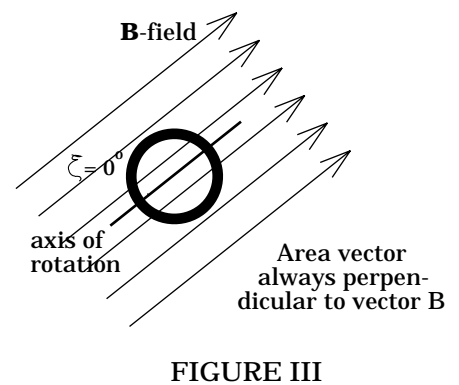
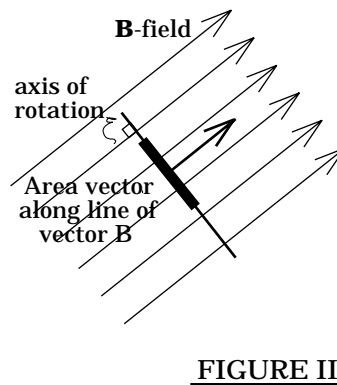
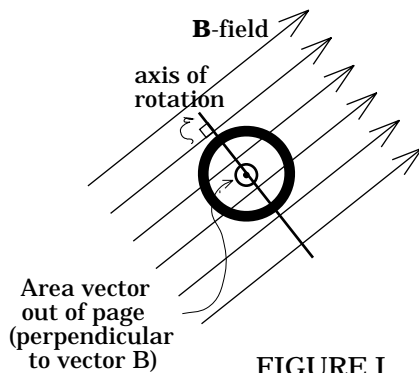


## CHAPTER 19 -- FARADAY'S LAW

19.1)

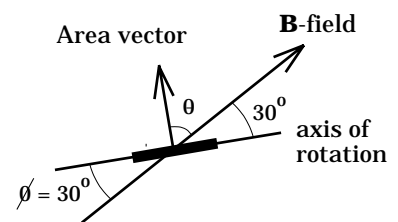
a.) To begin with, note that the angle  $\xi$  is not the angle between  $A$  and  $B$ --it is the angle between  $B$  and the coil's axis. With that in mind: The definition of magnetic flux is  $B \cdot A$ , where  $B$  is the magnetic field vector and  $A$  is the area vector ( $A$ 's direction is PERPENDICULAR to the coil's face). The only way we can get no flux through the coil as the coil rotates about its axis is if  $A$  and  $B$  are always perpendicular to one another. By observation, if the angle between the axis and  $B$  is  $\xi = 90^\circ$ ,  $A$  and  $B$  will be perpendicular as shown in Figure I below, but will not be perpendicular in Figure II. On the other hand, if the angle between the axis and  $B$  is  $\xi = 0^\circ$  (i.e., if the axis is along the line of  $B$ ) as shown in Figure III,  $A$  and  $B$  will always be perpendicular. That is the situation we want (that is, we want  $\xi = 0^\circ$ ).



b.) The number of winds has nothing to do with the magnetic flux ( $\phi_m = B \cdot A$ ) through the coil's area (it has to do only with the induced EMF generated in the coil should the flux CHANGE). Noting how the angle  $\xi$  was defined in the problem, we get the sketch shown to the right. With that sketch, we can write the coil's magnetic flux as:

$$\phi_m = B \cdot A$$

$$\phi_m = BA \cos \theta,$$



where  $\theta$  is the angle between  $B$  and  $A$ . In this case, the angle is  $60^\circ$  (see Figure IV). Substituting in we get:

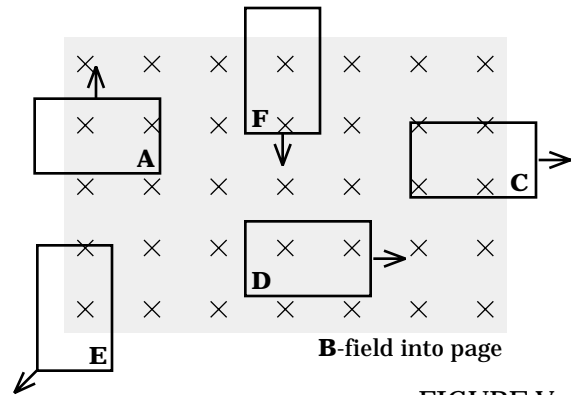
$$\begin{aligned}\phi_m &= (.065 \text{ T})[\pi(.15 \text{ m})^2] \cos 60^\circ \\ &= 2.3 \times 10^{-3} \text{ webers.}\end{aligned}$$

19.2)

a.) Using Lenz's Law:

--Loop A: No induced current as there is **NO CHANGING FLUX**.

--Loop C: The external flux is decreasing. A **CLOCKWISE** induced current will produce an induced  $B$ -field **INTO** the page through the coil's face, which in turn will produce an induced magnetic flux that will **OPPOSE** the decreasing external flux.



**FIGURE V**

--Loop D: No current as there is **NO CHANGING FLUX**.

