Chapter 18

MAGNETIC FIELDS

A.) A Small Matter of Special Relativity:

1.) Assume we have a particle of charge q moving with an initial velocity v_q parallel to a current-carrying wire as shown in Figure 18.1.

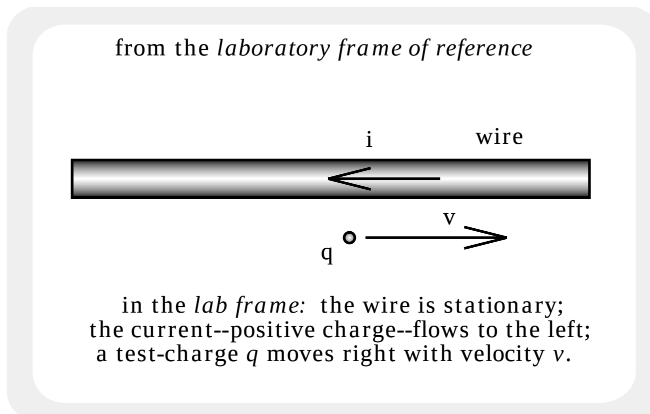

FIGURE 18.1

a.) Consider the situation from the perspective of the laboratory frame of reference (i.e., the frame in which you and I sit and in which the wire is motionless):

i.) The positive charges (the protons) are fixed in the wire while the negative charges (the electrons) have some non-zero average velocity v_e.

ii.) There are as many electrons as protons in the wire before the current begins (i.e., the wire is electrically neutral).

iii.) As many electrons leave the wire as come onto the wire while current flows. As such, the wire is perceived to be electrically neutral even when current is flowing.

b.) Consider now the situation from q's frame of reference:

Note: From this frame of reference, the charge q will be stationary while everything else is moving around it.

i.) In q's frame of reference (see Figure 18.2), the wire and all positive charges (protons) will move to the left with velocity v_q. Meanwhile, negative charges (electrons) will move to the left with velocity $v_q - v_e$ (we are assuming $v_q > v_e$). The action is summarized in Figure 18.2.

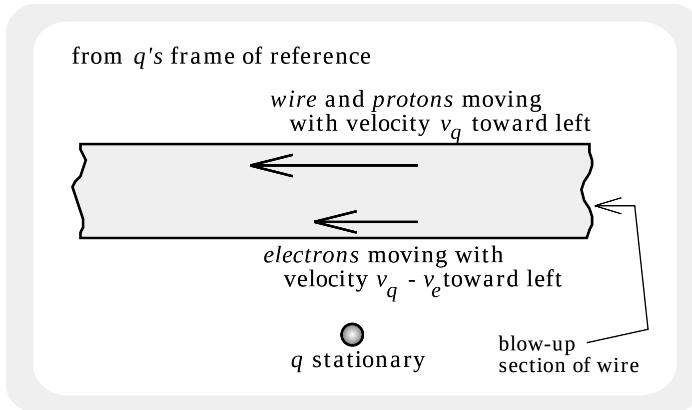

FIGURE 18.2

ii.) Notice that the protons move faster than electrons from this perspective.

2.) Einstein's Theory of Relativity suggests that when one object passes a second object, the second object will appear to the first to have contracted in length. Called "length contraction," the phenomenon is immediately evident only at very high speeds but does occur microscopically at low speeds.

3.) Because all of the charge in the wire moves relative to q's frame of reference, the distances between the charges should appear to be closer (relativistic length contraction) than would otherwise have been the case if viewed from the lab frame. What's more, the protons will appear to be more tightly packed because they are moving faster than the electrons (see Figure 18.3).

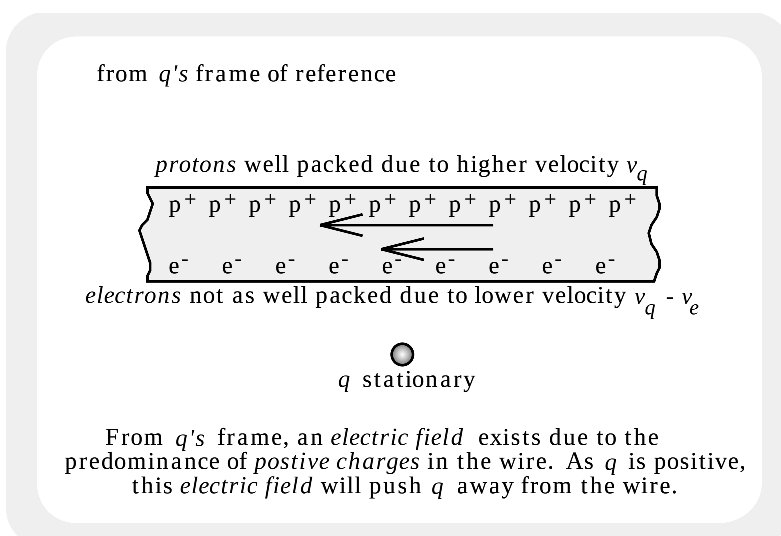

FIGURE 18.3

a.) In other words, the wire will appear to have more protons than electrons on it. That means charge q will perceive an electric field due to the predominance of positive charge, and that electric field will motivate q to accelerate away from the wire.

b.) If we set up an experiment in which a positive charge is made to move parallel to a current-carrying wire and opposite to the current's direction, we will observe a force on q pushing it away from the wire. The force is due to the relativistic effect we have been discussing, but observers in the previous century did not know that (Einstein's Theory of Relativity wasn't published until 1905). Working strictly from empirical observation, they assumed there must exist a new kind of force--a magnetic force--acting on the moving charge. The theory developed on behalf of that belief is today called "the classical theory of magnetism." It is the subject we are about to consider.

B.) Some Early Observations:

There are a number of observed phenomena that led early scientists to formulate the classical theory of magnetism. In no particular order:

1.) When suspended, certain metallic ores are found to have the peculiar ability to orient themselves north/south. They evidently align themselves with some sort of field, a field that in the early days of "modern science" was eventually called a magnetic field.

a.) In experimenting with a piece of such ore, it has been observed that this north/south orientation is always the same. That is, the same face always aligns itself to the north while the opposite face always aligns to the south. To distinguish between the two, one is called "the North Seeking Magnetic Pole N" and the other is called "the South Seeking Magnetic Pole S."

These observations were, in early times, the basis for what is today called a compass.

2.) When a compass is put in the vicinity of a "magnetized" piece of metallic ore, the compass is found to point in different directions at different places.

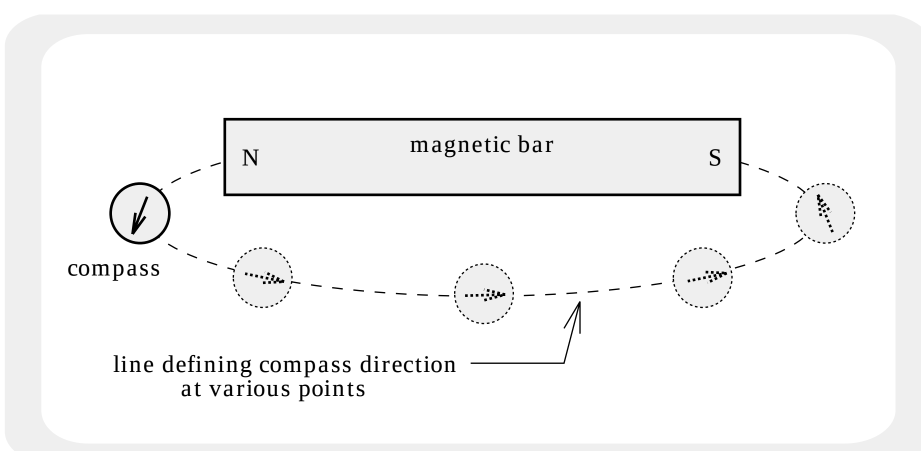

FIGURE 18.4a

a.) Following the needle direction for the various sample points shown in Figure 18.4a on the previous page, a line can be drawn. Doing this for a number of different positions around the bar allows us to sketch what are called "magnetic field lines" (see Figure 18.4b).

