

MULTIPLE CHOICE SOLUTIONS--E&MTEST I

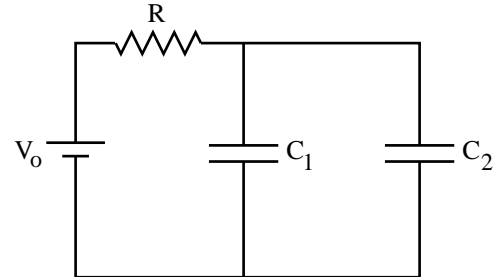
1.) The capacitors in the circuit are fully charged. At  $t = 0$ , the dielectric between the plates of  $C_1$  is quickly removed and that capacitance is halved. As a consequence:

a.) After a long period of time, the charge on  $C_2$  will increase. [After a long period of time, the voltage across  $C_2$  will again be  $V_0$ . As the value of  $C_2$  hasn't changed, its charge  $Q_2 = C_2V_0$  will not have changed. This response is false.]

b.) After a long period of time, the charge on  $C_2$  will decrease. [From above, this is false.]

c.) Just an instant after the dielectric is removed, the voltage across  $C_1$  will go to  $2V_0$ . [The initial charge on  $C_1$  is  $Q_1 = C_1V_C = C_1V_0$ . At  $t = 0$  when the dielectric is removed, there will not yet have been time for the free charge on  $C_1$ 's plates to move, but the reverse-oriented induced charge on the dielectric surface will be gone (reverse-oriented in the sense that positive charge is induced on the dielectric surface next to the capacitor's negative plate--the presence of this induced charge creates an electric field that fights the electric field generated by the free charge on the plates). That removal will effectively increase both the magnitude of the electric field and voltage difference between the plates. At just a hair past  $t = 0$ ,  $(C_1/2) = (Q_1)/(2V_0)$ , and the plate voltage will have temporarily doubled. This response is true.]

d.) None of the above. [Nope.]



2.) The charges shown in the configuration form an equilateral triangle. Where will a negative charge most likely feel a net force in the  $-j$  direction?

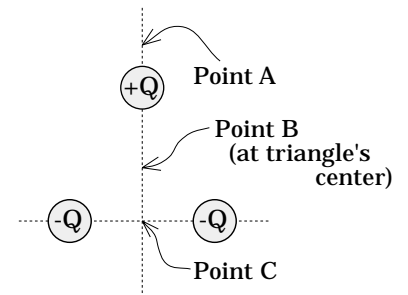
a.) Point A. [A negative charge at Point A will be attracted to the  $+Q$  charge and repulsed by the two  $-Q$  charges. Because the two  $-Q$  charges are farther away and at an angle, their net effect will be less. As Point A is relatively close to the  $+Q$  charge, it will hold sway and the net force at A will be in the  $-j$  direction. Other possibilities?]

b.) Point B. [At Point B, the electric field will be almost zero. Nevertheless, the  $+Q$  charge and the two  $-Q$  charges will apply force in the  $+j$  direction, so this option is definitely not a possibility.]

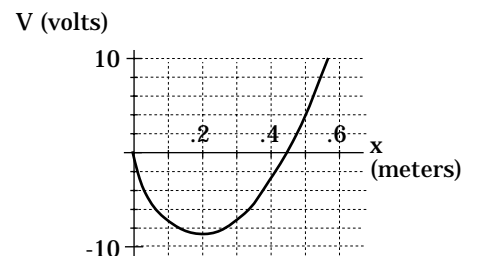
c.) Point C. [At Point C, the electrical effect due to the two  $-Q$  charges will cancel leaving a negative charge feeling attraction to the  $+Q$  charge. This force will be in the  $+j$  direction. This response is false.]

d.) Both Points A and C. [Nope.]

e.) None of the labeled points. [Nope.]



3.) An electrical potential field along the  $x$ -axis is defined by the graph shown. The associated electric field is:



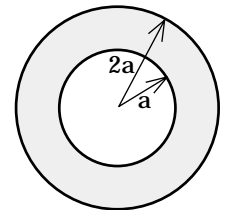
a.) Initially negative, then positive. Also, the field is zero at  $x = .45$  meters. [The relationship between an electric field  $E$  and its associate electrical potential field  $V$  is summarized by the relationship  $E = -\nabla V$ . In one dimension, the relationship states that minus the rate at which the electrical potential function changes (i.e.,  $V$ 's slope) is equal to the electric field function. For our electrical potential graph, the slope is first negative, then positive. That means that the electric field is first positive, then negative. The turnaround point where both the slope of  $V$  and the electric field  $E$  are zero is at  $x = .2$  meters. This response is false.]

b.) Initially positive, then negative. Also, the field is zero at  $x = .45$  meters. [Nope.]

c.) Initially positive, then negative. Also, the field is zero at  $x = .2$  meters. [This is the one.]

d.) None of the above. [Nope.]

4.) A hollow sphere of inside radius  $a$  and outside radius  $2a$  has a volume charge density shot through it of  $\frac{k_3 e^{-k_4 r}}{r^2}$ , where  $k_3$  and  $k_4$  are constants. The electric flux through a sphere whose radius is  $2a$  will be:



a.)  $\left(-\frac{4\pi k_3}{k_4 \epsilon_0}\right)(e^{-k_4 r})$ , and that function would have been different if the inside

radius of the hollow had been  $(1/2)a$ . [This is wrong for a number of reasons. First, the charge enclosed is positive. This means the field direction will produce a positive electric flux. Also, the charge enclosed calculation should be evaluated from  $a$  to  $2a$ . The function shown is an indefinite integral that hasn't been evaluated. If you didn't happen to notice either of those problems, you probably did the math, which follows:

$$\begin{aligned} \text{flux} = \phi_{\text{electric}} &= \int \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{enclosed}}}{\epsilon_0} \\ \Rightarrow \phi_{\text{electric}} &= \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{\int_{c=a}^{2a} \rho dV}{\epsilon_0} \\ &= \frac{\int_{c=a}^{2a} \left(\frac{k_3 e^{-k_4 c}}{c^2}\right) (4\pi c^2 dc)}{\epsilon_0} \\ &= \frac{4\pi k_3 \int_{c=a}^{2a} (e^{-k_4 c}) dc}{\epsilon_0} \\ &= \frac{4\pi k_3 \left(-\frac{1}{k_4}\right) \left[(e^{-k_4 c})\right]_{c=a}^{2a}}{\epsilon_0} \\ &= \frac{4\pi k_3 \left(-\frac{1}{k_4}\right) (e^{-k_4(2a)} - e^{-k_4 a})}{\epsilon_0} \\ &= \left(-\frac{4\pi k_3}{k_4 \epsilon_0}\right) (e^{-k_4(2a)} - e^{-k_4 a}). \end{aligned}$$

b.)  $\left(-\frac{4\pi k_3}{k_4 \epsilon_0}\right)(e^{-k_4(2a)} - e^{-k_4 a})$ , and that function would have been different if the inside

radius of the hollow had been  $(1/2)a$ . [From above, this is the correct expression. Additionally, halving the inside radius would have increased the charge enclosed and, as a consequence, the flux through a sphere of radius  $2a$ . This response is true.]

c.)  $\left(\frac{4\pi k_3}{k_4 \epsilon_0}\right)(e^{-k_4 r})$ , and that function would not have been different if the inside radius of

the hollow had been  $(1/2)a$ . [On the surface, at least the sign appears to be correct here (though appearances can be deceiving). This unevaluated integral will not do, though. ALWAYS evaluate integrals when you can. This response is false.]

d.)  $\left(-\frac{4\pi k_3}{k_4 \epsilon_0}\right)(e^{-k_4(2a)} - e^{-k_4 a})$ , and that function would not have been different if the

inside radius of the hollow had been  $(1/2)a$ . [Right expression, wrong analysis of what would happen if the inside radius had been smaller. This response is false.]

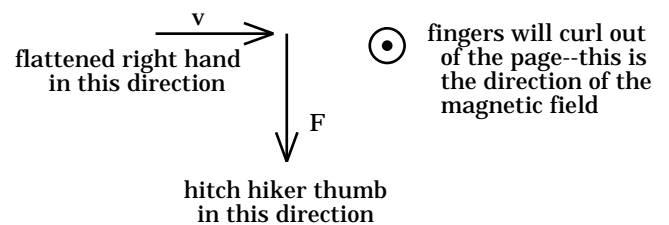
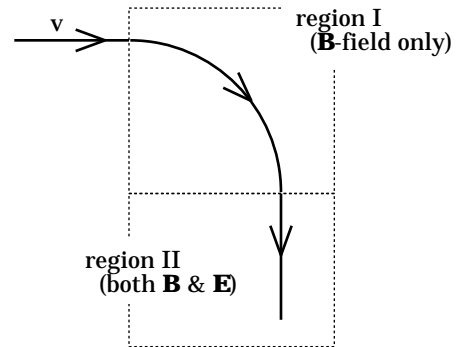
e.) The integral involved in this problem is not standard, hence the problem cannot be solved without access to a table of integrals. [This is a throw-away for those who looked at the problem and thought what are the odds? This response is false.]