--Loop E: The external flux is decreasing. A **CLOCKWISE** current will generate an induced flux that will **OPPOSE** the decreasing external flux.

--Loop F: The externally produced flux is increasing. A **COUNTER-CLOCKWISE** current will produce an induced flux that will **OPPOSE** the increasing external flux.

b.) Using Faraday's Law:

--Loop A: As there is no changing magnetic flux, the induced EMF in that coil will be **ZERO**.

--Loop C: We need to determine the loop's area-change  $\Delta A$  over a given amount of time  $\Delta t$ . In general, if the loop travels a distance  $d$  moving with velocity  $v$ , we can write:

$$v = d/\Delta t \quad \Rightarrow \quad \Delta t = d/v.$$

To travel, say, .05 meters going .28 m/s, it will take time:

$$\begin{aligned}\Delta t &= (.05\text{m})/ (.28 \text{ m/s}) \\ &= .179 \text{ seconds.}\end{aligned}$$

During that time the area of the coil inside the magnetic field goes from  $A_o = (.08 \text{ m})(.15 \text{ m})$  to  $A_f = (.08 \text{ m})(.1 \text{ m})$ , or  $\Delta A = A_f - A_o = (.08 \text{ m})(-.05 \text{ m}) = -4 \times 10^{-3} \text{ m}^2$ . We know that the induced EMF will equal:

$$\begin{aligned}\epsilon_c &= -N_c [\Delta\phi_m / \Delta t] \\ &= -N_c [B(\Delta A)(\cos 0^\circ) / \Delta t] \\ &= -(1)[(3 \times 10^{-2} \text{ T})(-4 \times 10^{-3} \text{ m}^2)(1) / (.179 \text{ s})] \\ &= 6.7 \times 10^{-4} \text{ volts.}\end{aligned}$$

According to the current direction we determined in Part a, a positive induced EMF evidently corresponds to a clockwise induced current.

--Loop F: Following logic similar to that used on Loop C, and noting that in this case the change of area goes from  $(.08 \text{ m})(.15 \text{ m})$  to  $(.08 \text{ m})(.2 \text{ m})$ , or  $\Delta A = 4 \times 10^{-3} \text{ m}^2$ , we can write:

$$\begin{aligned}\epsilon_F &= -N_F [\Delta\phi_m / \Delta t] \\ &= -N_F [B(\Delta A)(\cos 0^\circ) / \Delta t] \\ &= -(1)[(3 \times 10^{-2} \text{ T})(4 \times 10^{-3} \text{ m}^2)(1) / (.179 \text{ s})] \\ &= -6.7 \times 10^{-4} \text{ volts.}\end{aligned}$$

As the positive EMF in Loop C corresponds to a CLOCKWISE current, the negative EMF in Loop F should correspond to a COUNTERCLOCKWISE induced current. According to Part a, that is exactly what happens.

Note: The EMF in Loop C and in Loop F have the same magnitude because the change of flux during the .179 second time interval was the same in both cases.

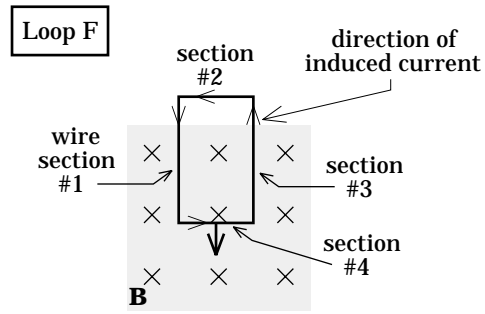
c.) To get the force on a current-carrying wire that is bathed in a magnetic field, we must apply the expression:

$$F = iL \times B$$

to each section of the wire in the B-field (see Figure VI on the next page), then add up all the forces acting as shown below:

$$F_{\text{net}} = F_1 + F_2 + F_3 + F_4.$$

We know that the magnitude of  $L$  in, say, wire section #1 is equal to that portion of the wire in the  $B$ -field, or .15 meters. We also know that the magnitude of  $B$  is  $3 \times 10^{-2}$  teslas. What we don't know is the magnitude of the induced current  $i$ . To get that, we must determine the induced EMF, then use  $i = \varepsilon_B/R$ . Using the



**FIGURE VI**

MAGNITUDE of the induced EMF from Part b (we just want the magnitude for the current calculation--we already know the current's direction is counterclockwise from Part a), we get:

$$\begin{aligned} i &= \varepsilon_B/R \\ &= (6.7 \times 10^{-4} \text{ v})/(4 \Omega) \\ &= 1.68 \times 10^{-4} \text{ amps.} \end{aligned}$$

Noting that there is no magnetic force being applied to wire section #2 because it is not in the magnetic field, we get:

$$\begin{aligned} F_{\text{net}} &= F_1 + F_2 + F_3 + F_4 \\ &= iL_1 B \sin 90^\circ(+i) + 0 + iL_3 B \sin 90^\circ(-i) + iL_4 B \sin 90^\circ(+j) \\ &= iL_4 B \sin 90^\circ(j) \\ &= (1.68 \times 10^{-4} \text{ A})(.08 \text{ m})(3 \times 10^{-2} \text{ T})(1) (j) \\ &= (4.032 \times 10^{-7} \text{ nts}) (j). \end{aligned}$$

Does this make sense? Certainly! The induced force will always oppose the motion. As the motion is downward, the net induced force should be upward in the  $+j$  direction. That is exactly what we have calculated.