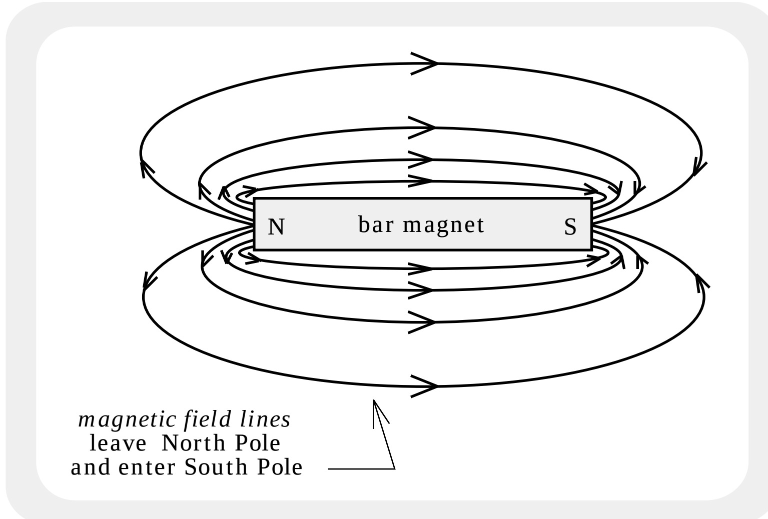

FIGURE 18.4b

b.) Magnetic field lines are similar to electric field lines in the sense that where the field lines are close together, the field is said to be large, but:

c.) Magnetic field lines are DIFFERENT from electric field lines in one very important way. The direction of an electric field line is defined as the direction a positive test charge will accelerate if released in the electric field. In other words, electric fields are really nothing more than slightly modified force-field lines ($E = F/q$).

The direction of a magnetic field line is defined as the direction a compass will point if a magnetic field is present. As will be shown shortly, magnetic fields are NOT modified force fields (though they are distantly related to force).

d.) A constant magnetic field is denoted by field lines that are equidistant and parallel as shown in Figure 18.4c.

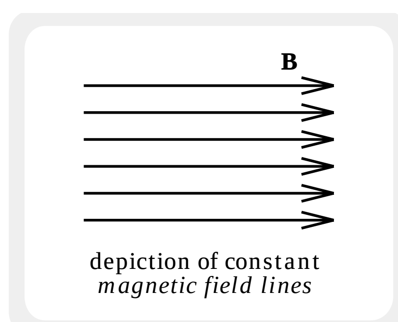

FIGURE 18.4c

3.) The strength of a magnetic field coupled with the direction of the magnetic field is combined together to define the magnetic field vector B. More will be said shortly about B, its relationship to the force on a charge moving in a magnetic field, and its units.

4.) While experimenting with electrical circuits in 1820, a man named Oersted observed that when a compass was placed near a current-carrying wire, the compass responds. Experimenting further:

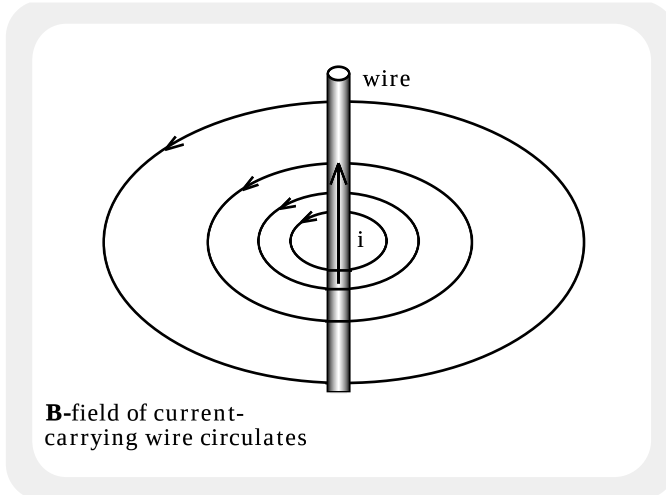

FIGURE 18.5a

a.) Oersted found that magnetic field lines CIRCLE around a current-carrying wire (see Figure 18.5a).

Notice that the direction of a current-produced magnetic field can be determined by using the following "weird" right-hand rule (from here on, this rule will be termed the right-thumb rule): Position the thumb of the right hand so that it follows the direction of current flow--the direction the fingers curl is the direction of the magnetic field's circulation around the wire (Figure 18.5b).

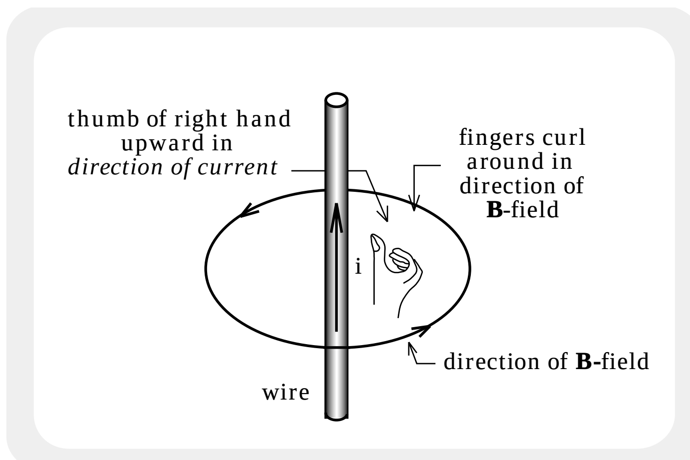

FIGURE 18.5b

b.) Oersted concluded that magnetic fields are somehow related to CHARGE IN MOTION.

5.) From experimentation, it has been observed that if a positive charge q is placed in a magnetic field B:

a.) The charge will feel NO FORCE due to the presence of the magnetic field if the charge is stationary (see Figure 18.6a);

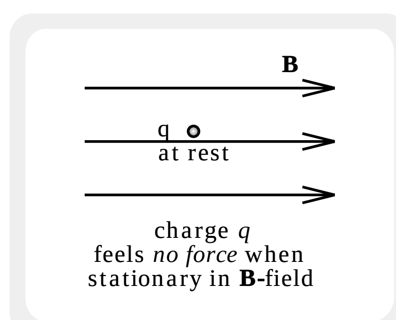

FIGURE 18.6a

b.) The charge will feel NO FORCE due to the presence of the magnetic field if the charge is moving with velocity v along the magnetic field lines (Figure 18.6b);

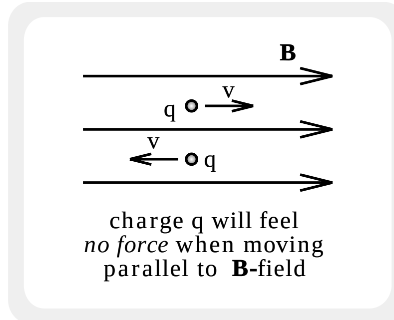

FIGURE 18.6b

c.) The charge WILL FEEL A FORCE due to the presence of the magnetic field if the charge's velocity vector is oriented at any angle other than zero or 180^{o} relative to the magnetic field vector B (see Figure 18.6c).