5.) A positive charge moves with known velocity  $v$  into region I in which exists an unknown B-field. It accelerates as shown in the sketch, then enters region II in which there exists not only B but also an unknown electric field E.

a.) The direction of the B-field is toward the bottom of the page, and there is no need for the presence of an electric field to keep the charge moving in the direction shown in region II,  $E = 0$ . [There clearly is an initial downward force on the charge as it enters region I, but magnetic forces do not orient themselves in the direction of magnetic fields. The direction of a magnetic force will be PERPENDICULAR to both the direction of B and the direction of  $v$ . In short, if B had been directed toward the bottom of the page, the force  $qv \times B$  would have pushed the charge into the page. This response is false.]

b.) The direction of the B-field is into the page, and the direction of the E-field is to the left. [Instead of trying each of the possibilities listed in the various responses, the easiest way to do this part of the problem is to use the cross product associated with  $qv \times B$  backwards. That is, run your flattened right hand in the direction of the initial  $v$ , then orient your thumb hitch hiker style in the direction of the required force (in this case, downward). The direction in which your curled fingers end up is the direction of the magnetic field. The direction that satisfies this criterion for this problem is out of the page. That is the direction of the magnetic field. As such, this response is false.]

c.) The direction of the B-field is into the page, and the direction of the E-field is to the right. [At the very least, the direction for the B-field is wrong. This response is false.]



d.) The direction of the B-field is out of the page, and the direction of the E-field is to the left. [This has the correct direction for the magnetic field. What about the direction of the electric field? With a B-field directed out of the page, the charge moving in the vertical will feel a magnetic force oriented to the left ( $qv \times B$ ). Because the charge is positive, the direction of the electric force and the direction of the electric field will be the same. To counteract a magnetic force to the left, therefore, the electric field direction must be to the right. This response is false.]

e.) The direction of the B-field is out of the page, and the direction of the E-field is to the right. [This is the one.]

f.) None of the above. [Nope.]

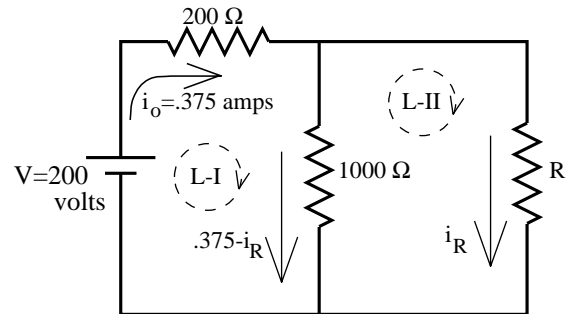
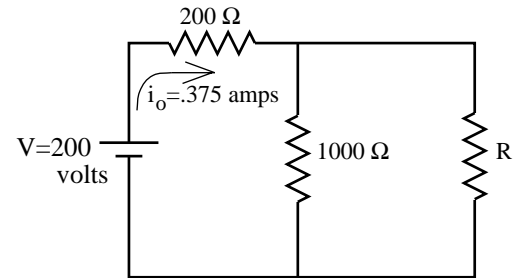
6.) Given the circuit information shown in the circuit, how large must the resistor R be?

a.)  $500 \Omega$ . [For practice, let's use Kirchoff's Laws on this one. The currents and loops are defined in the accompanying sketch (note that a node equation was used to define the current through the  $1000 \Omega$  resistor--this is nice because it means we have only one current unknown along with the unknown R). Summing the voltage differences around L-I yields  $(200 \text{ volts}) - (200 \Omega)(.375 \text{ amps}) - (1000 \Omega)(.375 - i_R) = 0$ . Solving yields  $i_R = .25$  amps. Summing the voltage differences around L-II yields  $(.375 - i_R)(1000 \Omega) - i_R R = 0$ . Substituting in for  $i_R$  yields a resistance  $R = 500 \Omega$ . This response is true.]

b.)  $1000 \Omega$ . [Nope.]

c.)  $2000 \Omega$ . [Nope.]

d.) None of the above. [Nope.]



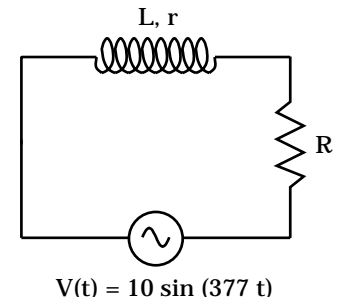
7.) The RMS voltage across the  $10 \text{ k}\Omega$  resistor in the RL circuit shown is 2 volts. The approximate RMS voltage across the inductor is:

a.) 3 volts. [The amplitude of the power supply is 10 volts, so the RMS value across the power supply is  $.707(10 \text{ volts}) = 7.07$  volts. If the RMS voltage across the resistor is 2 volts, the RMS voltage across the only other element in the circuit must be 5.07 volts. This response is false.]

b.) 5 volts. [This is the one.]

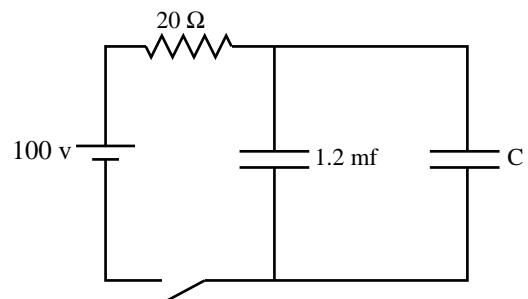
c.) 8 volts. [If you mistook the RMS voltage across the power supply for 10 volts, you got this incorrect response.]

d.) None of the above. [Nope.]



8.) The capacitors in the circuit shown are initially uncharged. At  $t = 0$ , the switch is closed. At  $t = .2$  seconds, it is observed that the current being drawn from the battery is approximately 2 amps. The capacitance of C is approximately:

a.) 4.4 mf. [The initial current in the circuit, given the fact that both capacitors will initially act like shorts, is  $i = V/R = (100 \text{ v})/(20 \Omega) = 5$  amps. The fact that the



current has dropped by 60% means that approximately one time constant has passed during the .2 second period. For this circuit, one time constant equals  $RC_{eq}$ . The equivalent capacitance is  $C + 1.2 \times 10^{-3}$  farads, so we can write  $(.2 \text{ seconds}) = (20 \ \Omega)(C + 1.2 \times 10^{-3})$ , or  $C = 8.8 \times 10^{-3}$  farads. This response is false.]

- b.) 8.8 mf. [This is the one.]
- c.) 13.3 mf. [Nope.]
- d.) None of the above. [Nope.]

9.) A coil of resistance R faces a uniform B-field coming out of the page that doubles at a constant rate every 10 seconds. At  $t = 0$ , the field strength is .5 teslas. As time progresses:

a.) The EMF generated in the coil will be constant, and the induced current in the coil will be clockwise. [Because the magnetic field increases

uniformly, the magnetic flux will increase uniformly,  $\frac{\Delta\phi_m}{\Delta t}$  will be

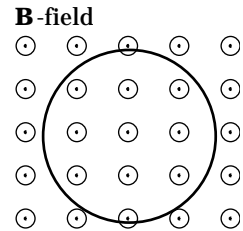
constant, and the induced EMF will be constant. The first part of the response is true. As for the direction of induced current flow, with an increasing magnetic flux, the current will flow in such a direction as to produce an induced magnetic field through the coil that is opposite the direction of the external field. The induced current that produces a B-field into the page will flow in a clockwise direction. This response is true.]

b.) The EMF generated in the coil will increase, and the induced current in the coil will be counterclockwise. [The EMF is constant. This response is false.]

c.) The EMF generated in the coil will decrease, and the induced current in the coil will be clockwise. [The EMF is constant. This response is false.]

d.) The EMF generated in the coil will increase, and the induced current in the coil will be clockwise. [The EMF is constant. This response is false.]

e.) There will be no EMF or induced current in the coil. [Nope.]



10.) A solid cylinder of radius a has a volume charge density shot through it of  $\frac{k_3 e^{-k_4 r}}{r}$ , where  $k_3$  and  $k_4$  are constants. The electric field function for  $r < a$  is:

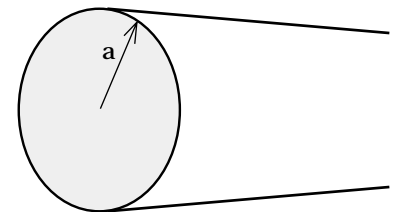
a.)  $\left(-\frac{k_3}{k_4 r \epsilon_0}\right) e^{-k_4 r}$ . [Defining a Gaussian cylinder of length L

and radius  $r > a$ , we need to determine the total charge inside the charged area. Using a cylindrical shell of radius c, thickness dc, circumference  $2\pi c$ , and differential volume  $dV = (2\pi c \ dc)L$ , Gauss's Law yields:

$$\int \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\Rightarrow E(2\pi r L) = \frac{\int \rho dV}{\epsilon_0}$$

$$\Rightarrow E = \frac{\int_{c=0}^r \left(\frac{k_3 e^{-k_4 c}}{c}\right) (2\pi c L dc)}{(2\pi r L) \epsilon_0}$$



$$\Rightarrow E = \frac{k_3 \int_{c=0}^r (e^{-k_4 c}) dc}{r \epsilon_0}$$

$$\Rightarrow E = \frac{k_3 \left( -\frac{1}{k_4} \right) \left[ (e^{-k_4 c}) \right]_{c=0}^r}{r \epsilon_0}$$

$$\Rightarrow E = \left( -\frac{k_3}{k_4 r \epsilon_0} \right) (e^{-k_4 r} - 1).$$

The response in this section is that of an indefinite integral. Because we know the limits of integration, we must do the evaluation. This response is false.]

- b.)  $\left( -\frac{k_3}{k_4 r \epsilon_0} \right) (e^{-k_4 r} - 1)$ . [This looks like the one.]
- c.)  $\left( \frac{-k_3}{k_4 \epsilon_0 r} \right) (e^{-k_4 r} - e^{-k_4 a})$ . [Nope. The limits are messed up.]
- d.) None of the above. [Nope.]

11.) A 1 amp fuse is placed in series with an RC circuit in which the AC voltage amplitude is 1500 volts. The net resistance is 140  $\Omega$ , and the capacitance is 20 mf. Approximately what is the largest frequency at which the power supply can operate without blowing the fuse?