Isn't this fun?

d.) The net force on Loop A and Loop D will be zero as the induced EMF in those loops is zero (hence the induced currents are zero). The induced force in Loop C will have the same magnitude as that of Loop F, but the direction will be different. How do you determine that direction?

The direction of the cross product  $iL \times B$  yields the direction. Try using it. Notice that the direction of the force is always such that it opposes the physical motion of the coil. In the case of Loop C, the coil is moving in the  $+i$  direction, so the force will be in the  $-i$  direction.

19.3)

a.) The magnetic flux is:

$$\begin{aligned}\phi_m &= B \cdot A \\ \phi_m &= BA \cos \theta \\ &= (2.3 \text{ T})[\pi(.03 \text{ m})^2] \cos 0^\circ \\ &= 6.5 \times 10^{-3} \text{ webers.}\end{aligned}$$

b.) The area vector is not changing. The magnetic field vector is changing and we know the rate at which that change occurs (i.e.,  $dB/dt$ ). Noting that  $d\phi_m = A(dB)$  and, for this case,  $d\phi_m/dt = A(dB/dt)$ , Faraday's Law can be written as:

$$\begin{aligned}\varepsilon &= -N [d\phi_m/dt] \\ &= -N [ A (dB/dt) (\cos 0^\circ)] \\ &= -(6) [\pi(.03 \text{ m})^2 (.6 \text{ T/sec}) (1) ] \\ &= -1 \times 10^{-2} \text{ volts.}\end{aligned}$$

Using  $i = \varepsilon/R$ , we get a current magnitude of:

$$\begin{aligned}i &= (10^{-2} \text{ v})/(12 \Omega) \\ &= 8.33 \times 10^{-4} \text{ amps.}\end{aligned}$$

According to Lenz's Law, the current should flow **CLOCKWISE**.

c-i.) The frequency of the AC current will be the same as the frequency of the coil's rotation. As  $\omega = 2\pi\nu$ , we can write:

$$\begin{aligned}\nu &= \omega/2\pi \\ &= (55 \text{ rad/sec})/(2\pi) \\ &= 8.75 \text{ hz.}\end{aligned}$$

c-ii.) In this case,  $B$  and  $A$  are constant while the angle between the two vectors changes with time. We can write the angle as a function of time by noting that  $\theta = \omega t$ . With this, Faraday's Law yields:

$$\begin{aligned}\varepsilon &= -N \frac{d\phi_m}{dt} \\ &= -N \frac{d(BA \cos \omega t)}{dt}\end{aligned}$$

$$\begin{aligned}
&= -NBA \frac{d(\cos \omega t)}{dt} \\
&= -NBA\omega(-\sin \omega t) \\
&= NBA\omega(\sin \omega t).
\end{aligned}$$

Putting in the numbers yields:

$$\varepsilon = 2.15 \sin(55t) \text{ volts.}$$

Note: We could have added a phase shift factor in the original cosine function if we wanted the magnetic flux to be something other than maximum at  $t = 0$ . In fact, if we had made the phase shift  $-\pi/2$ , we would have ended up with the magnetic flux acting like a sine function and an EMF that was acting like a cosine function.

19.4) In this case, A is constant while B changes.

a.) Noting that the angle between B and A is  $0^\circ$ , Faraday's Law yields:

$$\begin{aligned}
\varepsilon &= -N[\Delta\phi_m/\Delta t] \\
&= -N [(B_f - B_o) A \cos 0^\circ] / \Delta t \\
&= -(80) [(0.04 \text{ T} - .12 \text{ T})(.12 \text{ m})^2] / (2.4 \text{ sec}) \\
&= 3.84 \times 10^{-2} \text{ volts.}
\end{aligned}$$

Note: This approach appears to be considerably different than the one used in Problem 19.3. How does one know which approach to use when? Look to see what is given. If you know the rate at which B changes (i.e.,  $dB/dt$ ), the approach used in Problem 19.3 will do. If you are given specific B values at specific times, then  $(B_2 - B_1)/t$  will do. LOOK TO SEE WHAT IS GIVEN.

b.) Using Lenz's Law: The external field is decreasing INTO the page. Only a CLOCKWISE current will produce an induced flux that will diminish that decrease (i.e., add to the already existing, decreasing, external flux).

c.) Using  $i = \varepsilon/R$ , we get a resistance magnitude of:

$$\begin{aligned}
R &= (3.84 \times 10^{-2} \text{ v}) / (.15 \text{ A}) \\
&= .256 \Omega.
\end{aligned}$$