Furthermore, the direction of the force will be perpendicular to the plane defined by the magnetic field vector and the velocity vector. In the case shown in Figure 18.6c, that direction is perpendicular to the plane of the page.

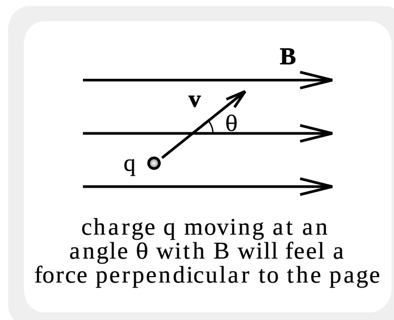

FIGURE 18.6c

d.) The charge will feel a maximum force if the velocity vector v is perpendicular to the magnetic field vector B.

e.) Putting all of the above information together, the experimentally determined relationship that exists between the magnitude of the force F_B on a charge q moving with velocity vector v at an angle θ with the magnetic field B is:

$$F_B = q|v||B| \sin\theta.$$

This is the magnitude of a cross product, which implies:

$$F_B = q\, v \times B.$$

Note: The direction of a cross product is always perpendicular to the plane defined by the two vectors being crossed. That is exactly the direction we needed for our magnetic force vector.

f.) IMPORTANT CONCLUSION: Magnetic fields are centripetal in nature--they change the direction of charged bodies but do not make them speed up or slow down. Additionally, the relationship between magnetic fields as defined (i.e., having a direction determined by the way a compass orients itself) and magnetic forces as observed experimentally is not an obvious one. More about this later.

C.) Continuing With Observations--The Bar Magnet:

1.) If a magnetic field is created by charges in motion, what kind of motion creates the magnetic field in an apparently motionless bar magnet? Possibilities: Electrons confined to the atom are constantly in motion-- they both orbit about the nucleus and spin about its axis. Let's consider both:

a.) <u>Orbital Motion</u>: While the orbital motion of electrons around the nucleus surely produces a magnetic field, the direction of an electron's motion will be "this way" as much as "that way" (electrons travel around the atom at speeds upward of 150,000 miles per second). Consequently, the net magnetic field produced by electron orbital motion is, on average, zero.

b.) <u>Spinning On Axis</u>: Electron spin also produces a magnetic field. Due to quantum mechanical effects, electrons spin in only one of two directions. These directions are usually referred to as "spin up" and "spin down." In most elements, there are as many electrons spinning up as spinning down which means the net magnetic field generated by all the spinning electrons is zero.

i.) There are some elements whose number of electrons spinning in one direction is noticeably different from the number spinning in the opposite direction. Iron, for instance, has six more electrons spinning one way than the other. As a consequence, the net magnetic field due to electron spin in an iron atom is not zero. Put another way, every iron atom is a mini-magnet unto itself.

ii.) Elements that exhibit this magnetic characteristic are called ferromagnetic materials. The most common are iron, nickel, and cobalt.

2.) Ferromagnetic materials do not always exhibit magnetic effects. Iron nails, for instance, do not usually attract or repulse one another as would be expected if they were magnetized. The question is, "Why?"

a.) Take a structure made of iron (a steel bolt, for example). Within it, there exist microscopic sections called domains. A domain is a volume in which each atom has aligned its magnetic field in the same direction as all the other atoms in the section.

b.) Figure 18.7a (next page) shows a side-view blow-up of the domains that reside on the face of a piece of iron. Notice that each domain has its magnetic field in some arbitrary direction. Because none of the domain-fields are aligned, the net (read this average) magnetic field on the face is essentially zero.

This is an example of a ferromagnetic material that does not appear to be magnetized.

c.) If the bolt is placed in a relatively strong magnetic field, the domains will align themselves with the external field and, in doing so, will align themselves with one another (see Figure 18.7b). In that case, each face of the bolt will either be a North Pole or South Pole. That is, we end up with a magnetized piece of iron.

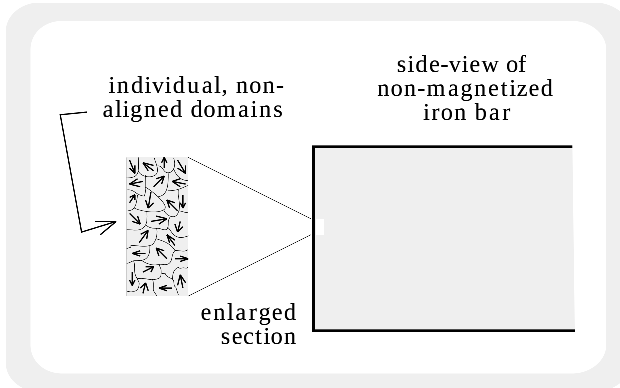

FIGURE 18.7a

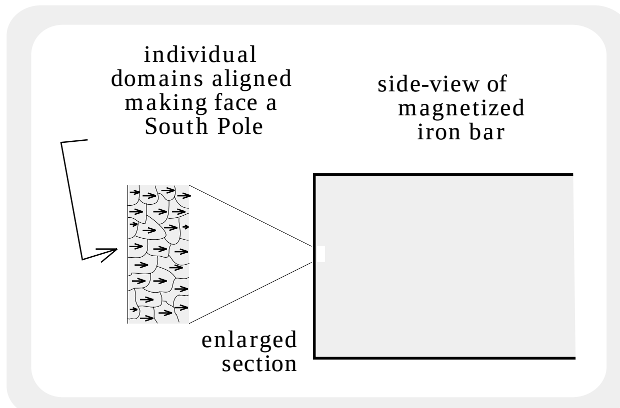

FIGURE 18.7b

D.) Observations--The Earth's Magnetic Field:

1.) Through experimentation, it was found that North Seeking Magnetic Poles always attract South Seeking Magnetic Poles. Like poles (i.e., N-N or S-S poles) repulse. One of the consequences of this is the peculiar situation we have with respect to the earth's magnetic field.

2.) By definition, the North Seeking Magnetic Pole of a compass points toward the northern geographic region of the earth. But if North Seeking Magnetic Poles are attracted to South Seeking Magnetic Poles, there must exist a South Magnetic Pole in the northern geographic hemisphere. In fact, that is exactly the case. The earth's magnetic field lines leave Antarctica and enter the Arctic (they actually enter in the Hudson Bay region--see Figure 18.8a on the next page for the theoretical distribution of magnetic field lines around the earth).

3.) Solar winds are streams of high energy sub-atomic particles that are constantly being emitted by the sun. Due to these solar winds, the earth's magnetic field lines are actually compressed in toward the earth on the earth's sun-side while being extruded out away from the earth on the earth's dark side. See Figure 18.8b.

