Chapter 20
AC Circuit Chapter Review

EQUATIONS:

- $V(t) = V_o \sin(2\pi \nu t)$ [This expression represents the voltage in an AC circuit, where V_o is the maximum voltage (the amplitude of the voltage function) and ν is the frequency of the AC source.]

- $i(t) = i_o \sin(2\pi \nu t)$ [This expression represents the current in an AC circuit, where i_o is the maximum current (the amplitude of the current function) and ν is the frequency of the AC source.]

- $i_{RMS} = .707 i_o$ [In a DC circuit, charge flows in one direction. The amount of charge flow per unit time (i.e., $\Delta q/\Delta t$) in a given branch is defined as the branch's DC current. When current moves through a resistor (a light bulb, for instance), electrons crash into the lattice structure of the wire giving up energy as they go. A measure of the amount of energy dissipated by the resistor per unit time is called the power rating. In an AC circuit, charge doesn't flow in one direction, it instead jiggles back and forth in response to the changing electric field provided by the AC voltage. In other words, the flow rate in an AC circuit changes from instant to instant. A sine wave characterizes the variation. Nevertheless, the charge flow does do work, giving up energy just as would if in a DC circuit. Put a little differently, power is dissipated by resistors in an AC circuit. If we want to use i^2R to determine the amount of power dissipated by such a resistor, what current value do we use (after all, the AC current is constantly changing)? We need a single value that yields a power rating that truly reflects the amount of work per unit time the resistor actually does. That value is called the RMS current. It is equal to .707 of the maximum current (i.e., $i_{RMS} = .707 i_o$). AC ammeters read RMS values. This means that the power relationship associated with an AC circuit is more accurately written as $P = (i_{RMS})^2 R$. Please note that I occasionally call RMS current the DC equivalent current. This is because it is equal to the single DC current value that would dissipate the same amount of power as does the AC current across a given resistor.]

- $V_{RMS} = .707 V_{max}$ [This is the relationship between the RMS voltage across an element in an AC circuit and the amplitude of the AC voltage. The RMS voltage is the quantity that AC voltmeters read. It is also the value that should be used in the power expression $P = i_{RMS} V_{RMS}$.]

- $X_L = 2\pi \nu L$ [Called the inductive reactance, this is a measure of the frequency-dependent resistive nature of an inductor (a coil). Notice that at high frequency, this value is high. That suggests that inductors do not allow high frequency currents to flow, which is the case. The unit of inductive reactance is an ohm.]

- $X_C = \frac{1}{2\pi\nu C}$ [Called the capacitive reactance, this is a measure of the frequency-dependent resistive nature of a capacitor. Notice that at low frequency, this value is high. This suggests that capacitors do not allow low frequency currents to flow, which is the case. The unit of capacitive reactance is an ohm.]

- $Z = \left[R^2 + (X_L - X_C)^2\right]^{\frac{1}{2}} = \left[R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2\right]^{\frac{1}{2}}$ [Called the impedance Z, this expression quantifies the total amount of resistance to charge flow that exists at a given frequency in an AC circuit (remember, elements like capacitors and inductors have a frequency-dependent resistive nature). OHM'S LAW should really be written in terms of impedance. That is, the most general form of Ohm's Law is V = iZ. Note that when resistors are the only elements in a circuit, Ohm's Law then becomes V = iR.]

- $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R_{total}}\right)$ [This expression quantifies the phase shift ϕ between the voltage and current in an AC circuit. Note that the phase shift will NEVER be $\pi/2$ radians as there will always be resistor-like resistance somewhere in the circuit. Notice also that when the phase shift is negative, the voltage lags the current by ϕ.]

- $\nu_{resonance} = \frac{1}{2\pi}\left(\frac{1}{LC}\right)^{\frac{1}{2}}$ [This is the expression that defines the resonance frequency for an RLC circuit. In such circuits, the inductor will try to eliminate high frequency signals, and the capacitor will try to eliminate low frequency signals. The only frequency that survives (actually, there is a range of frequencies, but that range is centered on this one frequency) is called the resonance frequency. At resonance, the effect of the capacitor and inductor cancels, the resistive nature becomes as small as it will ever be for the circuit, and the current becomes large. This characteristic of RLC circuits is very important when it comes to tuning in a radio station.]

COMMENTS, HINTS, and THINGS to be aware of:

- Alternating current (i.e., AC) is produced by rotating a coil in a magnetic field. The rate of rotation defines the frequency of the AC, and the amplitude depends upon the size of the coil and the size of the B field.

- Because inductors are resistant to high frequency signals, they are called low pass filters.

- Because capacitors are resistant to low frequency signals, they are called high pass filters.

- When hooking together a number of independent electrical entities (a tuner to a CD player to speakers, etc.), power is transferred most efficiently when the impedance of each entity is the same. As the electrical requirements of each device are different, the internal impedance of each will be different. To deal with this problem, a transformer can be used to impedance match each device to its neighbor.

- If the frequency of a power supply is 60 hertz (i.e., the frequency of the power provided by your wall sockets at home), the $2\pi\nu$ quantity becomes 377 radians and the voltage function becomes $V(t) = V_o \sin 377t$. It is interesting to note that when people say a wall socket provides 110 volts AC, they are really identifying the RMS voltage. In fact, the maximum voltage is just a little less than 156 volts (remember, $V_o = V_{RMS}/(.707)$).

- The resonance frequency for an RLC circuit can have a fairly broad side band (i.e., spread on either side of the actual resonance frequency). That means there is actually a range of frequencies at which current can proliferate.

- When resonance occurs in an RLC circuit, there is a very delicate interaction that occurs between the capacitor and inductor in the circuit. Consider an RLC circuit that doesn't include a power supply. Assume the capacitor is initially charged and a switch is thrown at t = 0 connecting all of the elements into a one branch circuit. When this is done, the capacitor will begin to discharge sending current through both the resistor and the inductor as charge flows from one capacitor plate to the other. The inductor will set up an EMF that initially fights this increase of current. This will last until the current hits a maximum. When the current begins to diminish, the inductor will produce an EMF that ultimately forces charge to flow even longer than normally would have been the case. That is, even when the capacitor has completely discharged, there will still be current flowing in the circuit. This current recharges the capacitor plates, but with the plate polarity reversed (i.e., the plate that was negative becomes positive, and vice versa). When the current finally dies out, the capacitor (newly charged going the other way) will begin to discharge as would be expected of a charge capacitor. This charging, discharging, charging, discharging process occurs at a natural frequency, called the resonance frequency, which is dependent upon the size of L and C. So when a power supply that is tuned to an RLC's resonance frequency is placed in the circuit, it helps this oscillatory process and the amplitude of the current becomes large. When other frequencies are impressed upon the system, the power source and the natural oscillatory nature of the circuit are out of synch and little current flows. In short, resonance is a natural consequence of the way discharging capacitors and EMF inducing inductors interact in an RLC circuit.

- Don't forget, the voltage across a resistor and the current through a resistor are linearly proportional to one another (i.e., $V = iR$). A big voltage across a resistor in a given branch means a big current, relatively speaking, whereas a small voltage across the same resistor means a small current. With that in mind, if you are asked to speculate as to whether a current is high or low in a particular circuit, try to determine what the voltage drops are for the various other elements in the circuit, then decide how large the voltage drop across the resistor must be. If it is big, that means big current. If it is small, that means small current.