19.5)

a.) Before the current begins to change, the only voltage drop across the inductor is due to the internal resistance inherent within the inductor's wire. That means:

$$\begin{aligned} V_L &= ir_L \\ &= (2.5 \text{ a})(6 \Omega) \\ &= 15 \text{ volts.} \end{aligned}$$

b.) The time constant for an inductor/resistor circuit tells us how long it takes for the current to reach .63 of its maximum (assuming the switch closes at  $t = 0$ ) or, if the system has been turned off as is the case here, the time it takes for the current to FALL to .37 of its maximum. In other words, knowing that it took .05 seconds to hit approximately one-third of its original value after the switch is opened means the time constant for the RL circuit is approximately .05 seconds. Noting that the total resistance in the circuit is  $(r_L + R)$  and remembering that the time constant for an RL circuit is  $\Gamma_{RL} = L/(R + r_L)$ , we can write:

$$\begin{aligned} L / (R + r_L) &= .05 \\ \Rightarrow R + (6 \Omega) &= [(1.5 \text{ H}) / (.05)] \\ \Rightarrow R &= 24 \Omega. \end{aligned}$$

c.) The energy stored in a current-carrying inductor is equal to  $(1/2)Li^2$ . If that value decreases, which it will as the current decreases, the "lost" energy goes into driving current in the circuit even longer than expected (remember, inductors are coils and coils hate to have the flux through their cross-section change). The amount of energy provided to the circuit is equal to  $(1/2)Li_f^2 - (1/2)Li_o^2$  (this number will actually be negative--the negative telling you that the inductor is losing that amount of energy to the circuit). As POWER is defined as the work done (read this "energy given up") per unit time, then:

$$\begin{aligned} P &= [(1/2)Li_f^2 - (1/2)Li_o^2]/\Delta t \\ &= [.5(1.5 \text{ H})[.33(2.5 \text{ a})]^2 - .5(1.5 \text{ H})(2.5 \text{ a})^2]/(.05 \text{ sec}) \\ &= -83.5 \text{ watts.} \end{aligned}$$

d.) The energy goes into driving current through the circuit even after the battery has been taken out of the circuit by the switch. That is, if the resistor is a light bulb, it will "burn" very bright with the initial change, then dampen out over some period of time (how long this takes depends upon the resistance in the circuit).

19.6)

a.) As the induced magnetic force will oppose the acceleration, the induced current must be clockwise (a clockwise current will feel a net upward force as it moves through the magnetic field).

b.) At terminal velocity, N.S.L. implies:

$$\begin{aligned} \Sigma F_y: \\ -mg + F_{\text{induced}} &= ma_y \\ &= 0 \end{aligned}$$

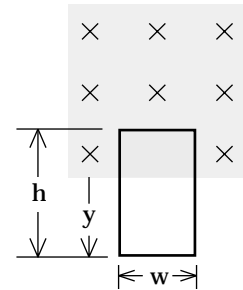
as terminal velocity requires that  $a_y = 0$ .

We know that  $F_{\text{induced}} = i_{\text{induced}} L \times B$ . The forces on the two side wires will add collectively to zero. The upper, horizontal wire is in the B-field. It will feel an upward force as it attempts to leave the field in a downward direction (the direction of this force is evident either by evaluating the force-defining cross product or by remembering that induced forces are always directed OPPOSITE the direction of the coil motion that causes the changing flux).

We know that the magnitude of the upper length of wire is  $w_{\text{upper}} = .6$  meters and that the magnitude of B is .85 teslas. To finish the problem, all we additionally need to determine is  $i$ . To get this, we will use Faraday's Law to determine  $\epsilon$ , then use  $i = \epsilon/R$ .

To get  $\epsilon$ , we need to determine a general expression for the area of the coil inside the magnetic field at an arbitrary point in time. In this case, this expression will be  $A = (h - y)w$ --see the sketch to the right.

Knowing all of this, Faraday's Law for the general case becomes:



area of coil inside **B**-field at arbitrary time is  $A = (h - y)w$

**FIGURE VII**

$$\begin{aligned} \epsilon &= -N \frac{d\phi_m}{dt} \\ &= -N \frac{d(BA \cos 0^\circ)}{dt} \\ &= -NB \frac{d[(h - y)w]}{dt} \\ &= NBw \frac{dy}{dt} \\ &= NBwv_y. \end{aligned}$$

Using  $i = \varepsilon/R$ , we get a current magnitude of:

$$\begin{aligned} i &= \frac{\varepsilon}{R} \\ &= \frac{NBwv_y}{R}. \end{aligned}$$

With the current, we can determine the magnitude of the induced magnetic force  $F_{\text{ind}}$  at any point in time as:

$$\begin{aligned} F_{\text{induced}} &= iwB \sin 90^\circ \\ &= \left( \frac{NBwv_y}{R} \right) wB(1) \\ &= \left( \frac{NB^2w^2v_y}{R} \right). \end{aligned}$$