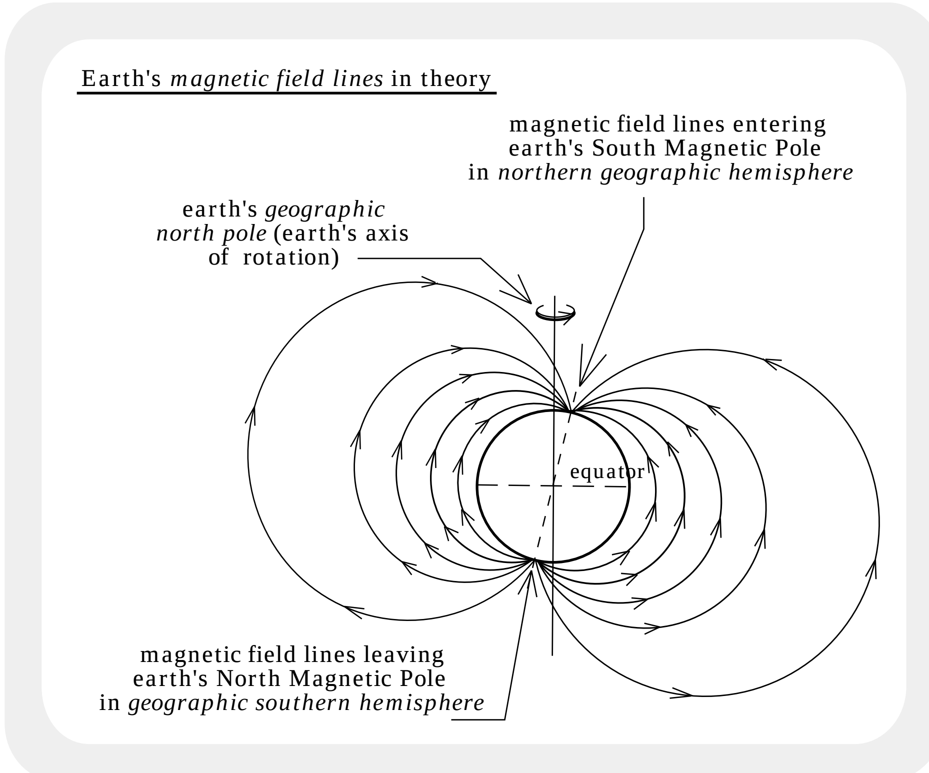

FIGURE 18.8a

4.) The earth's magnetic field is believed to be caused by motion of molten iron at the earth's core. By looking at core samples of the earth's geological history over long periods of time, it has been found that the earth's magnetic field changes direction periodically (sometime between 200,000 to 400,000 years per cycle). Although scientists are not completely sure why, the current theory is that long-period oscillatory variations in the motion of the earth's iron-rich molten interior create this effect.

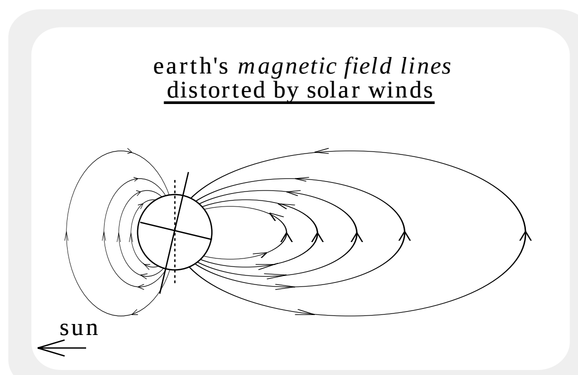

FIGURE 18.8b

E.) Approach for Determining a Magnetic Field--Ampere's Law:

1.) We have made our observations; now it is time to examine the math that has grown up around those observations.

Theoreticians have developed a number of ways for determining magnetic field functions for current configurations. In the 1820's, one of these approaches was created by Andre Ampere.

2.) Ampere's Law states the following:

a.) Define a closed path in a region in which a B-field exists.

b.) Define a differential length-vector dl over a differential section of the path.

c.) Determine the dot product between the magnetic field B evaluated at the differential section and the vector dl.

d.) Sum all such dot products around the closed path.

e.) That sum will always be proportional to the amount of current that passes through the face of the path (the face of the path is the area enclosed within the boundaries of the path).

f.) Putting this all in mathematical terms, we get:

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_o i_{thru} \ ,$$

where μ_o is the proportionality constant called the permeability of a vacuum and is equal to $4\pi x10^{-7}$ teslas·meter/amp (i.e., $1.26x10^{-6}$ T·m/A).

Note 1: The integral symbol $\oint$ denotes an integration around a closed path.

Note 2: The term i_{thru} is used to denote the current passing through the face of the defined Amperian path.

3.) As a point of order, and because it puts things in perspective, it should be noted that Ampere's Law does for magnetic fields what Gauss's Law did for electric fields. That is:

a.) In Gauss's Law, we defined an arbitrary closed surface that had the right geometry and symmetry; in Ampere's Law we define an arbitrary closed path that has the right geometry and symmetry.

b.) In Gauss's Law, we defined a differential surface area vector dS at an arbitrary position on the surface. In Ampere's Law, we define a differential length vector dl at an arbitrary position on the path.

c.) In Gauss's Law, we dotted the unknown electric field function E (assumed to be evaluated at dS) into the differential surface area vector dS. In Ampere's Law, we dot the unknown magnetic field function B (assumed to be evaluated at dl) into the differential length vector dl.

d.) In Gauss's Law, we summed all the E·dS quantities over the entire surface (we used a surface integral $\int_S$ to do this) to determine the total electric flux through the surface. In Ampere's Law, we sum all the B·dl quantities over the entire path (we use a line integral $\oint$ to do this) to determine the total magnetic circulation around the path.

e.) In Gauss's Law, the electric flux through the Gaussian surface is proportional to the charge enclosed within the surface. In Ampere's Law, the magnetic circulation around the Amperian path is proportional to the current passing through the path's face.

F.) Ampere's Law and a Straight, Infinite, Current-Carrying Wire:

1.) Consider an infinitely long current-carrying wire:

a.) As mentioned above, Oersted found that a current-carrying wire produces a magnetic field that circulates around the wire according to the right-thumb rule (thumb of right hand in direction of current; fingers curl in direction of the B-field).

b.) To determine the magnitude of the magnetic field, we will use Ampere's Law.

2.) The geometry in which Ampere's Law is most easily used is one in which the magnitude of the magnetic field is the same at every point along the arbitrarily defined path. This happens to be just such a case.

a.) Due to symmetry, the Amperian path of choice here is a circle of arbitrary radius r centered on the wire. Figure 18.9a shows just such a path while Figure 18.9b views the situation looking along the line of the wire.

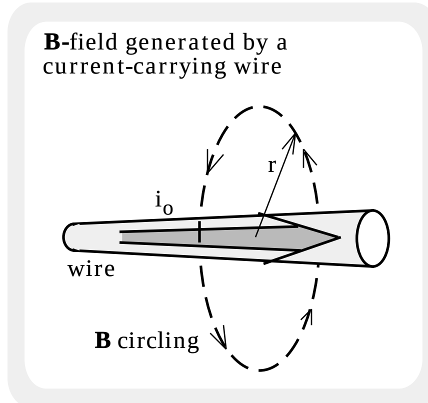

FIGURE 18.9a

b.) Just as was the case with Gauss's Law, Ampere's Law has two parts to deal with: the right-hand side of the equation and the left-hand side of the equation. We will deal with both separately, then put it all together.

c.) The left-hand side of Ampere's Law is:

$$\oint \mathbf{B} \bullet d\mathbf{l},$$

or the integral sum of the dot product of the magnetic field vector (evaluated at a point on the Amperian path) and the differential displacement along the path at that point.

