Chapter 19
Faraday's Law Chapter Review

EQUATIONS:

- $\Phi_m = \int_S \boldsymbol{B} \bullet d\boldsymbol{S}$ [This is the most general expression for the magnetic flux Φ_m generated by a magnetic field B passing through the face of a closed path. It instructs us to define a differential surface area vector dS perpendicular to the face of the closed path, multiply that differential area by the component of B parallel to dS (that is what the dot product does), then sum all such products over the entire face via integration. THIS EXPRESSION IS ONLY USED WHEN B VARIES over the face. The units of magnetic flux are teslas·m^2, commonly called webers.]

- $\Phi_m = \boldsymbol{B} \bullet \boldsymbol{A}$ [This magnetic flux expression is used when B is constant over a path's face. Note that the expression is equal to $BA\cos\theta$, where B is the magnitude of the magnetic field, A is the total area of the face, and θ is the angle between B and A. USE THIS EXPRESSION WHENEVER YOU CAN. There is no reason to make a problem harder than it has to be by unnecessarily messing with the integral expression presented just above.]

- *EMF* [An electromotive force (EMF), symbolized by an ε in a circuit, has the unit of volts and is the factor within an electrical circuit that motivates charge to move as a current. In a typical DC circuit, the EMF is provided by a battery or power supply.]

- $\varepsilon = -N\dfrac{d\Phi_m}{dt}$ [This is Faraday's Law. It states the following: If you take a wire coil through which an external magnetic field B passes, there will be a magnetic flux Φ_m through the face of that coil (i.e., through the area bounded by the loops). If the magnetic flux through that face changes, an induced EMF will be set up that will motivate charge in the coil to flow. The size of the induced EMF is proportional to the rate of change of flux $\dfrac{d\Phi_m}{dt}$ through the face, with the proportionality constant being the opposite of the number of winds in the coil, or -N. Note that THIS EXPRESSION IS USED WHEN Φ_m VARIES IN A NON-LINEAR WAY. The expression that follows is easier to use if the change is linear.]

- $\varepsilon = -N\dfrac{\Delta\Phi_m}{\Delta t} = -N\dfrac{\Delta(BA\cos\theta)}{\Delta t}$ [This is Faraday's Law for a situation in which the change of magnetic flux is linear. Note that there are a lot of ways in which the magnetic flux can change. If, say, the magnetic field produces the change, the expression becomes $\varepsilon = -N\left(\dfrac{\Delta B}{\Delta t}\right)A\cos\theta$. Note also that the information required to determine the $\left(\dfrac{\Delta B}{\Delta t}\right)$ term can be given in at least two ways. Specifically, you could be given two magnetic field values corresponding to two specified times (example: B = 2 teslas at t = 3 seconds and B = 14 teslas at t = 5 seconds), or you could be given the rate at which B changes with time (example: 6 teslas/second). In any case, this form of Faraday's Law is easier to use than the more formal, derivative-laden version.]

- $\varepsilon = iR$ [In a coil in which an induced current is generated by an induced EMF, the relationship between the current and the EMF is still summarized by Ohm's Law.]

- $\varepsilon = -L\frac{di}{dt}$ [If you change the magnetic flux down the axis of a coil, you will get an induced EMF across the coil's leads. At some point, someone realized that a change in the externally driven current in the coil causes a magnetic field change that causes a magnetic flux change that causes an induced EMF in the coil. As such, one could write the induced EMF in terms of the rate of change of current with time, or di/dt. This expression requires a proportionality constant that is called the coil's inductance. The symbol for inductance is L, and its MKS unit is henrys, though most coils have an inductance in the millihenry (i.e., mH) range. As is the case with a resistor's resistance R or a capacitor's capacitance C, an inductor's inductance L is the parameter that identifies the size of the entity. Note that due to this, coils are often referred to as inductors when in an electrical circuit. Other common colloquial terms are choke and solenoid.]

- $i(t) = i_o\left(1 - e^{-(R/L)t}\right)$ [This is the solution to Kirchoff's Laws written for an RL circuit. YOU WILL NEVER USE THIS EXPRESSION. It has been included here because it mathematically highlights the fact that currents in RL circuits rise slowly FROM ZERO (versus immediately jumping to maximum current as in a straight resistor or RC circuit).]

- $\tau = \left(\frac{L}{R}\right)$ [One time constant τ for an RL circuit is the amount of time it takes after power is supplied for the current in the RL circuit to rise to 63% of its maximum. Just as was the case with capacitors, two time constants (i.e., 2τ) is associated with 87%. Note that this function is reciprocal. That is, it will take one time constant for an established current in an RL circuit to drop 63% of its original value (i.e., down to 37% current) when power is removed from the circuit.]

- $Energy\ stored = \frac{1}{2}Li^2$ [A current carrying inductor stores energy in the magnetic field that resides down its axis. The amount of energy is equal to $\frac{1}{2}Li^2$. Note that this is also the amount of energy required to get i's worth of current flowing through the coil in the first place, assuming no energy loss occurs in the process.]