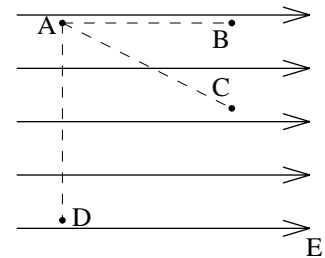
a.) 350 Hz. [Under normal circumstances, Ohm's Law should be used with RMS values. In this case, though, the .707 factor in the  $V_{\text{RMS}} = i_{\text{RMS}} R$  expression will simply cancel out. As such, we will use amplitude values instead (this is actually useful here as the current of interest is not 10 amps RMS but 10 amps maximum). Using the appropriate expression for impedance of an RC

circuit, we can write Ohm's Law as  $V_{\text{max}} = i_{\text{max}} Z = i_{\text{max}} \sqrt{R_{\text{net}}^2 + \left( \frac{1}{2\pi\nu C} \right)^2}$ . Putting in the appropriate values, we get [1500 volts] = [1 amp][[140  $\Omega$ ]<sup>2</sup> + 1/[2 $\pi$  $\nu$ (2x10<sup>-3</sup> farads)]<sup>2</sup>]<sup>1/2</sup>. Solving yields  $\nu = 3.52 \times 10^2$  hertz. This response is true.]

- b.) 500 Hz. [Nope.]
- c.) 5000 Hz. [Nope.]
- d.) None of the above. [Nope.]

12.) An electric field is set up as shown.

a.) The field will do more work when a positive charge goes from A to B than when the same charge goes from A to C. [As  $W = q\Delta V$ , the amount of work the field does is related solely to the size of the charge and the electrical potential difference between the two points. The potential difference between A and B will be the same as the potential difference between A and C as both B and C are on the same equipotential line (draw a line between B and C--that line will be perpendicular to the electric field lines, a characteristic of equipotential lines). In short, the field will do the same amount of work on a charge moving from A to B as it does on a charge moving from A to C. This response is false.]



b.) The field will do the same amount of work when a positive charge goes from A to B as when the same charge goes from A to C. [From above, this is true.]

c.) The field will do more work when a positive charge goes from A to B than when the same charge goes from A to D because the distance between A and D is greater. [The line between A and D is an equipotential line--the work done by the field as the body moves from A to D will be ZERO. This response is false.]

d.) None of the above. [Nope.]

13.) Approximately 1 amp flows into the box. The circuit that exists inside the box will be:

a.) Four  $2\ \Omega$  resistors in series. [From Ohm's Law,  $i = V/R$ . If the total current  $i$  drawn from the battery is to be approximately equal to 1 amp, and if  $V = 10.1$  volts,  $R_{eq}$  must be somewhere around  $10\ \Omega$ 's.

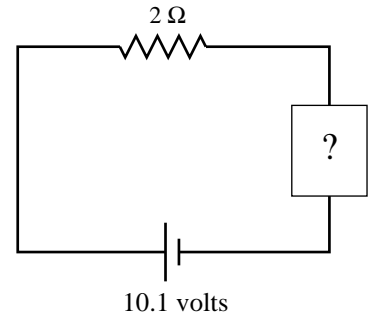
Knowing that the external resistor is  $2\ \Omega$ 's, the equivalent resistance from inside the box must be approximately  $8\ \Omega$ 's. Four  $2\ \Omega$  resistors in series will provide  $8\ \Omega$ 's to the system, and this response is true. Are there other true responses?]

b.) Four  $32\ \Omega$  resistors in parallel. [Four  $32\ \Omega$  resistors in parallel will have an equivalent resistance of  $(32\ \Omega)/4 = 8\ \Omega$ 's (note that I've used a shortcut to determine the equivalent resistance of the parallel combination--if all of the resistors in the combination are equal,  $R_{eq} = R/n$ , where  $R$  is the value of the common resistance and  $n$  is the number of resistors in the combination--if you don't believe that this will work, try it on a few examples). This will do the trick, also. Are there other true responses?]

c.) An  $8\ \Omega$  resistor in parallel with a  $10,000\ \Omega$  resistor. [For parallel circuits,  $R_{eq}$  will always be smaller than the smallest resistor in the combination. If there is an enormous difference in resistor size amongst the resistors in the combination, it is the small resistors that determine  $R_{eq}$ . This should be clear, but if it is not, see for yourself. For this situation,  $R_{eq} = [1/(8\ \Omega) + 1/(10000\ \Omega)]^{-1} = [.125 + .001]^{-1} = [.126]^{-1} = 7.995\ \Omega$ s. This is, to a very good approximation, equal to  $8\ \Omega$ 's. This response is true.]

d.) Both a and b. [Not quite.]

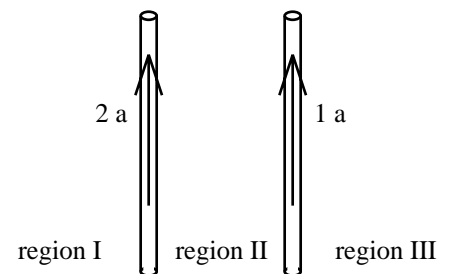
e.) Responses a, b, and c. [This is the one.]



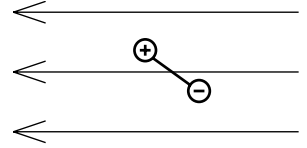
14.) In what region is the net magnetic field equal to 2 teslas directed out of the page?

a.) In region I. [As the direction of the magnetic field produced by the 2 amp current in region I is out of the page, and as the 1 amp current produces a field that is also out of the page in that region, region I will have a place where the net magnetic field is out of the page and equal to 2 teslas. This response is true. Are there others?]

b.) In region II. [The temptation is to assume that because the 2 amp wire produces a big magnetic field into the page in region II, there will be no point where our criterion is satisfied. That is not true. The 1 amp wire produces a magnetic field that is out of the page in region II. Very close to that wire, its B-field will predominate over that of the 2 amp wire. Furthermore, there will be a place where the net magnetic field will equal 2 teslas. This statement is true.]



- c.) In region III. [Both wires produce magnetic fields into the page in region III. This response is false.]  
 d.) Both in regions I and II. [This is the one.]  
 e.) All of the above. [Nope.]

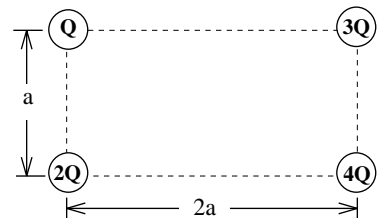


15.) A dipole is placed in an electric field as shown. Over time, the dipole will:

- a.) Experience a constant acceleration of its center of mass toward the right and will experience a constant torque that motivates it to angularly accelerate in a clockwise direction. [The electric field in this situation is constant. That means that the force on the positive charge directed to the left (i.e., in the direction of the electric field lines) will equal the force on the negative charge directed to the right. In other words, there will be no net force on the dipole and it will not translationally accelerate at all. That, in itself, makes this statement false.]  
 b.) Experience no acceleration of its center of mass but will experience a varying torque that motivates it to angularly accelerate in a clockwise direction. [The first part of this response is true. As for the second part: There will be a torque on the dipole that will vary in magnitude, depending upon the angular position of the dipole relative to the electric field (when the dipole is aligned with the electric field, the torque will be zero; when the dipole is perpendicular to the electric field, the torque will be a maximum). That torque will be in the counterclockwise direction, and this response is false.]  
 c.) Experience a varying acceleration of its center of mass toward the left and will experience a varying torque that motivates it to angularly accelerate in a clockwise direction. [Nope.]  
 d.) Experience no acceleration but will experience a varying torque that motivates it to angularly accelerate in a counterclockwise direction. [This is the one.]  
 e.) None of the above. [Nope.]

16.) Charges are placed as shown at the corners of a rectangle.

- a.) At the center of the rectangle, the x-component of the electric field will be positive (to the right) and the y-component will be negative (downward). [All of the charges are positive. That means that a positive test charge will be repulsed by each. As most of the charge is on the right side of the system, the x-component of the electric field at the center will be to the left in the negative direction. As such, this response is false.]



- b.) At the center of the rectangle, the x-component of the electric field will be negative (to the left) and the y-component will be negative (downward). [The first part is correct. In the y-direction, the electric field component due to 2Q will overpower the field component due to Q, and the field component due to 4Q will overpower the field component of 3Q. As 2Q and 4Q are on the bottom, the net y-component should be upward in the positive direction, and this response is false.]  
 c.) At the center of the rectangle, the x-component of the electric field will be positive (to the right) and the y-component will be positive (upward). [Nope.]  
 d.) At the center of the rectangle, the x-component of the electric field will be negative (to the left) and the y-component will be positive (upward). [This is the one.]

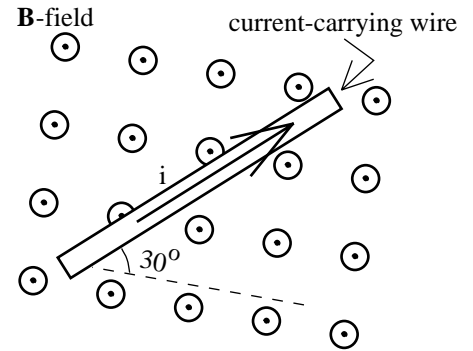
17.) A 2 meter long wire carries a .5 amp current as shown. If the wire is bathed in a  $10^{-2}$  tesla magnetic field, the magnitude of the force on the wire is:

a.) Zero newtons. [Using  $F = iL \times B$ , the magnitude of the force will be  $F = (.5 \text{ a})(2 \text{ m})(10^{-2} \text{ T})(\sin 90^\circ) = 10^{-2}$  newtons. This response is false.]

b.)  $.5 \times 10^{-2}$  newtons. [If you mistakenly took the angle between  $L$  and  $B$  to be  $30^\circ$ , you got this incorrect answer. NOTE: When either  $L$  or  $B$  is in the plane of the paper and the other is perpendicular to that plane, the angle between the two vectors will ALWAYS be  $90^\circ$ .]

c.)  $10^{-2}$  newtons. [This is the one.]

d.) None of the above. [Nope.]