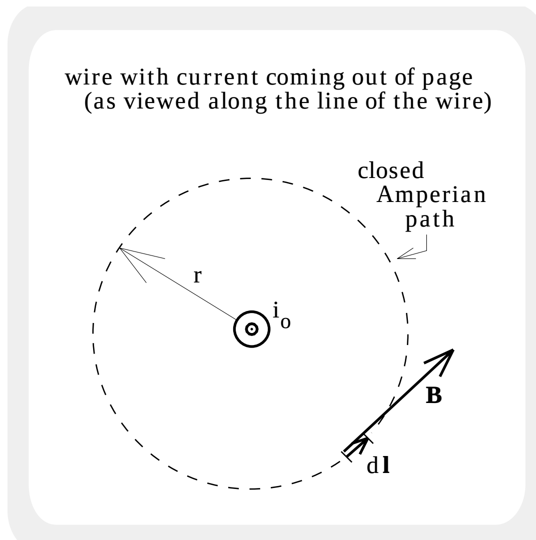

FIGURE 18.9b

i.) As the vector dl is oriented in the same direction as the vector B (the direction of dl was defined that way), and as the path was chosen so that the magnitude of B would be constant at every point along the path, the dot product can be treated as:

$$\begin{aligned}\oint B \bullet dl &= \oint B(dl)\cos 0^\circ \\ &= B\oint dl \\ &= B(2\pi r).\end{aligned}$$

d.) The right-hand side of Ampere's Law requires that we determine the amount of current that breaks through the face of the area defined by the path.

i.) In this case, the total current breaking through the face is simply the current through the wire, or i_o.

3.) Putting everything together and presenting it the way you will be expected to present it on a test, we write:

$$\oint B \bullet dl = \mu_o i_{thru}$$

$$\Rightarrow \quad \oint B(dl)\cos 0^o = \mu_o i_{thru}$$

$$\Rightarrow \quad B\oint dl = \mu_o i_o$$

$$\Rightarrow \quad B(2\pi r) = \mu_o i_o$$

$$\Rightarrow \quad B = \frac{\mu_o i_o}{2\pi r}.$$

G.) A Note About the Vector Nature of Magnetic Fields:

1.) Consider the two wires shown in Figure 18.10a. One carries a .5 amp current out of the page (the circle with dot depicts an arrowhead coming out of the page) while the other carries a .25 amp current into the page (the circle with cross depicts an arrowhead going into the page). Assuming the distances are as shown in the sketch, what is the net magnetic field generated at Point P?

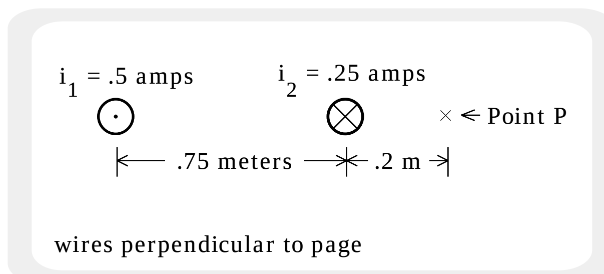

FIGURE 18.10a

a.) As for the B-field directions (see Figure 18.10b for the bottom line):

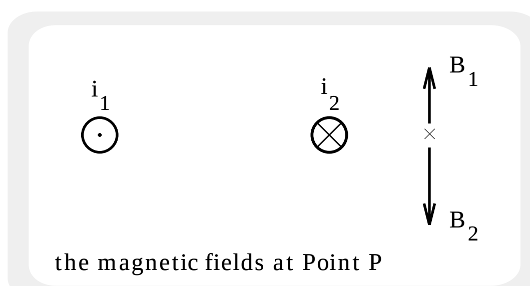

FIGURE 18.10b

i.) Current i_1 produces a B-field whose direction at Point P is toward the top of the page (+j direction) while current i_2 produces a B-field whose direction at Point P is toward the bottom of the page (-j direction). Note that both are tangent to circles centered on their respective current carrying wires, just as suggested by the right thumb rule and the Figure 18.9b.

b.) The magnitude of the magnetic fields are:

i.) For current i_1:

$$B_1 = \mu_o i_1/2\pi r_1$$
$$= [(1.26\text{x}10^{-6}\ \text{kg}\cdot\text{m/coul}^2)(.5\ \text{A})]/[2\pi(.95\ \text{m})]$$
$$= 1.06\text{x}10^{-7}\ \text{teslas.}$$

ii.) For current i_2:

$$B_2 = \mu_o i_2/2\pi r_2$$
$$= [(1.26\text{x}10^{-6}\ \text{kg}\cdot\text{m/coul}^2)(.25\ \text{A})]/[2\pi(.2\ \text{m})]$$
$$= 2.51\text{x}10^{-7}\ \text{teslas.}$$

c.) The net B-field will be the vector sum of the two fields:

$$B_{net} = B_1 + B_2$$
$$= (1.06\text{x}10^{-7}\ \text{teslas})(+j) + (2.51\ \text{x}10^{-7}\ \text{teslas})(-j)$$
$$= (1.06\text{x}10^{-7}\ \text{teslas})(j) - (2.51\ \text{x}10^{-7}\ \text{teslas})(j)$$
$$= (-1.45\text{x}10^{-7}\ \text{teslas})\ j.$$

Note: In the third line, the negative sign belongs to the second term's unit vector (i_2's B-field direction was toward the bottom of the page). It has been brought out in front of the magnitude value to make it easier to track.

2.) For the wire configuration shown in Figure 18.11a, what is the net magnetic field generated at Point P?

a.) As for the B-field directions (see Figure 18.11b on the next page for the bottom line):

i.) Current i_1 produces a B-field whose direction at Point P is toward the top of the page (+j direction).

ii.) Current i_2 produces a B-field whose direction at Point P is toward the bottom-left of the page at a 45^o angle to the left (i.e., in the -i + (-j) direction).

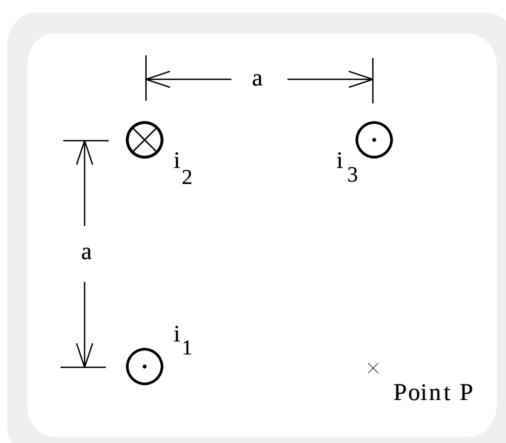

FIGURE 18.11a

ii.) Current i_3 produces a B-field whose direction at Point P is toward the right of the page (+i direction).

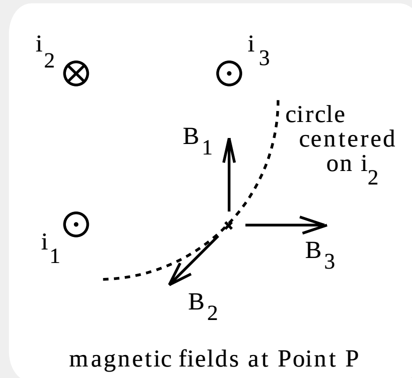

FIGURE 18.11b

b.) The magnitudes of the magnetic fields are:

i.) For current i_1:

$$B_1 = \mu_o i_1/(2\pi r_1) = \mu_o i_1/(2\pi a).$$

ii.) For current i_2:

$$B_2 = \mu_o i_2/(2\pi r_2) = \mu_o i_2/[2\pi(\sqrt{2}\,a)].$$

iii.) For current i_3:

$$B_3 = \mu_o i_3/2\pi r_3 = \mu_o i_3/(2\pi a).$$

c.) Noting that B_2 must be broken into its components, the net B-field will be the vector sum of the three individual fields, or:

$$B_{net} = B_1 + B_2 + B_3$$

$$= \left[\left(\frac{\mu_o i_1}{2\pi a}\right)(+\mathbf{j})\right] + \left[\left(\frac{\mu_o i_2}{2\pi(\sqrt{2}a)}\right)(\cos 45^o)(-\mathbf{i}) + \left(\frac{\mu_o i_2}{2\pi(\sqrt{2}a)}\right)(\sin 45^o)(-\mathbf{j})\right] + \left[\left(\frac{\mu_o i_3}{2\pi a}\right)(+\mathbf{i})\right]$$

$$\Rightarrow \quad \mathbf{B}_{net} = \frac{\mu_o}{2\pi a}\left[\left[(i_3) - \left(\frac{i_2}{\sqrt{2}}\right)\cos 45^o\right]\mathbf{i} + \left[(i_1) - \left(\frac{i_2}{\sqrt{2}}\right)\sin 45^o\right]\mathbf{j}\right].$$