Using this with our N.S.L. expression for the terminal velocity situation, we can write:

$$\begin{aligned} F_{\text{induced}} &= mg \\ \Rightarrow \frac{NB^2w^2v_{\text{terminal}}}{R} &= mg \\ \Rightarrow v_{\text{terminal}} &= \frac{mgR}{NB^2w^2}. \end{aligned}$$

Noting that  $N = 1$ ,  $w = .6$  meters, and  $B = .85$  teslas, terminal velocity is found to be  $v_{\text{terminal}} = 39.6$  m/s.

c.) According to the analysis done above, the velocity of the loop at any arbitrary point in time will be related to the induced EMF by:

$$\varepsilon = NBwv_y,$$

with the current in the loop at any instant being:

$$\begin{aligned} i &= \varepsilon / R \\ &= \frac{NBwv_y}{R}. \end{aligned}$$

With gravity and the force due to the magnetic field unequal, the acceleration will be non-zero. Setting  $v_y = v$  and using N.S.L. yields:

$\Sigma F_y$ :

$$\begin{aligned} |\mathbf{i} \mathbf{w} \times \mathbf{B}| - mg &= -m \frac{dv}{dt} \\ \Rightarrow iwB \sin 90^\circ - mg &= -m \frac{dv}{dt} \\ \Rightarrow \left[ \frac{NBwv}{R} \right] wB \sin 90^\circ - mg &= -m \frac{dv}{dt}. \end{aligned}$$

Expanding, we get:

$$\begin{aligned} \frac{NB^2 w^2 v}{mR} - g &= -\frac{dv}{dt} \\ \Rightarrow \left[ v - \frac{mR}{NB^2 w^2} g \right] &= -\frac{mR}{NB^2 w^2} \frac{dv}{dt} \\ \Rightarrow -\frac{NB^2 w^2}{mR} dt &= \frac{dv}{\left[ v - \frac{mR}{NB^2 w^2} g \right]}. \end{aligned}$$

Integrating from  $t = 0$  (i.e., from when the velocity is  $v = 0$ ) to some arbitrary point in time, we get:

$$\begin{aligned} -\frac{NB^2 w^2}{mR} \int_{t=0}^t dt &= \int_{v=0}^v \frac{dv}{\left[ v - \frac{mRg}{NB^2 w^2} \right]} \\ \Rightarrow -\frac{NB^2 w^2}{mR} t &= \ln \left[ v - \frac{mRg}{NB^2 w^2} \right] \Big|_{v=0} \\ \Rightarrow -\frac{NB^2 w^2}{mR} t &= \ln \left[ v - \frac{mRg}{NB^2 w^2} \right] - \ln \left[ -\frac{mRg}{NB^2 w^2} \right] \\ \Rightarrow -\frac{NB^2 w^2}{mR} t &= \ln \left[ \frac{v - \frac{mRg}{NB^2 w^2}}{-\frac{mRg}{NB^2 w^2}} \right]. \end{aligned}$$

To get rid of the ln function and unembed the velocity term, each expression must be made into the exponent of the exponential e. That is:

$$\begin{aligned}
e^{\frac{-NB^2w^2}{mR}t} &= e^{\ln \left[ \frac{v - \frac{mR}{NB^2w^2}g}{\frac{-mR}{NB^2w^2}g} \right]} \\
\Rightarrow e^{\frac{-NB^2w^2}{mR}t} &= \left[ \frac{v - \frac{mRg}{NB^2w^2}}{\frac{-mRg}{NB^2w^2}} \right] \\
\Rightarrow v &= \frac{-mRg}{NB^2w^2} e^{\frac{-NB^2w^2}{mR}t} + \frac{mRg}{NB^2w^2}.
\end{aligned}$$

Does this make sense? Yes! At  $t = 0$ , the expression reduces to zero. At  $t = \infty$ , the expression reduces to  $mgR/(NB^2w^2)$ --the terminal velocity. The equation works nicely at the extremes--always a good sign!

19.7)

a.) As was done in Problem 19.3c, we can take care of the rotation by writing the time-varying angle between A and B as  $\theta = \omega t$  (we can put in the  $\omega = 2\pi\nu$  later). That makes the magnetic flux look like:

$$\begin{aligned}
\phi_m &= BA_o \cos \theta \\
&= (B_o e^{-kt}) A_o \cos \omega t.
\end{aligned}$$