18.) In the system shown, the switch has been set on contact A for a long time. At  $t = 0$ , the switch flips from contact A to contact B. The power dissipated by the resistor during the discharge will be:

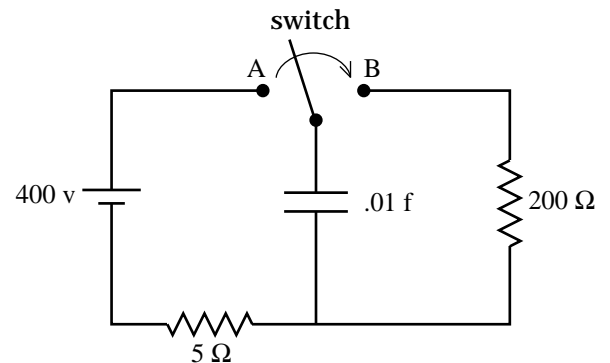
a.) 200 watts. [This is a tricky question.]

When a problem refers to power dissipated, it is referring to the amount of work per unit charge being done at a particular point in time (in a resistor, for instance, that quantity is equal to  $i^2 R$ ). If the question had asked for the average power dissipated during the discharge, we might have had a shot, but as worded, this question makes no sense. Note that if the problem had asked for the amount of energy dissipated by the resistor during discharge, that value would have equaled the total energy stored in the capacitor. That quantity is  $.5CV^2 = .5(.01)(400 \text{ v})^2 = 800$  joules. In any case, this response is false.]

b.) 400 watts. [Nope.]

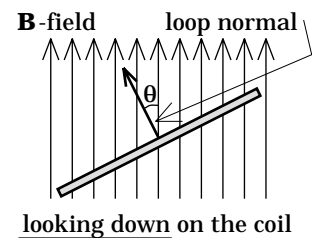
c.) 800 watts. [As this is the amount of energy stored in the capacitor, it is also the amount of energy dissipated by the resistor during the discharge. Unfortunately, that wasn't the question and this response is false.]

d.) None of the above. [This is the one.]



19.) The magnetic flux through a square loop is measured and found to be  $10^{-4}$  webers when the loop's normal is oriented at an unknown angle  $\theta$  relative to the direction of the B-field (see sketch). When the normal is parallel to the direction of the B-field, the magnetic flux through the loop is  $10^{-3}$  webers. The angle  $\theta$  is:

a.)  $22^\circ$ . [The magnetic flux in the first orientation is equal to  $BA \cos \theta = 10^{-4}$  webers. When the coil is facing the magnetic field, we have  $BA \cos 0^\circ = 10^{-3}$  webers. As the magnetic field  $B$  is constant and the area  $A$  is



constant, the product of the two will always be equal to  $10^{-3}$  webers. Using that information in the first equation yields  $BA \cos \theta = (10^{-3} \text{ teslas}) \cos \theta = 10^{-4} \text{ webers}$ , or  $\theta = \cos^{-1}(.1)$ . According to my calculator, that equals  $84.3^\circ$ . This response is false.]

- b.)  $47^\circ$ . [Nope.]
- c.)  $78^\circ$ . [Nope.]
- d.) None of the above. [This is the one.]

20.) An electric field is defined by  $E = kr^2\mathbf{r}$ , where  $\mathbf{r}$  is a unit vector in the radial direction.

a.) The units of  $k$  must be kilograms/(meter $\cdot$ second $^2$  $\cdot$ coulomb). [k's units must be such that when they are multiplied by meters $^2$  (i.e.,  $\mathbf{r}^2$ 's units), we get the electric field units (newtons/coulomb). That means that k's units are newtons/coulomb $\cdot$ meter $^2$ . As the units for newtons are kilogram $\cdot$ meters/second $^2$ , the units for  $k$  must be kilograms/(meter $\cdot$ second $^2$  $\cdot$ coulomb). This response is true. Are there others?]

b.) The units of  $k$  must be volt/meter $^3$ . [Electric fields are related to electrical potential fields by the expression  $E\cdot d = -\Delta V$ . From this, the units for  $E$  can be written as volts/meter. k's units must be such that when they are multiplied by meters $^2$ , we get the electric field units (volts/meter). Those units will be volts/meter $^3$ , and this response is true. Note that the overall correct response for this problem will probably be all of the above, but to be sure, we ought to examine Response c to be sure.]

c.) The units of  $k$  must be newtons/(meter $^2$  $\cdot$ coulomb). [Reiterating what has been said above, k's units must be such that when they are multiplied by meters $^2$ , we get the electric field units (newtons/coulomb). Combining, we get k's units as newtons/coulomb $\cdot$ meter $^2$ , and this response is true.]

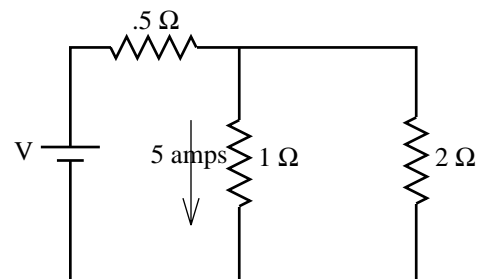
- d.) All of the above. [Looks like this is the one.]
- e.) None of the above. [Nope.]

21.) The voltage across the  $.5 \Omega$  resistor is:

a.) 12.5 volts. [If we knew the voltage  $V_0$  across the battery, we could subtract from  $V_0$  the voltage across the  $1 \Omega$  resistor ( $V = iR$ ) to get the voltage across the  $.5 \Omega$  resistor. Unfortunately, we don't have that information. What we do know is that the current through the  $.5 \Omega$  resistor is equal to the current into the parallel combination, and the voltage across the parallel resistors is the same. That parallel voltage equals (5 amps)( $1 \Omega$ ) = 5 volts. The current through the  $2 \Omega$  resistor is that voltage divided by  $2 \Omega$ 's, or  $i = (5 \text{ volts})/(2 \Omega) = 2.5$  amps, and the total current into the parallel combination (i.e., the current through the  $.5 \Omega$  resistor) is  $2.5 \text{ amps} + 5 \text{ amps} = 7.5$  amps. With that, we can write:  $V_{.5\Omega \text{ res}} = (7.5 \text{ amps})(.5 \Omega) = 3.75$  volts. This response is false.]

b.) 7.5 volts. [This is the numerical value for the current through the  $.5 \Omega$  resistor, not the voltage across the  $.5 \Omega$  resistor. This response is false.]

- c.) 5 volts. [Nope.]
- d.) None of the above. [This is the one.]



22.) A .5 kg mass has a 10 coulomb charge on it. It is placed in an electrical potential field at  $x = 2$  meters where the voltage is 2 volts. Released from rest, the mass is allowed to accelerate freely. At  $x = 3$  meters, its velocity is 12 m/s.

a.) The voltage at  $x = 3$  meters is 1.6 volts. [An easy way to look at this problem is through the conservation of energy. That theorem states that  $KE_1 + U_1 + W_{\text{ext}} = KE_2 + U_2$ . Remembering that a charge  $q$  at a point whose electrical potential is  $V$  has potential energy  $qV$ , we can write:  $qV_{x=2} = .5mv^2 + qV_{x=3}$ , or  $(10 \text{ coul})(2 \text{ volts}) = .5(.5 \text{ kg})(12 \text{ m/s})^2 + (10 \text{ coul})V_{x=3}$ . Solving, we get  $V_{x=3} = -1.6$  volts. As such, this response is false, though one might be tempted to ignore the negative sign and assume it is correct. In fact, the negative sign isn't a problem. It is changes of electrical potential that are important (just as it is changes in potential energy that are important). The electrical potential at a specific point has significance only in its relationship to the electrical potential of other points.]

b.) The voltage at  $x = 3$  meters is -1.6 volts. [As nonsensical as a negative electrical potential may seem, this is the answer. If you think about it, it isn't that outrageous. The electric field is fairly big (the body accelerated from rest to 12 m/s over a mere one meter), which suggests that the electrical potential difference between  $x = 2$  and  $x = 3$  must be fairly big. Positive charges accelerate from higher to lower electrical potential which means that the electrical potential at  $x = 3$  must be smaller than the electrical potential at  $x = 2$ . It isn't outside the realm of possibility that that smaller electrical potential could range into the negative numbers . . . which it did. In any case, this response is true.]

c.) The voltage at  $x = 3$  meters is -1.7 volts. [If you forgot to square the velocity term, you came out with this. It is incorrect.]

d.) None of the above. [Nope.]

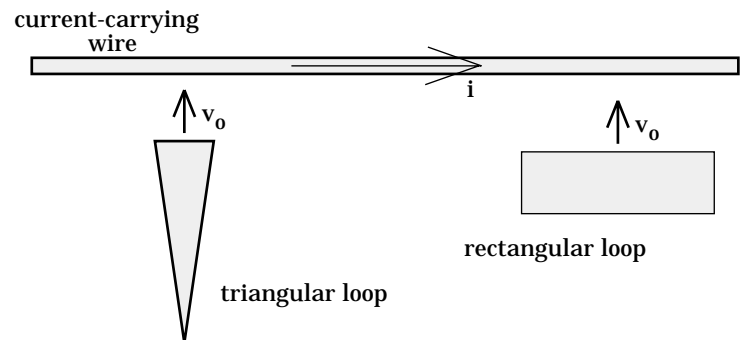
23.) A triangular loop and a rectangular loop each have the same area. Each is forced to approach a current-carrying wire with the same constant velocity (see sketch and ignore gravity).

a.) The direction of the induced current in both is clockwise, and the induced current in the triangle is greater than the induced current in the rectangle.