H.) Ampere's Law and the B-Field Produced in an Infinitely Long Coil:

1.) Consider a very long (read this infinitely long) wire coil having n winds-per-unit-length (see Figure 18.12a). If we spread the coils out in order to see how the current-produced magnetic field acts at various places within the geometry, we will note some interesting things. Specifically:

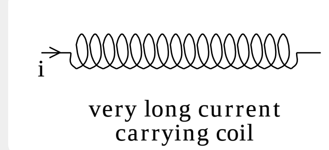

FIGURE 18.12a

a.) The net B-field perpendicular to the coil's axis adds to zero. That is, the B-field generated by the charge flow in the upper-left-side section of wire (see Figure 18.12b) generates a magnetic field perpendicular to the coil's axis that is equal and opposite to the magnetic field generated by charge flow in the upper-right-side section of wire. The two fields simply add to zero.

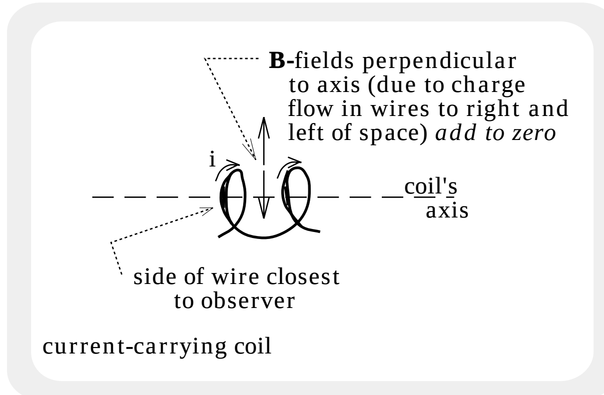

FIGURE 18.12b

b.) For an infinitely long coil, the B-field parallel to the axis and outside the coil will be zero, as the magnetic field lines along the central axis will never leave that axis (again, this is for an infinite coil).

Note: To a very good approximation, this is true even for finite-length coils (see Figure 18.12c). The reasoning is as follows: Assuming we are a fair distance from the coil, the B-field generated by charge flow on the top side of a given coil is almost equal and opposite to the B-field generated by charge flow on the bottom side of the coil (see Figure 18.12d). A similar situation exists for the net field below the wire and, in fact, for all points outside the perimeter of the coil.

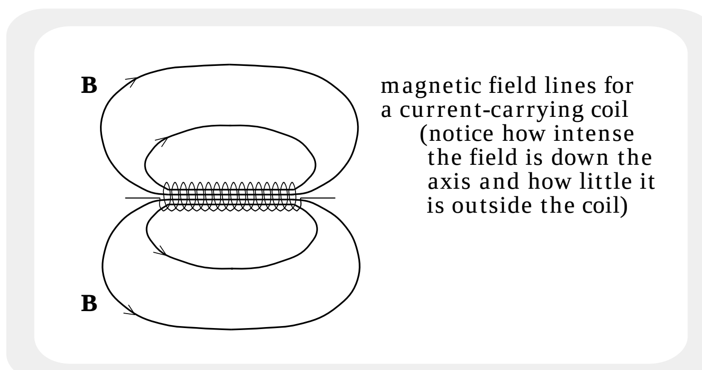

FIGURE 18.12c

c.) For an infinitely long coil, the net B-field is all down the axis of the coil.

2.) To determine the Amperian path:

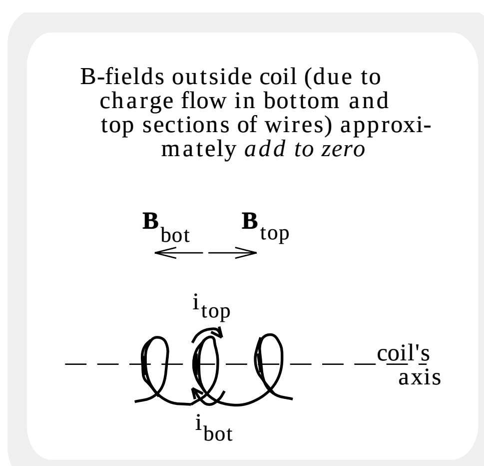

FIGURE 18.12d

a.) In GAUSS'S LAW, the trick was to find a closed surface through which the electric field was either a constant (i.e., on the cylindrical part of a Gaussian cylinder), zero (i.e., inside a conductor), or such that E·dS was zero (i.e., on the flat end of a cylindrical Gaussian surface).

b.) Ampere's Law is similar. We want a path upon which the magnetic field is either a constant, zero, or such that over the path, B·dl is zero.

c.) The path that works for this case is a rectangle, as shown in Figure 18.13.

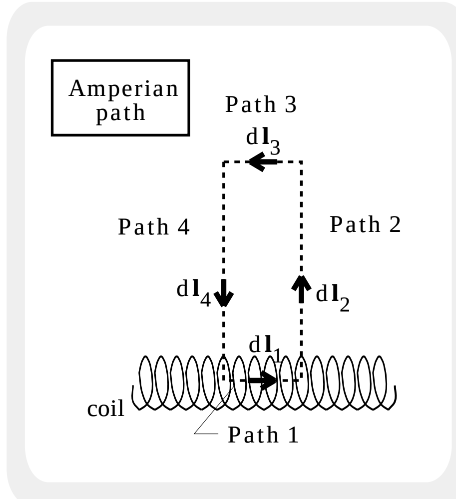

FIGURE 18.13

3.) With our Amperian path defined, we are ready to use Ampere's Law.

a.) The first thing to notice is that the path has four sides and, hence, will require four dot products (each path section is shown on the sketch for convenience).

i.) As there is no magnetic field perpendicular to the coil, the magnetic field along Paths 2 and 4 is zero. As such, the integrals associated with those paths are zero.

ii.) As there is no magnetic field outside the infinitely long coil, the magnetic field along Path 3 is also zero.

Note: Even if we had been working with a finite coil, the magnetic field outside the coil is very small, especially if we allow the path to extend out a considerable way from the coil's axis.