The induced EMF is:

$$\begin{aligned}
\varepsilon &= -N \frac{d\phi_m}{dt} \\
&= -N \frac{d((B_o e^{-kt}) A_o \cos \omega t)}{dt} \\
&= -NB_o A_o \frac{d((e^{-kt}) \cos \omega t)}{dt} \\
&= -NB_o A_o [(-k)(e^{-kt}) \cos \omega t + (e^{-kt})\omega(-\sin \omega t)] \\
&= NB_o A_o e^{-kt} [k \cos \omega t + \omega \sin \omega t].
\end{aligned}$$

b.) To determine the maximum value of the EMF, we must determine the time when the rate of change of the EMF is zero (this is a standard maximization problem). Doing this process yields:

$$\begin{aligned}\frac{d\varepsilon}{dt} &= NB_oA_o \frac{d(e^{-kt}[k \cos \omega t + \omega \sin \omega t])}{dt} \\ &= NB_oA_o \frac{d(e^{-kt}[k \cos \omega t + \omega \sin \omega t])}{dt} \\ &= NB_oA_o [(-k)e^{-kt}[k \cos \omega t + \omega \sin \omega t] + e^{-kt}[-k\omega \sin \omega t + \omega^2 \cos \omega t]].\end{aligned}$$

The time-derivative of the EMF expression yields the slope of the EMF function. As maxima or minima have tangent-slopes equal to zero, we can write:

$$\begin{aligned}\frac{d\varepsilon}{dt} &= NB_oA_o [(-k)e^{-kt}[k \cos \omega t + \omega \sin \omega t] + e^{-kt}[-k\omega \sin \omega t + \omega^2 \cos \omega t]] \\ &= 0.\end{aligned}$$

Canceling out the  $NB_oA_o e^{-kt}$  terms and multiplying by -1, we can write:

$$\begin{aligned}k^2 \cos \omega t + \omega k \sin \omega t + k\omega \sin \omega t - \omega^2 \cos \omega t &= 0 \\ \Rightarrow (+\omega k + k\omega) \sin \omega t + (k^2 - \omega^2) \cos \omega t &= 0 \\ \Rightarrow (2\omega k) \sin \omega t = -(k^2 - \omega^2) \cos \omega t \\ \Rightarrow \frac{\sin \omega t}{\cos \omega t} = \frac{(-k^2 + \omega^2)}{2\omega k} \\ \Rightarrow \tan \omega t = \frac{(-k^2 + \omega^2)}{2\omega k} \\ \Rightarrow \omega t = \tan^{-1} \left[ \frac{(-k^2 + \omega^2)}{2\omega k} \right] \\ \Rightarrow t = \frac{1}{\omega} \tan^{-1} \left[ \frac{(-k^2 + \omega^2)}{2\omega k} \right].\end{aligned}$$

Be impressed. The units of the inverse tangent are radians; the units of the coefficient are seconds (remember, because radians is a

generic term, the units of  $\omega$  is technically seconds<sup>-1</sup>). Everything seems to be working, at least as far as the units go.

19.8)

a.) Evaluating  $B = 12t^3 - 4.5t^2$  at  $t = .2$  seconds yields  $B = -.084$  teslas. The negative sign implies that  $B$  is into the page at  $t = .2$  seconds.

b.) The general expression for the magnetic flux is:

$$\begin{aligned}\phi_m &= \mathbf{B} \cdot \mathbf{A} \\ &= BA \cos \theta \\ &= (12t^3 - 4.5t^2)(\pi R^2) \cos 0^\circ \\ &= (12t^3 - 4.5t^2)(\pi R^2).\end{aligned}$$

c.) The induced EMF is:

$$\begin{aligned}\varepsilon &= -N \frac{d\phi_m}{dt} \\ &= -N \frac{d((12t^3 - 4.5t^2)(\pi R^2))}{dt} \\ &= -N\pi R^2 \frac{d(12t^3 - 4.5t^2)}{dt} \\ &= -N\pi R^2 [36t^2 - 9t].\end{aligned}$$

d.) The EMF will equal zero when  $36t^2 - 9t = 0$ . This will occur at  $t = 0$  and at  $t = 9/36 = 1/4 = .25$  seconds.

e.) How the magnetic flux changes is what governs the direction of the induced current. Although it isn't always true, in this problem the changing flux is due solely to the changing magnetic field ( $A$  and the angle between  $A$  and  $B$  are both fixed). In other words, for the just before  $t = .25$  seconds part, we need to know:

--How the external magnetic field is changing just before  $t = .25$  seconds (this will tell us if the induced magnetic field adds to or subtracts from the external magnetic field) and;

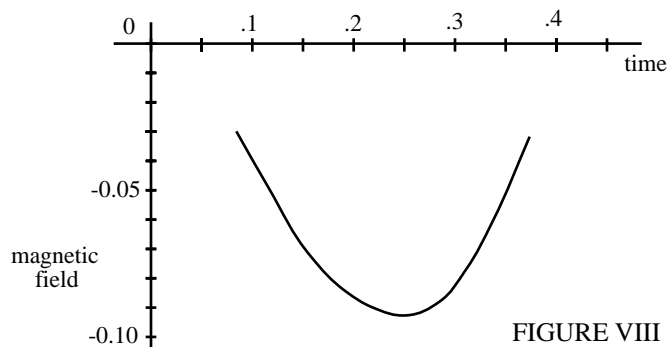
--The direction of the external magnetic field just before  $t = .25$  seconds (this tells us in which direction the addition or subtraction must occur).