[The long wire is producing a magnetic field that is into the page in the region of the loops (use the right-thumb rule to determine this). The coils are approaching the long wire, so the magnetic flux through each is increasing. That means the induced current will set up so as to create an induced magnetic field through the coils that is out of the page. To do that, the current must be in the counterclockwise direction. From that alone, this response is false.]

b.) The direction of the induced current in both is counterclockwise, and the induced current in the triangle is greater than the induced current in the rectangle. [The induced current direction is correct here. What about the second part? Due to its shape, the magnetic flux change will be greater in the rectangle than it is in the triangle. As such, this response is false.]

c.) The direction of the induced current in both is clockwise, and the induced current in the rectangle is greater than the induced current in the triangle. [Nope.]



- d.) The direction of the induced current in both is counterclockwise, and the induced current in the rectangle is greater than the induced current in the triangle. [This is the one.]
- e.) The induced currents are in opposite directions. [Nope.]

24.) For the circuit shown:

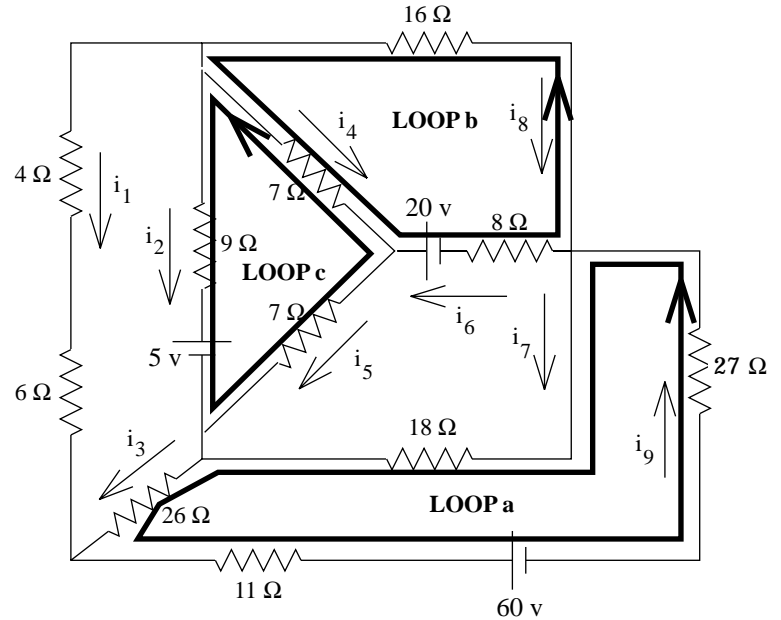
a.)  $-38i_9 - 18i_7 - 26i_3 = 60$ . [The loops used for all of the equations written for this question are highlighted in the sketch. Summing the voltage changes around LOOP a yields  $-27i_9 - 18i_7 - 26i_3 - 11i_9 - 60 = 0$ . Combining like terms and placing the 60 volt quantity on the right side of the equal sign, we get  $-38i_9 - 18i_7 - 26i_3 = 60$ . This response is true. Are there others?]

b.)  $16i_8 - 7i_4 + 8i_6 = -20$ . [LOOP b yields  $16i_8 - 7i_4 - 20 + 8i_6 = 0$ . This response would work if the sign of the 20 volt term was correct. This response is false.]

c.)  $7i_4 - 9i_2 + 7i_5 = 5$ . [LOOP c yields  $7i_4 - 9i_2 - 5 + 7i_5 = 0$ . This matches and this response is true. Are there other possibilities?]

d.) There are at least two correct loop equations above. [A nasty thing to do, making you check each loop instead of finding the first correct one and stopping there. In any case, this response is true.]

e.) None of the above. [Nope.]



25.) The capacitance of the capacitor in the circuit is 10 nf and the resistance is  $R = 1000 \Omega$ . At 200 cycles per second, the approximate capacitive reactance of the circuit is:

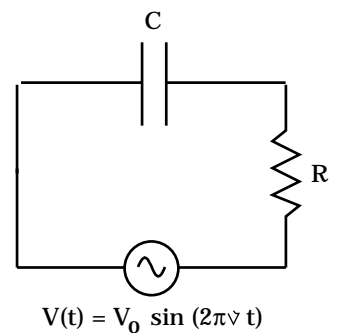
a.)  $1.25 \times 10^{-4} \Omega$ . [The circuit's capacitive reactance is  $X_C = \frac{1}{2\pi\nu C} =$

$\frac{1}{[(2\pi)(200 \text{ Hz})(10 \times 10^{-9} \text{ f})]} = 7.96 \times 10^4 \Omega$ . This response is false.]

b.)  $2000 \Omega$ . [Nope.]

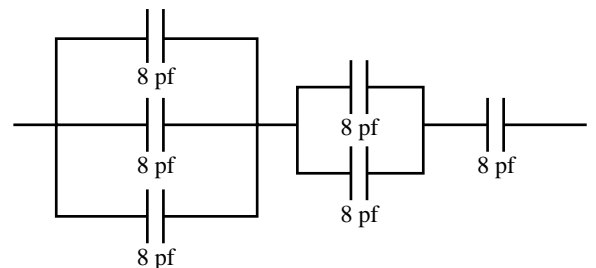
c.)  $80,000 \Omega$ . [This is the one.]

d.) None of the above. [Nope.]



26.) The equivalent capacitance for the capacitor combination shown is:

a.)  $11/48$  picofarads. [The two parallel combinations have equivalent capacitances of 24 pf and 16 pf respectively. Adding the inverse of 24 pf, 16 pf, and

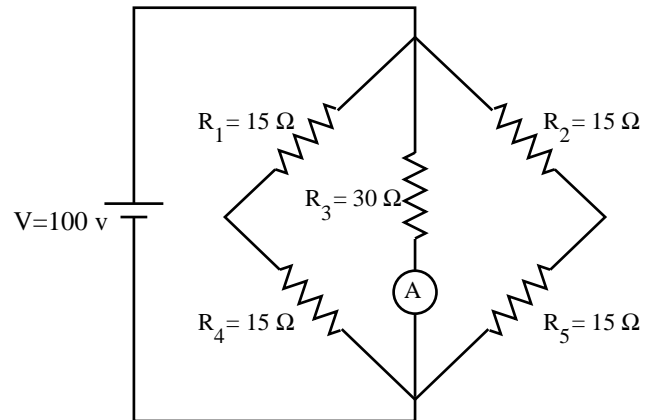


8 pf , then inverting that result yields an equivalent capacitance of  $(48/11)$  pf . This response is false.]



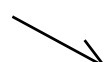

- b.)  $48/11$  picofarads. [This is the one.]  
 c.) 14.67 picofarads. [If you reversed the equivalent capacitance relationships treating the parallel combinations like series combinations etc., you got this incorrect answer.]  
 d.) None of the above. [Nope.]

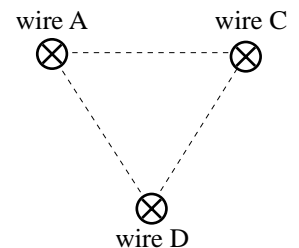
27.) In the circuit shown, what will the ammeter read?

- a.) Zero amps. [There is obviously a voltage difference across the  $30\ \Omega$  resistor, so the current through that resistor and that branch will not be zero and this response is false.]  
 b.) 3.3 amps. [In a way, this is so easy it's tricky. Although the circuit looks awful, the only thing that is required to determine the reading of the ammeter is knowing the current through  $R_3$ . That feat can be easily done if you know the voltage across  $R_3$  and the magnitude of  $R_3$ . The resistor's magnitude was given. The voltage across  $R_3$  is simply the voltage across the battery (note that there is nothing between  $R_3$  and either the high voltage or low voltage terminal of the battery). As such,  $i_3 = (100\ \text{volts})/(30\ \Omega) = 3.3$  amps. This response is true.]  
 c.) 10 amps. [Nope.]  
 d.) None of the above. [Nope.]



28.) Three current-carrying wires oriented perpendicular to the page are positioned at the corners of a triangle as shown. Assume the current magnitudes are the same for all of the wires. Wires C and D will produce a magnetic force on wire A. In what direction will that force be?

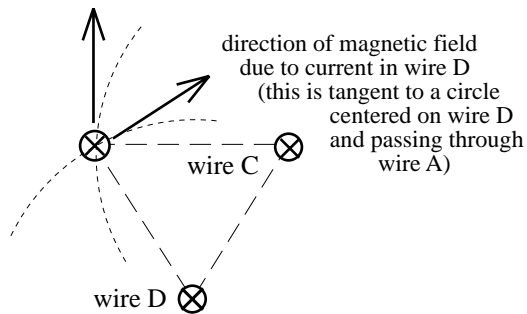
- a.)  b.)  c.)  d.)  e.) None of these.