Bottom line: B·dl along Path 3 will be zero (if not exactly, then to a good approximation) whether the coil is infinitely long or not.

b.) The total current passing through the area bounded by the path is equal to the current in one wire times the number of wires inside the Amperian path (see Figure 18.14 on the next page).

i.) The number of wires inside the Amperian path will equal the number of wires per unit length--call this n--multiplied by the length of the path defined in the sketch (next page) as L_1, or nL_1.

ii.) That means that the net current passing through the boundary defined by the path will be:

$$i_{thru} = (\text{n winds/meter})(L_1 \text{ meters})(i_o \text{ amps/wind}) = nL_1 i_o.$$

4.) Putting it all together, we can write:

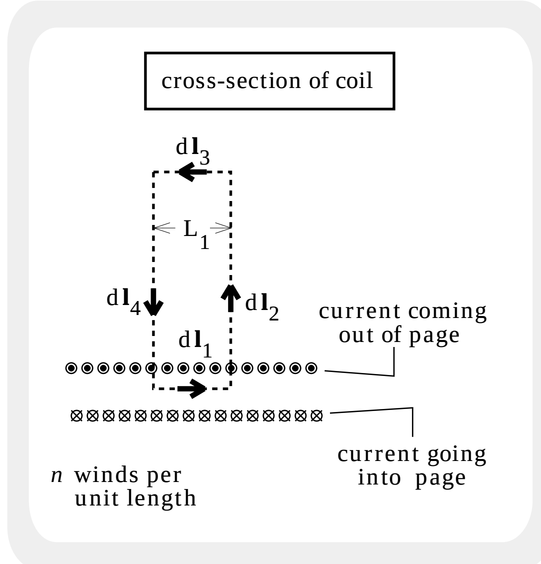

FIGURE 18.14

$$\oint B \bullet dl = \mu_o i_{thru}$$

$$\Rightarrow \quad \int_{L_1} B \bullet dl_1 + \int_{L_2} B \bullet dl_2 + \int_{L_3} B \bullet dl_3 + \int_{L_4} B \bullet dl_4 = \mu_o(nL_1 i_o)$$

$$\Rightarrow \quad B\int_{L_1} dl_1 + (0) + (0) + (0) = \mu_o(nL_1 i_o)$$

$$\Rightarrow \quad BL_1 = \mu_o(nL_1 i_o)$$

$$\Rightarrow \quad B = \mu_o n i_o.$$

I.) The Law of Biot Savart:

1.) Ampere's Law is useful when there is symmetry (i.e., when the magnitude of B is a constant over an Amperian path), or when the magnitude of B is a constant over part of an Amperian path while B·dl is zero over the rest of the path. When a more general situation is at hand, a more general approach is needed. In such cases, we turn to the Law of Biot Savart.

2.) The Law of Biot Savart states that a differential section of wire will produce a differential magnetic field dB at some point near the wire such that:

$$d\mathbf{B} = \frac{\mu_o i}{4\pi}\frac{d\mathbf{l} x \hat{\mathbf{r}}}{r^2}.$$

In this expression:

a.) dB is the differential magnetic field vector due to the current in dl as evaluated at the point of interest (see Figure 18.15);

b.) i is the current in the wire;

c.) dl is a vector whose magnitude is equal to the length of the differential section of wire and whose direction is in the direction of the current;

d.) r is the magnitude of a position vector drawn <u>from</u> the differential section dl <u>to</u> the point of interest (call this Point P); and

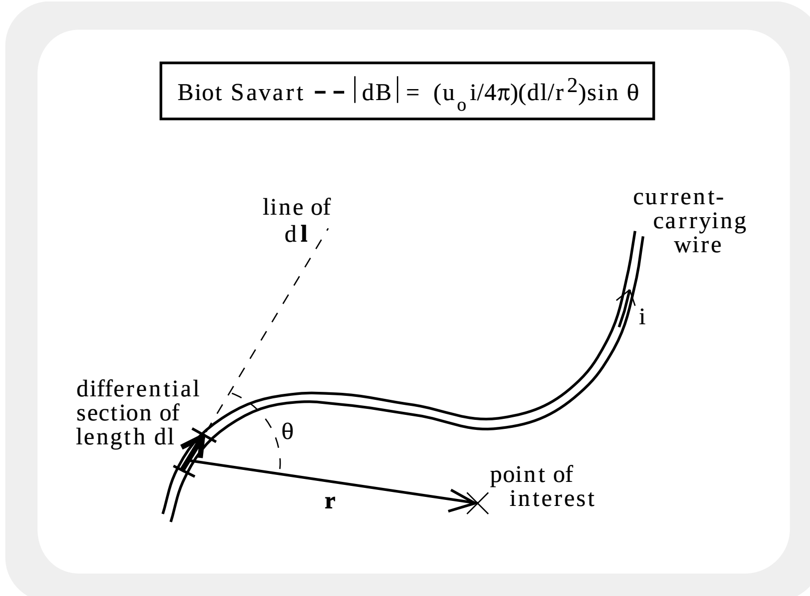

FIGURE 18.15

e.) $\hat{r}$ is a UNIT VECTOR in the direction alluded to in Part d directly above.

Note: A number of texts write Biot Savart as:

$$d\mathbf{B} = \frac{\mu_o i}{4\pi}\frac{d\mathbf{l}x\mathbf{r}}{r^3},$$

where r is the entire position vector, both direction and magnitude, and an r^3 is placed in the denominator to compensate for the fact that r includes the vector's magnitude. Either presentation is correct, although in practice you will find the first one easier to use.

Big Note: The symbols dl and r are defined <u>differently</u> in Biot Savart and in Ampere's Law. KNOW AND UNDERSTAND THE DIFFERENCES.

3.) Although the cross product operation defines the direction of the magnetic field at Point P, there are instances when the magnitude of the magnetic field is of sole interest. In such cases, the magnitude of the cross product yields a differential magnetic field equal to:

$$dB = \frac{\mu_o i}{4\pi}\frac{dl}{r^2}\sin\theta,$$

where θ is the angle between dl and r (again, see Figure 18.15).

J.) Example of an Easy Biot Savart Problem:

1.) Consider the current-carrying wire shown in Figure 18.16a (the power supply has been added, but we will ignore its presence in the magnetic field calculation). What is the net magnetic field (as a vector) at the common center of the semicircles?

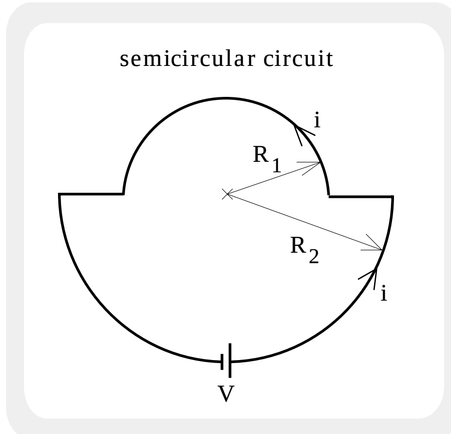

FIGURE 18.16a

a.) Figure 18.16b shows dl vectors for each section of the system.

b.) Using Biot Savart on the top semicircle, we can determine the differential magnetic field due to the charge flow in the segment dl_1. That expression is:

$$dB_1 = \frac{\mu_o i}{4\pi} \frac{dl_1}{r_1^{\,2}} \sin(90^\circ),$$

where $r_1 = R_1$.