To make things easier, let's begin by graphing the magnetic field function. To do so:

--We will use the magnetic field expression given in the problem for points in time around  $t = .25$  seconds, and;

--We can use the fact that the EMF is zero at  $t = .25$  seconds (that means the slope of the magnetic flux must be zero at that point in time which, in this case, means the slope of the magnetic field expression must be zero at that point in time).

--Putting it all together, we get the graph shown in the sketch. Using the graph:



i.) Just before  $t = .25$  seconds:

--From the graph, the magnetic field is negative and, hence, into the page just before  $t = .25$  seconds.

--The magnetic field is getting bigger (i.e., more negative) just before  $t = .25$  seconds.

--An increasing magnetic field (hence, magnetic flux) will induce a current that fights the increase.

--The induced magnetic field that fights an increasing external magnetic field directed into the page will itself be directed out of the page.

--The induced current that produces such an induced field will be in the counterclockwise direction. That is the direction of the induced current before  $t = .25$  seconds.

ii.) Just after  $t = .25$  seconds:

--From the graph, the magnetic field is still negative and, hence, into the page just after  $t = .25$  seconds.

--The magnetic field is getting smaller (i.e., it's proceeding back toward zero) just after  $t = .25$  seconds.

--A decreasing magnetic field (hence, magnetic flux) will induce a current that fights the decrease.

--The induced magnetic field that fights a decreasing external magnetic field directed into the page will itself be directed into of the page.

--The induced current that produces such an induced field will be in the clockwise direction. That is the direction of the induced current after  $t = .25$  seconds.

Note: The EMF at  $t = .2$  seconds is  $-.36N\pi R^2$  while the EMF at  $t = .3$  seconds is  $.54N\pi R^2$ . As the EMF and the change in flux are essentially the same, this tells us that the changes are different on either side of  $t = .25$  seconds and, hence, that the induced currents will be in different directions. This is really the only generalization we can make from the EMF information.

f.) The relationship that is important here is:

$$N \frac{d\phi_m}{dt} = -\oint \mathbf{E} \cdot d\mathbf{l},$$

where  $E$  is the electric field evaluated along a differential path-length  $d\mathbf{l}$ . In most problems, the magnitude of  $E$  is assumed to be constant and in the direction of  $d\mathbf{l}$ , so the above equation becomes:

$$N \frac{d\phi_m}{dt} = -E \oint d\mathbf{l}.$$

In this case, the integral is simply adding differential sections around a closed circular loop (i.e., the integral equals  $2\pi r$ , where  $r$  is the radius of the circular path). Using this, we get:

$$\begin{aligned} N \frac{d\phi_m}{dt} &= -E \oint d\mathbf{l} \\ \Rightarrow N\pi R^2 [36t^2 - 9t] &= -E(2\pi R) \\ \Rightarrow E &= \frac{-NR[36t^2 - 9t]}{(2)}. \end{aligned}$$

g.) N.S.L. maintains that  $F = ma$ . As the force in this case is generated by the electric field  $E$ , we can also write  $F = qE$ . Combining the two, we get  $a = qE/m$ . Substituting  $R/2$  for  $R$  in the general electric field expression derived above, we get:

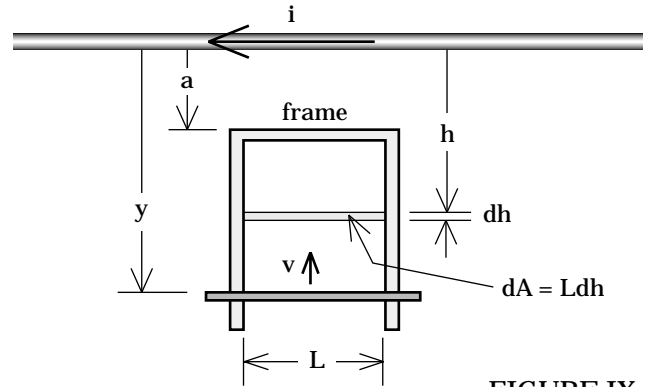
$$\begin{aligned} a &= \frac{q}{m} E \\ &= \frac{q}{m} \left[ \frac{-N(R/2)[36t^2 - 9t]}{(2)} \right] \\ &= \frac{(1.6 \times 10^{-19} \text{ coul})}{(9.1 \times 10^{-31} \text{ kg})} \left[ \frac{-(15)(.2 \text{ meters} / 2)[36(3.3 \text{ sec})^2 - 9(3.3 \text{ sec})]}{2} \right] \\ &= -4.78 \times 10^{13} \text{ m/s}^2. \end{aligned}$$

Note: The electric field at  $t = 3.33$  seconds is only  $4.14 \times 10^{-3}$  nt/C. What makes this acceleration so large is the charge to mass ratio  $q/m$ .

