Preamble to the Constant Alpha

In the dual theory of light, light is sometimes viewed as a wave and sometimes viewed as a particle.

When light is viewed as a wave, the parameters that characterizes the action is the wave's wavelength or frequency.

The relationship between the wavelength λ of a wave and its frequency ν is:

$$c = \lambda \nu,$$

where "c" is the velocity of the wave ($c = 3 \text{x} 10^8 \text{ m/s}$).

When light is viewed as a particle, the parameters that characterizes the action is the particles energy E.

Note that a bundle of light energy--a "particle" of light--is called a *photon.*

The relationship between the energy wrapped up in a light's photon and the frequency of that same light when viewed as a wave is:

$$E = h\nu,$$

where h is Planck's constant ($h = 6.63x10^{-34}\ m^2 \bullet kg / s$).

Written in terms of wavelength (with $\nu=c/\lambda$), this becomes:

$$E = hc / \lambda$$

Let's look at a mildly relativistic electron (i.e., an electron that is moving close to the speed of light). If light--something that's normally associated with a wave--can act like a particle, it stands to reason that a particle like an electron can, under the right circumstances, act like a wave. And in fact, that is the case.

What we know about the electron as a particle is that its momentum p (mass times velocity) approximately equals:

$$p = m_{electron}c.$$

If the size of the electron-as-wave is the wavelength λ of its wave packet (called the de Broglie wavelength), then Heisenberg Uncertainty Principle $\Delta x \Delta p \geq \hbar$ suggests that:

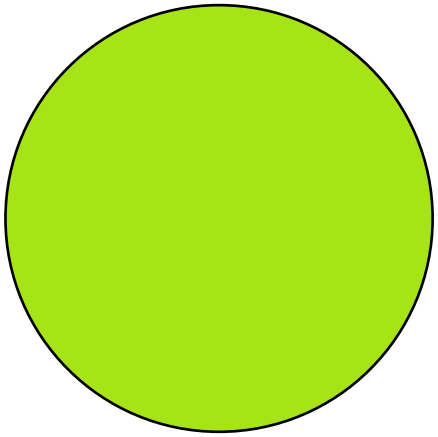
$$\lambda p = \hbar$$

where $\hbar = h / 2\pi$.

Combining the two relationships, we find that the "*de Broglie wavelength*" (the size of the wave packet) for an electron moving close to the speed of light is:

$$\lambda = \frac{\hbar}{m_{electron}c}$$

New topic: For the amusement of it, let's think of an electron as being a sphere of small radius that has its mass and charge shot uniformly throughout its volume.



If this was the case, the negative charge on the right side of the sphere would be repulsed by the negative charge on the left side of the sphere. Put a little differently, when the electron was created, it would take energy to force together the bits of charge that would ultimately make up the electron's structure.

The question is, how much energy is that?

To answer that, consider the following: The presence of a charge "q" in space produces what is called a voltage field (symbolized by a "V") in the region around the charge. That field, as evaluated at a point a distance "r" units from the charge," will numerically equal the size of the charge "q" divided by the distance "r" between the charge and the point. That is, it is equal to:

$$V = \frac{q}{r}.$$

Minor Note: This assumes we are in the CGS system of units where the proportionality constant between "V" and the right side of the equation is "1." For those of you who are in the Honors or AP Physics class, the proportionality constant in the MKS system of units would be $1/4\pi\varepsilon_o$.

The amount of energy U required to force a second, like charge "q" to a distance "r" units from the field producing charge turns out to be:

$$\begin{aligned} U &= qV \\ &= q\left(\frac{q}{r}\right) \\ &= \frac{q^2}{r}. \end{aligned}$$

If the two charges happen to be electron, the amount of energy required to bring two electrons of charge "e" in close enough to touch, called the "binding energy," is therefore:

$$U = \frac{e^2}{r_{electron}}.$$

Note that as energy would have to be put *into* the system to force the two electrons together, the binding energy is in this case is *positive*.

As you've learned, the rest mass energy is the amount of energy that must be provided before an object of mass "m" can come into existence. That value is, according to Einstein, equal to $E = mc^2$.

One question you might want to ask is, "How close would you have to get two bits of charge before the binding energy (the energy you'd have to put into the system for it to assemble itself) would equal the rest energy (the amount of energy required to create a single particle)?"

We can figure that out by simply equating the two energy equations we've derived. Specifically,

$$\text{binding energy} = \text{rest mass}$$

$$\frac{e^2}{r} = mc^2$$

Solving this for the “r” term yields:

$$r = \frac{e^2}{mc^2}$$

In fact, this yields the classical radius of the electron.

What’s interesting is that we now have two lengths associated with the electron. The first is the classically derived radius, or:

$$r = \frac{e^2}{m_{electron}c^2}$$

The second is the *de Broglie wavelength* of an electron when the electron is exhibiting wave-like characteristics, or:

$$\lambda = \frac{\hbar}{m_{electron}c}$$

Taking the ratio of the two distances, we can write:

$$\frac{r}{\lambda} = \frac{\dfrac{e^2}{m_{electron} c^2}}{\dfrac{\hbar}{m_{electron} c}}$$

$$\Rightarrow \quad \frac{r}{\lambda} = \frac{e^2}{\hbar c}$$

This ratio is given a special symbol. It is designated ALPHA so that:

$$\alpha = \frac{e^2}{\hbar c}$$

This value is approximately 1/137, or the ratio of the strong to electromagnetic force strength.

In other words, alpha is the *scale* factor (i.e., is the proportionality constant between) the wavelength $\lambda_{\mathrm{deBroglie}}$ of an electron's wave packet when the electron is acting like a wave and the radius of the electron r_{electron} when it is acting like a particle. Mathematically, this is written as:

$$r_{\mathrm{electron}} = \alpha\, \lambda_{\mathrm{deBroglie}}$$

A *coupling constant* determines the strength of an interaction. Alpha is a dimensionless coupling constant that:

a.) determines the strength of the electromagnetic force on an electron;
b.) determines the strength between the interaction of electrons and photons;

Alpha first popped up when A. Sommerfeld in 1916 incorporated elliptical orbits and relativistic mass into Bohr's atom model to explain the "fine structure" splitting of the energy levels of the hydrogen atom. In his analysis, the quantity $\alpha = v_{n=1\ \text{Bohr electron}}/c$, where "c" is the speed of light in a vacuum and $v_{n=1\ \text{Bohr electron}}$ is the speed of the electron in the first Bohr orbital, appeared naturally. As it's value determined the size of the splitting of the hydrogen spectral lines, it was defined as the "fine structure constant."