- If $N_s > N_p$, then $\varepsilon_s > \varepsilon_p$ and $i_s < i_p$ [If the turns-ratio of a transformer is such that the number of turns N_s in the secondary coil is greater than the number of turns N_p in the primary coil, then induced EMF ε_s in the secondary will be greater than the EMF ε_p in the primary, and the current i_s in the secondary will be less than the current i_p in the primary. As the voltage increases between the primary and the secondary in this case, such transformers are called step up transformers. If the turns ratio had been the other way around, the transformer would have been called a step down transformer.]

- $\varepsilon_{induced} = NBav$ [DO NOT MEMORIZE THIS RELATIONSHIP. It is the derived expression for the EMF generated in a coil of width a and winds N as the coil is pulled from a constant magnetic field B with velocity v. It has been included here to remind you that motional EMF's are important, and that you should understand the theory well enough to be able to DERIVE the above stated expression. You should also be able to derive an expression for the current generated in the coil, and an expression for the force on the coil due to the induced current's interaction with the external magnetic field.]

- $N\frac{d\Phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l}$ [An induced EMF that is generated in a coil must be associated with an electric field (after all, an electric field is what makes charge move in a wire). The problem is that previously, we have related electric fields and electrical potential differences by the relationship $\Delta V = -\int \mathbf{E} \bullet d\mathbf{l}$. A difficulty shows itself when we try to use this relationship in conjunction with the closed path Faraday's Law requires (remember, Faraday's Law deals with the induced EMF set up around a closed path due to the presence of a changing magnetic flux). In such cases, ΔV should be zero as the electrical potential difference between a point and itself is zero. As that can't be the case (otherwise, $-\oint \mathbf{E} \bullet d\mathbf{l}$ would also be zero), there must be something wrong with our reasoning. The problem goes away when we realize that a changing magnetic flux does not produce a conservative force field. As such, there can be no potential energy function assigned to the force field, and the concept of ΔV becomes nonsense. The relationship that does exist between an electric field E that has been induced around a closed path due to the presence of an EMF inducing, changing magnetic flux, is $N\frac{d\Phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l}$.]

<u>COMMENTS, HINTS, and THINGS to be aware of</u>:

- The concept of an EMF is not new. When you put a voltmeter across the terminals of a power supply, the voltage measured is called the terminal voltage. Because the power supply will have an internal resistance that causes a voltage drop when current is drawn from the source, the terminal voltage is not equal to the power supply's EMF. It is, instead, equal to $\varepsilon - ir_i$, where ε is the source's EMF, i is the current being drawn from the source, and r_i is the internal resistance within the source.

- If, AT A GIVEN INSTANT, the magnetic field is the same everywhere across the face of a bounded area, even if the magnetic field function is changing with time, you can use B·A to determine the general expression for the magnetic flux AT THAT POINT IN TIME. Use the integral expression ONLY if the magnetic field is different from point to point across the face AT A GIVEN INSTANT IN TIME. In short, don't use the integral form of the magnetic flux expression if it isn't absolutely necessary.

- Don't use the integral form of Faraday's Law if it isn't absolutely necessary. The Δ version works just fine, assuming the magnetic flux is changing linearly.

- An induced EMF generated by a coil leaving a magnetic field (a motional EMF problem) ALWAYS generates a force that OPPOSES the motion. That is, if the coil is made to leave the field, the induced current in the coil will interact with the external magnetic field (F = iLxB) producing a force on the wire that is opposite to the direction of the motion. The same is true if the coil enters a magnetic field.

- Eddy currents refer to the swirling motion of induced charge flow in a solid, metallic plate (the plate doesn't have to be magnetizable--aluminum works just fine) as part of the plate is forced into or out of a magnetic field. The interaction of these induced currents and the external magnetic field produces a force that ALWAYS slows the plate's motion. This is the basis of an eddy current brake.

- Whenever any material experiences a change of magnetic flux, whether the material be a conductor or insulator, there will ALWAYS be an induced electric field set up in the material such that $\frac{d\Phi_m}{dt} = -\oint \boldsymbol{E} \bullet d\boldsymbol{l}$. If the material happens to be a conductor, charge will move and you will get current. If the material is an insulator, no charge will move (ah say, it's an insulator), though the field will nevertheless be there.

- If you are asked to use Kirchoff's Laws on a circuit in which there exists at least one inductor, don't be put off. Just as the voltage drop across a resistor is iR and the voltage drop across a capacitor is q/C, the voltage drop across an inductor is $L\frac{di}{dt}$. Note, though, that because an inductor is essentially wire wrapped as a coil, there is also resistor-like resistance associated with it. That means there can also be a non-zero voltage drop across an inductor equal to ir_L, where i is the current through the inductor and r_L is the resistor-like resistance inherent within the wire itself. This voltage drop is sometimes ignored because it is often small, but don't be thrown off if you run into a problem that includes it.