19.9) This problem is similar to Problem 19.3c with one big exception--the magnetic field is produced by a current-carrying wire and, hence, is not constant throughout the area encompassed by the face.

a.) We need to determine the flux through a differential area within the face, then integrate to determine the net flux at a given instant.

The magnetic field is caused by the current-carrying wire. We could derive the B-field expression using Ampere's Law if we didn't already know it. As we have already derived that expression, we will assume we know it (it is  $B_{\text{wire}} = \frac{\mu_0 i_{\text{wire}}}{2\pi h}$ , where  $h$  is the distance between the wire and the position of interest). The sketch defines the differential quantities needed to execute the flux calculation. Using them and our magnetic field expression, we get:



**FIGURE IX**

$$\begin{aligned}
 \phi_m &= \int \mathbf{B} \cdot d\mathbf{S} \\
 &= \int_{h=a}^y \mathbf{B} \, dS \cos \theta \\
 &= \int_{h=a}^y \frac{\mu_0 i_{\text{wire}}}{2\pi h} (Ldh) \cos 0^\circ \\
 &= \frac{\mu_0 i_{\text{wire}} L}{2\pi} \int_{h=a}^y \frac{dh}{h} \\
 &= \frac{\mu_0 i_{\text{wire}} L}{2\pi} \ln \left[ \frac{y}{a} \right]
 \end{aligned}$$

With this, we can use Faraday's law to determine the induced EMF:

$$\begin{aligned}
\varepsilon &= -N \frac{d\phi_m}{dt} \\
&= -N \frac{d\left(\frac{\mu_0 i_{\text{wire}} L}{2\pi} \ln\left[\frac{y}{a}\right]\right)}{dt} \\
&= -\frac{\mu_0 i_{\text{wire}} LN}{2\pi} \frac{d(\ln[y] - \ln[a])}{dt} \\
&= -\frac{\mu_0 i_{\text{wire}} LN}{2\pi} \left[\frac{1}{y} \frac{dy}{dt}\right] \\
&= -\frac{\mu_0 i_{\text{wire}} LN}{2\pi y} v_y.
\end{aligned}$$

b.) With the induced EMF expression, we can determine the current  $i$  and, from  $i\mathbf{L}\times\mathbf{B}$ , determine the force on the bar as it moves in the magnetic field. Executing all that yields:

$$\begin{aligned}
i_{\text{induced}} &= \frac{\varepsilon}{R} \\
&= \frac{\left(-\frac{\mu_0 i_{\text{wire}} LN}{2\pi y} v_y\right)}{R}
\end{aligned}$$

Newton's Second Law yields:

$$\begin{aligned}
\underline{\Sigma F}_y: \\
-|i_{\text{induced}}\mathbf{L}\times\mathbf{B}| - mg + F_{\text{you}} = ma,
\end{aligned}$$

where  $\mathbf{B}$  is the magnetic field evaluated at the bar's position (i.e., at  $y$ ). For a constant velocity,  $a = 0$ . Noting this, using  $\mathbf{B}$  due to a wire, observing that  $N = 1$ , and substituting in for  $i_{\text{induced}}$ , we have:

$$\begin{aligned}
-\left(\frac{\mu_0 i_{\text{wire}} L}{2\pi y R} v_y\right) L \left[\frac{\mu_0 i_{\text{wire}}}{2\pi y}\right] \sin 90^\circ - mg + F_{\text{you}} &= 0 \\
\Rightarrow F_{\text{you}} &= \left(\frac{\mu_0^2 i_{\text{wire}}^2 L^2}{4\pi^2 y^2 R} v_y\right) + mg.
\end{aligned}$$

19.10)

a.) The inductor-induced EMF across the primary when there is no current change in the circuit is zero.

b.) When there is a current change in the primary circuit, the inductor-induced EMF setup across the primary coil is:

$$\begin{aligned}\varepsilon_p &= -L (\Delta i/\Delta t) \\ &= -(10^{-2} \text{ H})[(0 - 8.25 \text{ A})/(.04 \text{ sec})] \\ &= 2.06 \text{ volts.}\end{aligned}$$

c.) The only voltage in the circuit after the switch is opened is that due to the induced EMF across the primary coils of the transformer. As such:

$$\begin{aligned}i_p &= \varepsilon_p/R \\ &= (2.06 \text{ v})/(80 \Omega) \\ &= .0257 \text{ amps.}\end{aligned}$$

d.) In the secondary circuit, there is no power supply. There is also no changing flux before the switch is opened. Therefore, the induced EMF across the secondary will be ZERO before the switch is opened, and the induced current will also be zero.

e.) After the switch is opened, the secondary current will be such that:

$$\begin{aligned}N_p/N_s &= i_s/i_p \\ \Rightarrow i_s &= i_p N_p/N_s \\ &= (.0257 \text{ amps})[1200/25] \\ &= 1.23 \text{ amps.}\end{aligned}$$

f.) As  $N_s < N_p$ , the transformer must be a step-down type (the secondary voltage is stepped down relative to the primary voltage).