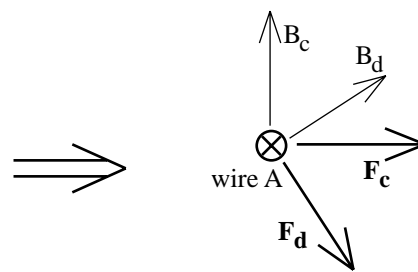
[Commentary: The net force on A will be the sum of the magnetic forces produced by the interaction between A's current and the magnetic fields generated by C and D. Working with C first: The direction of C's magnetic field at A is determined using the right-thumb rule. That is, orient your right thumb in the direction of current in C (i.e., into the page). The curl of your fingers will give you the sense of the circulation of C's B-field around C's wire. Evaluated at wire A, C's magnetic field direction is straight up. The direction of the force on A due to that field is determined using  $F = iL \times B$ . The right-hand rule used for cross products (with your flattened right hand pointing in the direction of the first vector ( $v$ ), rotate your flattened hand so that you can curl your fingers in the direction of the second vector ( $B$ )--your thumb extended hitch hiker style will point in the direction of the cross product ( $F$ )) suggests that the direction of this force will be directly horizontal and to the right. A similar analysis yields a magnetic field due to D at A that is to the right and up, and the force cross product for

that situation yields a force direction to the right and down. The vector sum of the two force vectors yields a net vector that is down and to the right. A graphic summary of all of this is shown below. The vector that best mimics this configuration is Response c.]

direction of magnetic field due to current in wire C  
(this is tangent to a circle centered on wire C and passing through wire A)



Note that the force on wire A is perpendicular to both the current direction and the direction of the magnetic field



29.) The net impedance in an RL circuit is  $2000 \Omega$ . The resistor-like resistance in that circuit is  $1000 \Omega$ . The inductor is removed and placed in a second circuit whose voltage amplitude is twice that of the original circuit and whose frequency is the same. The resistor-like resistance in that circuit is  $500 \Omega$ . The approximate impedance in that circuit will be:

a.)  $1000 \Omega$ . [To begin with, the amplitude of the power source in the circuit has no bearing on the resistive nature of the circuit. As such, the fact that the amplitude of the power source has been doubled means nothing. A measure of the resistive nature of the circuit is called the

impedance. The most general expression for the impedance is  $\sqrt{R_{net}^2 + (X_L - X_C)^2}$ . Because there

is no capacitor in our circuit, that expression becomes  $\sqrt{R_{net}^2 + (2\pi\nu L)^2}$  (note that the inductive reactance  $X_L$  has been replaced by its equivalent  $2\pi\nu L$ ). Plug in numbers for the original circuit

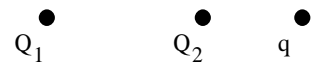
into that expression, we get  $(2000 \Omega) = [(1000\Omega)^2 + X_L^2]^{1/2}$ , or  $X_L = (3)^{1/2} \times 10^3 \Omega$ . Because the

frequency is the same in both circuits, the inductive reactance  $X_L$  will be the same in both

situations. This information coupled with the impedance expression as used on the second circuit yields  $Z = [(500\Omega)^2 + X_L^2]^{1/2} = [(500\Omega)^2 + [(3)^{1/2} \times 10^3 \Omega]^2]^{1/2} = [(250000\Omega) + (3000000 \Omega)]^{1/2} = 1804 \Omega$ .

This response is false.]

- b.)  $1800\Omega$ . [This is the one.]
- c.)  $4000 \Omega$ . [Nope.]
- d.) None of the above. [Nope.]



30.) Charges  $Q_1$  and  $Q_2$  are placed as shown. We don't know whether they are positive or negative. It is known that charge  $q$  is positive and that, as placed, it feels no electric force.

a.) Charge  $Q_1$  must be positive and larger in magnitude than  $Q_2$ , which must be negative. [For  $q$  to feel no force,  $Q_2$  must be smaller than  $Q_1$ . How so?  $Q_1$  is farther away than  $Q_2$ . The only way the force provided by  $Q_1$  can counteract the force provided by  $Q_2$  is if  $Q_1$  is oppositely charged and larger in magnitude than  $Q_2$ . As such, the first part of this statement is

true. As for the actual sign, it doesn't matter as long as the signs are opposite. That is, if  $Q_1$  were positive, it would produce a force on  $q$  in the  $+x$  direction. An equal and opposite force would have to be provided by  $Q_2$  for the zero net force requirement to be satisfied. If  $Q_1$  were negative, it would produce a force on  $q$  in the  $-x$  direction. An equal and opposite force would have to be provided by  $Q_2$  for the zero net force requirement to be satisfied. In either case, the zero force requirement is met and neither charge combination is necessary for the observed data to be true. This statement is false.]

b.) Charge  $Q_1$  must be negative and larger in magnitude than  $Q_2$ , which must be positive. [This is tricky. The "must be" part of the response makes it false, as it doesn't matter which charge is positive and which is negative.]

c.) Charge  $Q_1$  must be negative and smaller in magnitude than  $Q_2$ , which must be positive. [Again, the "must be" part of the response makes it false.]

d.) If  $q$ 's charge had been negative, it would have been necessary to place it on the other side of  $Q_1$  to find a point where it would feel no force. [Not so. The electric fields produced by  $Q_1$  and  $Q_2$  had to add to zero for the force on  $q$  to equal zero. Given what we have deduced about the charges involved, that will only happen at one place--to the right of  $Q_2$ . In fact, it doesn't matter whether  $q$  is positive or negative. When placed at a point where the E-field is zero, the force on any  $q$  will be zero. This response is false.]

e.) None of the above. [This is the one.]

31.) Two large, parallel plates are oppositely charged, then disconnected from the battery source that charged them up. The distance between the plates is then doubled.

a.) The electric field will halve as will the electrical potential difference between the plates. [If we use Gauss's Law to determine the electric field between oppositely charged parallel plates, we find an electric field that is constant and equal to  $\frac{\sigma}{\epsilon_0}$ , where  $\sigma$  is the charge per unit

area on one plate (the only assumption made here is that the dimensions of the plates are large in comparison to the distance between the plates; that is, the field expression is good in the region away from the edges of the plate--if the plate dimensions are on par with the distance between the plates, edge effects take over and the expression is no longer valid). Nowhere in that derivation is the plate separation used. As such, the electric field is not dependent upon plate separation. The first part of this response is false.]

b.) The electric field will halve but the electrical potential difference between the plates will stay the same. [Nope, the electric field stays constant.]

c.) The electric field will stay the same and the electrical potential difference between the plates will stay the same. [The first part is correct. As for the second part: The electrical potential in a constant electric field gets proportionally smaller as one moves downstream in the field.

That means that if the plate separation doubles and the electric field doesn't change (it is still  $\frac{\sigma}{\epsilon_0}$ ), the electrical potential difference between the plates will double. The second part of this response is false.]

d.) The electric field will stay the same but the electrical potential difference between the plates will double. [This is the one.]

e.) None of the above. [Nope.]

32.) A variable power supply produces an AC voltage equal to  $5 \sin(4\pi t)$  volts. What is the frequency of the source?

a.) 2 hertz. [The most general form for the voltage across a power source is

$V(t) = V_o \sin(\omega t + \phi) = V_o \sin((2\pi\nu)t + \phi)$ , where  $V_o$  is the amplitude of the voltage function,  $\phi$  is the phase shift (usually zero), and  $\nu$  and  $\omega$  are the source's frequency and angular frequency, respectively. As the angular frequency and frequency are related by  $\omega = 2\pi\nu$ , the frequency, in this case will be  $\nu = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2$  cycles/second. This response is true. Are there others?]

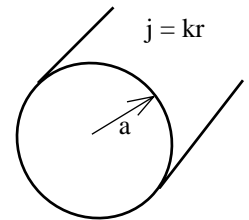
b.) 2 cycles/second. [This question is obviously checking to see if you know that hertz and cycles/second are the same thing. They are. This response is also true. Are there others?]

c.) 79 hertz. [If you multiplied the angular frequency by  $2\pi$  instead of dividing by that amount, you got this incorrect response.]

d.) Both a and b. [This is the one.]

e.) None of the above. [Nope.]

33.) A wire of radius  $a$  is made of an odd mixture of metals that effectively allows for a current flow that varies from point to point across a cut-away of the wire. In fact, the current along the central axis is zero, with the current magnitude getting larger as one moves out from there. The current density function (i.e., the current per unit area) is  $j = kc$ , where  $c$  is a distance from the wire's central axis to the point of interest. What is the magnitude of the magnetic field a distance  $r$  units from the central axis, where  $r < a$ ?



a.)  $k\mu_0 c^2/(2r)$ . [This is an Ampere's Law problem (see auxiliary sketch). If the current through a circular Amperian path of radius  $c$  was equally distributed throughout the cross-section of the wire, that current would equal the current density  $j$  (in amps per unit area) times the area  $A$  of the face bounded by the path. Mathematically, this would be  $i = jA = (j)(\pi c^2)$ . Unfortunately, the current density varies as one gets farther and farther away from the central axis. As such, we must define an imaginary loop of differential area  $dA$  located a distance  $r$  units from the central axis and through which the current density is  $j = kr$ . The differential current  $di$  through that differential area will equal  $di = j dA = (kr)(2\pi r dr)$ . Integrating that over the entire Amperian face will yield the total current through that face. Doing this in the context of Ampere's Law yields:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{thru}}$$

$$\Rightarrow \oint B(dl) \cos 0^\circ = \mu_0 \int di$$

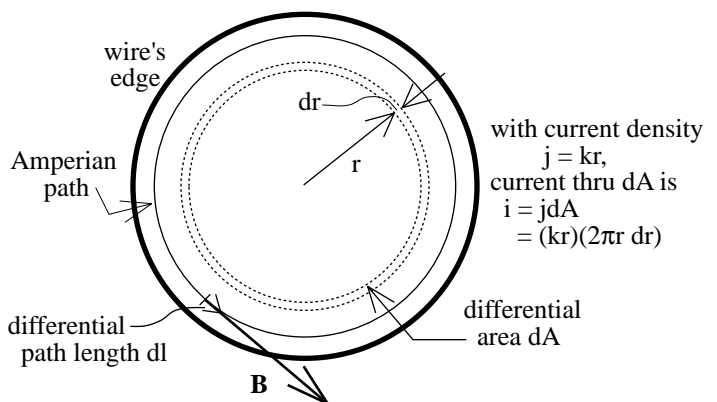
$$\Rightarrow B \oint (dl) = \mu_0 \int_{r=0}^c (kr)(2\pi r dr)$$

$$\Rightarrow B(2\pi r) = 2\pi k\mu_0 \int_{r=0}^c r^2 dr$$

$$\Rightarrow B = \frac{k\mu_0 c^3}{r^3}$$

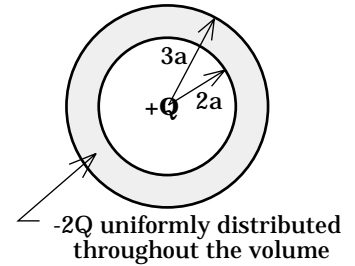
This response is false.]

cross-section of the wire  
(current out of page)



- b.)  $k\mu_0 c^3/(3r)$ . [This is the one.]  
 c.)  $k\mu_0 c^4/(4r)$ . [Nope. Please NOTE: If you hadn't known how to do this problem, a tricky way to approach it would have been to do a unit analysis (this is sometimes called a dimensional analysis) on the possible answers. You couldn't be sure the constants were correct, but at least a match in units would be a big start toward eliminating prospects.]  
 d.) None of the above. [Nope.]

