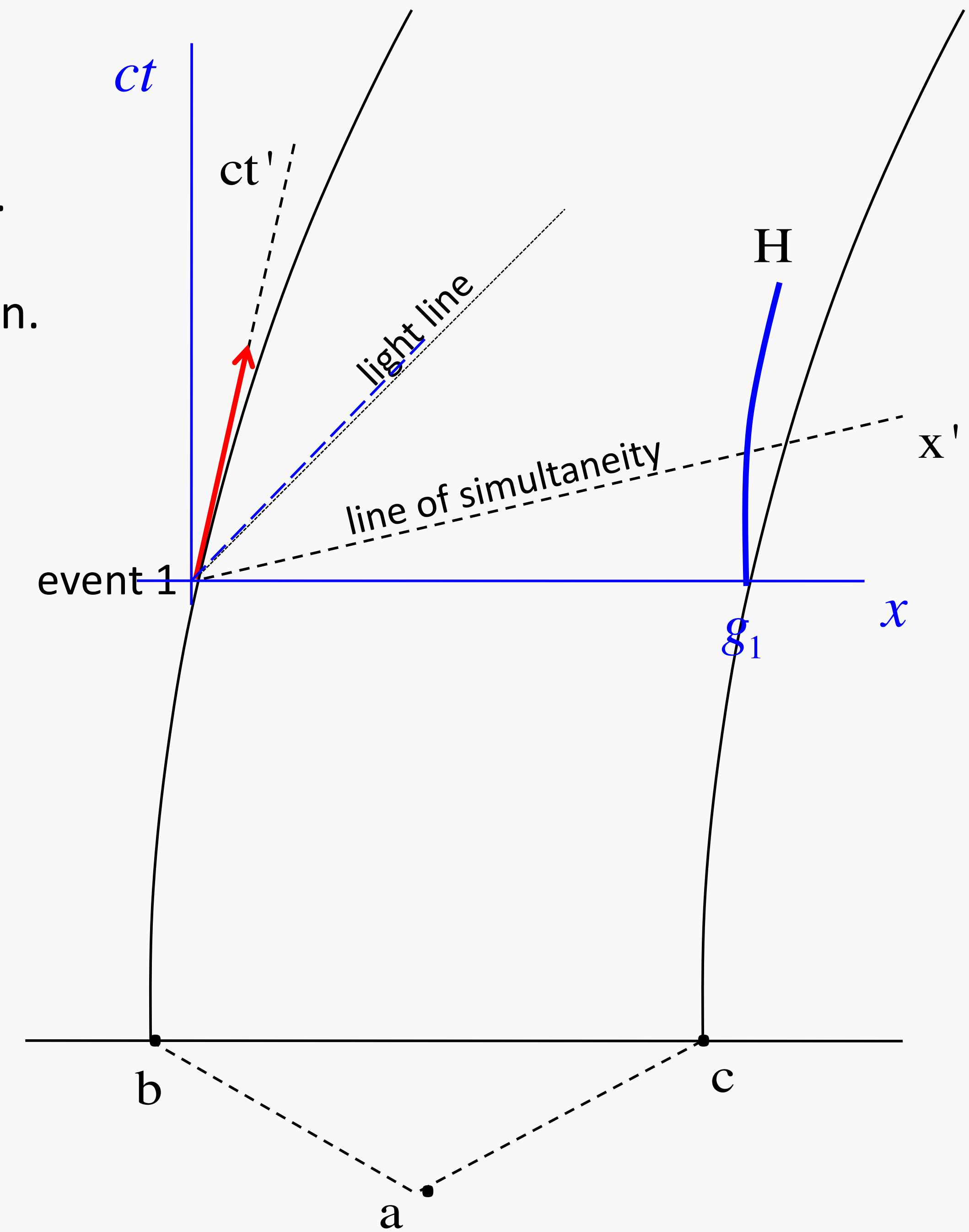
If we want the string to remain fixed in length, the interval along a line of simultaneity between *event 1* and ship C's position must conform to the constraints placed on it by our calibration curve H.

To see this, pick a point and call it event 1. Draw its velocity vector tangent to the world line at that event. This will define the ct’ axis.

Draw in the world line for a photon.

The ct’ axis will be symmetric with the x’ axis about that light-line, so once done, draw in the x’ axis. (Note that this defines a *line of simultaneity.*)

Once you have the primed axis, overlay the calibration curve so that its origin is at event 1 and the x = 0 coordinate of H is on ship C’s world line.



H intersects the line of simultaneity from event 1 where ship C would have to be if the interval *s* was to be constant.

Doing this process over and over again, we get the world-line (in red) that ship C would have to take for the string to keep from snapping. As that ship doesn't take that path, we have to deduce that the string will snap!