34.) A charge  $+Q$  is suspended at the center of a hollow sphere that is, itself, charged uniformly to  $-2Q$ . The sphere's inside radius is  $2a$  and its outside radius is  $3a$ .



a.) The electric field lines outside the sphere will be oriented radially outward, and there will be a place between  $2a$  and  $3a$  at which the electric field is zero. [The total charge enclosed inside a Gaussian surface that is outside the sphere will be  $-Q$ . That means that the electric field will be one of a negative charge. Electric field lines always enter negatively charged structures, so the electric field lines in this situation should be inward, not outward. This response is false.]

b.) The electric field lines outside the sphere will be oriented radially inward, and there will be a place between  $2a$  and  $3a$  at which the electric field is zero. [From above, the first part of this statement is true. As for the second part, there will be a Gaussian radius between  $2a$  and  $3a$  where the total charge enclosed inside the Gaussian surface will be zero. On that surface, the electric field must evaluate to zero and this statement is true. Are there others?]

c.) The electric field will be continuous across the boundary defined by the inside radius (i.e., at  $2a$ ). [We could guess at this, but it would probably be better to actually do the math and see what we get. We know that the function that defines an electric field generated by the point charge at the center of the hollow will be  $E = \frac{Q}{4\pi\epsilon_0 r^2}$ . Evaluating that at  $2a$  yields

$$E = \frac{Q}{4\pi\epsilon_0 (2a)^2} = \frac{Q}{16\pi\epsilon_0 a^2}.$$

Inside the region between  $2a$  and  $3a$ , the electric field can be determined using Gauss's Law. Create a Gaussian sphere in the region. The volume charge distribution inside the solid is the total charge divided by the total volume in which charge

resides, or  $\rho = \frac{-2Q}{(4/3)\pi(3a)^3 - (4/3)\pi(2a)^3} = \frac{-2Q}{(4/3)\pi(19a^3)}$ . From this, we can write:

$$\begin{aligned} \int \mathbf{E} \cdot d\mathbf{S} &= \frac{q_{\text{enclosed}}}{\epsilon_0} \\ \Rightarrow \int E(dS) \cos 0^\circ &= \frac{Q + \int_{2a}^r \rho dV}{\epsilon_0} \\ \Rightarrow E \int (dS) &= \frac{Q + \int_{2a}^r \left( \frac{-2Q}{(4/3)\pi(19a^3)} \right) (4\pi c^2 dc)}{\epsilon_0} \\ \Rightarrow E(4\pi r^2) &= \frac{Q - \left( \frac{6Q}{19a^3} \right) \int_{c=2a}^r (c^2 dc)}{\epsilon_0} \end{aligned}$$

$$\Rightarrow E = \frac{Q - \left(\frac{6Q}{19a^3}\right) \left[\frac{c^3}{3}\right]_{2a}}{4\pi r^2 \epsilon_0}$$

$$\Rightarrow E = \frac{Q - \left(\frac{2Q}{19a^3}\right) (r^3 - (2a)^3)}{4\pi r^2 \epsilon_0}$$

$$\Rightarrow E = \frac{Q - \left(\frac{2Q}{19a^3}\right) (r^3 - 8a^3)}{4\pi r^2 \epsilon_0}.$$

This moderately clumsy-looking expression does something unexpected. At  $r = 2a$ , the  $(r^3 - 8a^3)$  term goes to zero and the electric field at  $r = 2a$  is found to be  $E = \frac{Q}{4\pi\epsilon_0(2a)^2} = \frac{Q}{16\pi\epsilon_0 a^2}$ .

This is exactly what we got before, so evidently, in this case, the electric field is continuous across the boundary at  $r = 2a$ . Please note: This electric field continuous across a boundary situation is not always true. If the solid had been a conductor, the electric field at  $r = 2a$  due to the levitated point charge would have been  $E = \frac{Q}{4\pi\epsilon_0(2a)^2} = \frac{Q}{16\pi\epsilon_0 a^2}$ , but the electric field at  $r = 2a$  evaluated using the expression for the electric field inside a conductor would have been zero. In that case, the electric field is not continuous. Still, this response is true.]

d.) Both b and c. [This is the one.]

e.) None of the above. [Nope.]

35.) A 10 mf capacitor is charged by a 100 volt battery, then isolated (i.e., removed from the circuit). It is then connected in parallel with an uncharged capacitor C. After the charge on the 10 mf capacitor redistributes itself, the voltage across C is measured at 80 volts. The capacitance C is:

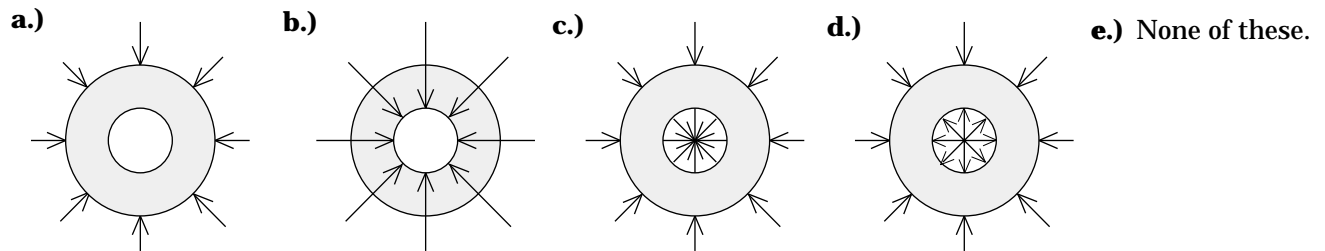
a.) 12 mf. [Note: This is a good example of a problem that isn't very difficult but that isn't exactly straightforward, either. You just have to diddle with it to get it. Diddling on: The initial charge on the 10 mf capacitor (note that this is  $10^{-2}$  farads) will be  $Q_1 = C_1 V_1 = (10^{-2} \text{ f})(100 \text{ v}) = 1 \text{ coulomb}$ . That is the total amount of charge in the system. When the 10 mf capacitor is connected across C, that charge redistributes itself between the two capacitors until the voltage across each is the same (in this case, 80 volts). Assume the new charge on C is Q. The new charge on the 10 mf capacitor will be  $(1 - Q)$ . That means that  $(1 - Q) = C_{10\text{mf}} V = (10^{-2} \text{ farads})(80 \text{ volts}) = .8 \text{ coulombs}$ . In turn, that means that  $Q = .2 \text{ coulombs}$ . Knowing the charge Q on C and the voltage across C, we can write  $C = Q/V = (.2 \text{ coulombs})/(80 \text{ volts}) = 2.5 \times 10^{-3} \text{ farads}$ . This response is false.]

b.) 8 mf. [Nope.]

c.) 2.5 mf. [This is the one.]

d.) None of the above. [Nope.]

36.) If you place a charge  $-Q$  on a hollow conducting sphere, the electric field lines for the situation will look like:



[Commentary: In a static electricity situation, free charge will always distribute itself so as to have no electric field inside a conductor. This eliminates Response b. In an attempt to get away from its kind, and because there is no additional charge levitating at the center of the sphere, free charge will migrate to the outside of a conducting surface (a Gaussian surface anywhere within the confines of the sphere will have zero charge enclosed). That means there will be electric field lines outside the sphere, but none inside the sphere (this eliminates Responses c and d). As the charge is negative, the electric field lines will be inward, which means that Response a does the task for us here.]

37.) A .5 kg mass has a 10 coulomb charge on it. It is placed at Point A in an electric field and released from rest, freely accelerating to Point B. The electrical potential of B is 40 volts.

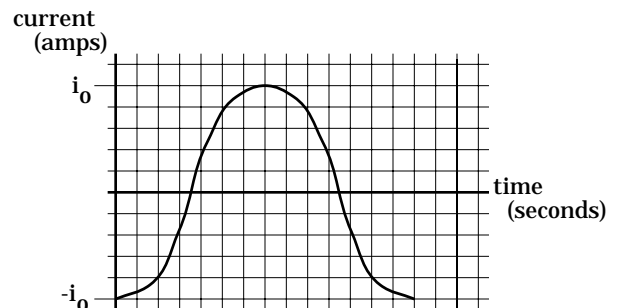
a.) If the field does 200 joules of work in the process, the velocity of the mass when at B is  $(200)^{1/2}$  m/s. [This is tricky. The temptation is to try to use the conservation of energy and write  $qV_A + 200 = .5mv^2 + qV_B$ . If you do, you will have inadvertently used the 200 joule quantity twice. How so? The amount of work the field does as the body moves from A to B can be taken into consideration in one of two ways. Either you can use potential energy quantities (you know  $V_B$  -- you can get  $V_A$  using  $W_{\text{field}} = -(qV_B - qV_A) = 200$  joules) and put  $W_{\text{extraneous}} = 0$ , or you can ignore the potential energy quantities and account for the work done by the field by setting  $W_{\text{extraneous}} = W_{\text{field}} = 200$  joules. YOU CANNOT DO BOTH. Using the latter approach, we get  $(200 \text{ joules}) = .5mv_B^2$ , or  $v_B = (800)^{1/2}$  m/s. This response is false.]

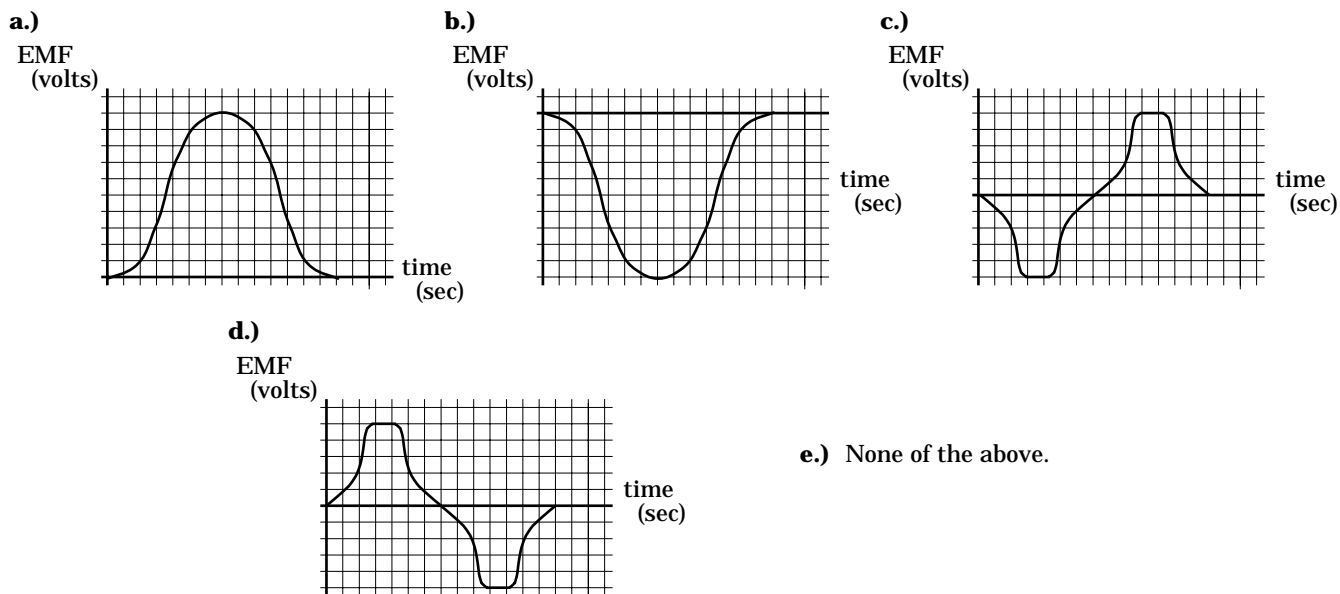
b.) If the field does 200 joules of work in the process, the velocity of the mass when at B is  $(800)^{1/2}$  m/s. [This is the one.]

c.) If the field does 200 joules of work in the process, there isn't enough information to determine the velocity at A because we don't have enough information to calculate the voltage at A. [This isn't the case.]

d.) None of the above. [Nope.]

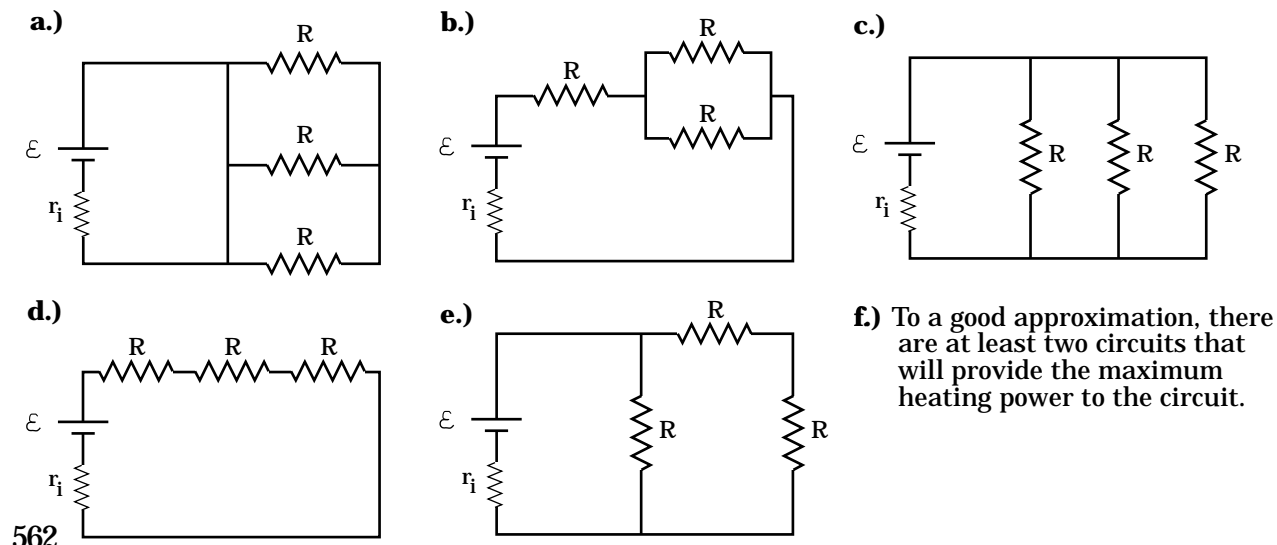
38.) A graph of the current through the primary coil of a transformer looks like the sketch shown to the right (note that at  $t = 0$ , the current is negative). A graph of the EMF set up in the secondary will look like:





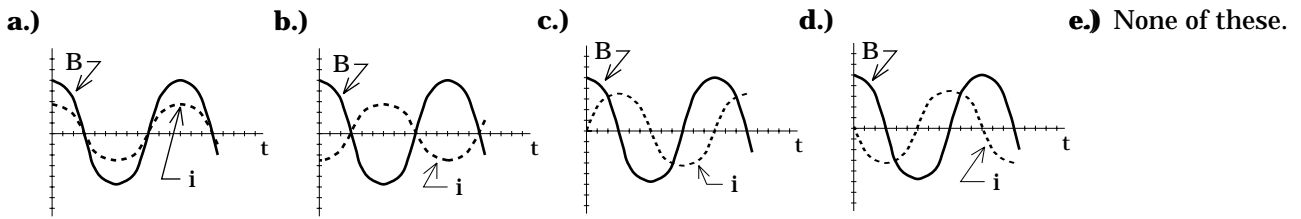
[Commentary: An induced EMF is set up whenever there is a changing magnetic flux through the cross-section of a coil. In the case of a transformer, the changing flux is essentially the same for both the primary and secondary coils. As such, a change of current in the primary creates a changing magnetic flux in the secondary. This induces an EMF in the secondary coil. The relationship between the current change in the primary and the EMF in the secondary is  $\varepsilon_s = -L(di_p/dt)$ . In other words, what we are looking for here is minus the slope of the current function. Looking at the current graph, the slope starts out at approximately zero, then gets progressively larger until it hits a section over which the slope is the same (i.e., the slope is a constant). The slope then gets less and less until, at the top, it is zero. Either of these two observations eliminates graphs a and b. Graphs c and d generally do a nice job of graphing the slope of the current function, but we want minus that slope, so graph c gets the nod.]

39.) In the circuit below, each of the resistors characterized by R is the same size. Assuming the power supply's EMF is  $\varepsilon$  while its very small internal resistance is  $r_i$ , which circuit will increase the temperature of water the fastest? Assume that all three heating resistors are immersed.



[Commentary: The circuit that will raise the temperature of water the fastest will be the circuit whose heating resistors dissipate the most power. As power dissipation in a resistor is governed by the current through the resistor, we are looking for the situation in which at least most (if not all) of the heating resistors have the greatest current possible through them. That will happen when the equivalent resistance of the circuit is a minimum. Circuit a is essentially a short circuit--no current will pass through the heating resistors--hence it is not a viable candidate. In examining the other circuits, remembering that the equivalent resistance of a parallel combination is always smaller than any of the individual resistors within the combination, it looks as though Circuit c will do the trick. NOTE: When in parallel, all three heating resistors will have the maximum possible voltage across them, hence the greatest power output for warming water.]

40.) A coil is placed in a changing magnetic field. A graph of the B-field is shown on each of the grids below. Due to the changing B-field, an induced current is generated in the coil. Which graph depicts the appropriate current function, given the B-field function?



[Commentary: The relationship between the induced EMF and the induced current is  $\varepsilon = iR$ , where  $R$  is the resistance in the coil. As EMF and induced current are proportional, the relationship between the changing magnetic field and the induced EMF will be the same as the relationship between the changing magnetic field and the induced current. The former of those relationships is  $\varepsilon = -(NA \cos\theta) \frac{dB}{dt}$ . As such, we need an induced current graph that looks like minus the slope of the magnetic field graph. Just after  $t = 0$ , the B-field graph has a negative slope the magnitude of which gets larger with time, then retreats toward zero. Minus that yields positive values. In this case, the graph that does the trick is Graph c.]