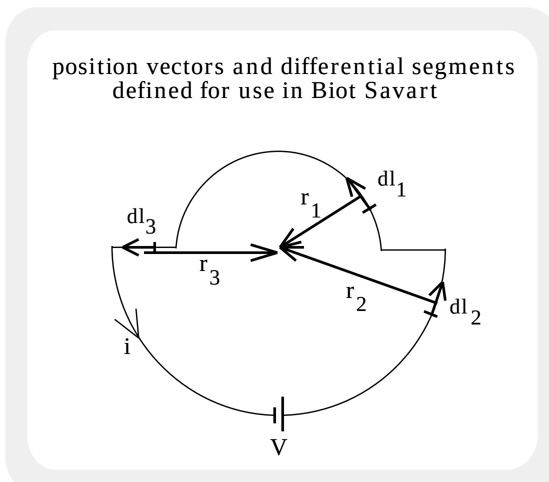

FIGURE 18.16b

c.) To determine the <u>total</u> magnetic field due to the entire upper semicircle, we must integrate dB_1. Substituting $r_1 = R_1$ into our expression and doing the integration, we get:

$$\begin{aligned} B_1 &= \int dB \\ &= \frac{\mu_o i}{4\pi R_1^{\,2}} \int_{\text{around semi-circle}} dl \\ &= \frac{\mu_o i}{4\pi R_1^{\,2}} \left(\frac{2\pi R_1}{2} \right) \\ &= \frac{\mu_o i}{4 R_1}. \end{aligned}$$

d.) Using the right-thumb rule on the wire, we find the magnetic field due to the upper semicircle is out of the page.

e.) A similar exercise generates a magnetic field expression due to current flowing in the lower semicircle. It equals:

$$B_2 = \frac{\mu_o i}{4R_2}.$$

f.) Examine Figure 18.16b again. The angle between dl_3 and r_3 is 180^o. As the sine of 180^o is zero, the magnetic field due to the left-hand, straight-wire section will be zero at the point of interest (it will not be zero at other places, but at the center of the semicircles it is zero). The same is true of the straight-line section on the right.

g.) This means that as a vector the net field equals:

$$\begin{aligned} B &= B_1 + B_2 \\ &= \left[\frac{\mu_o i}{4R_1} + \frac{\mu_o i}{4R_2}\right](+k). \end{aligned}$$

K.) A Second Example of Biot Savart:

1.) Reiteration: So far, we have dealt with situations in which the magnetic fields at a point of interest have all been in the same direction (i.e., in the previous problem, the field was out of the page for all dl segments). In such cases, the approach used was:

a.) Define dl (an arbitrary segment on the current-carrying wire).

b.) Define r (a vector <u>from</u> dl <u>to</u> the point of interest).

c.) Define the angle θ between the line of dl and the line of r.

d.) Use the right-hand rule on dlxr to determine the direction of the differential magnetic field at the point of interest.

e.) Use Biot Savart to determine the magnitude of the differential magnetic field at the point of interest due to current in dl.

f.) Integrate the differential magnetic field expression to determine the net magnetic field due to all of the dl's for which your derived dB expression is valid (in the previous problem, there were four such sections--the upper and lower semicircles and the right and left-hand straight-line sections).

g.) Add up all the derived B-field expressions to get the net magnetic field at the point of interest, complete with a statement of direction.

2.) There is another twist that hasn't yet been addressed. What happens if the major segments produce magnetic fields whose directions are different? To see how such a possibility might occur, consider the following situation:

3.) A circular wire of radius R rests in the x-z plane (i.e., in the horizontal) with its center at the origin of the coordinate system being used (see Figure 18.17a). Current i flows in the wire as shown in the sketch. Derive an expression for the net magnetic field a distance y units up the y-axis.

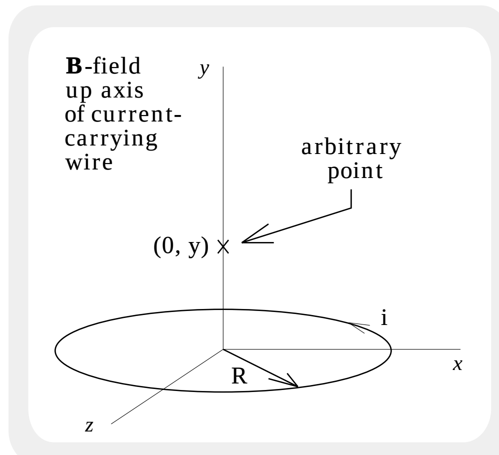

FIGURE 18.17a

a.) Proceeding with our approach: The vector dl is defined in the direction of current flow. It is supposed to be an arbitrarily defined segment of wire, but for simplicity and ease of viewing on the accompanying sketches, let's define it to be the segment that cuts through the x-y plane moving into the page (see Figure 18.17b).

b.) The vector r is defined as a vector from the segment to the point of interest. In this case, the point of interest is at an arbitrary point y units up the y-axis. Note that as defined, r is in the x-y plane.

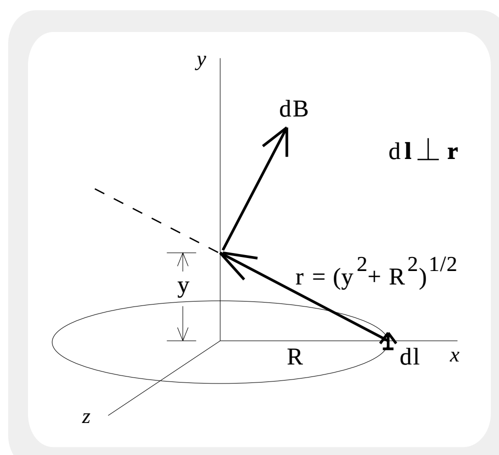

FIGURE 18.17b

c.) Using the right-hand rule to determine the direction of the cross product between dl and r, we find that the direction of the differential magnetic field dB produced by that differential segment of current is in the x-y plane as shown in Figure 18.17b.

d.) The angle between dl and r is 90^o (dl is into the page while r is in the plane of the page). Additionally observing that $r = (y^2 + R^2)^{1/2}$, we can use Biot Savart to write:

$$dB = \frac{\mu_o i}{4\pi}\frac{dl x \hat{r}}{r^2}$$
$$= \frac{\mu_o i}{4\pi}\frac{dl}{\left[\left(y^2+R^2\right)^{1/2}\right]^2}\sin 90^o$$
$$= \frac{\mu_o i}{4\pi}\frac{dl}{\left(y^2+R^2\right)}.$$

e.) The magnitude of dB will be the same for any given dl, but the direction of dB will be different from segment to segment. That means we could break dB into its components and integrate each component separately, or we could be clever.

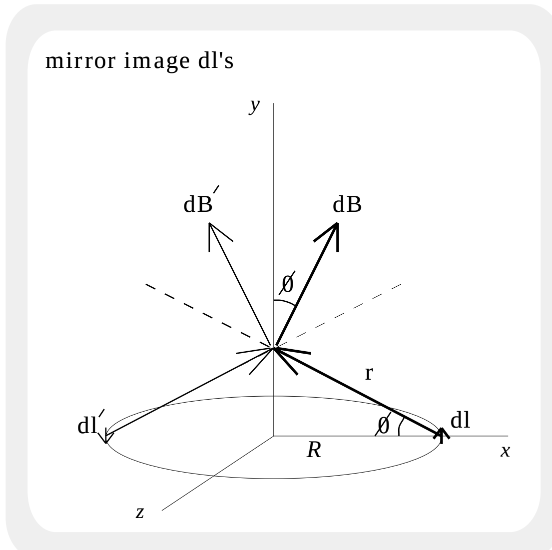

FIGURE 18.17c

f.) Being clever, examine the direction of the magnetic field produced by a segment that is 180° from our defined dl (see Figure 18.17c). The horizontal component of that vector is equal and opposite to the horizontal component of dB produced by dl. As all such components will add to zero, we can ignore the horizontal component and deal solely with the vertical components.

Note: If you don't believe that we can ignore the horizontal component, determine dB sin ϕ (i.e., dB's horizontal component) and do the integral. You will find that it evaluates to zero.

g.) With the horizontal component ignored, $B_{net} = \int dB_y = \int dB \cos \phi$. Using the geometry of a circle and Figure 18.17c, we can see that $\cos \phi = R/r$. As such, we can write:

$$
\begin{aligned}
B_{net} &= \int dB_y \\
&= \frac{\mu_o i}{4\pi}\int\left[\frac{dl}{\left(y^2+R^2\right)}\right](\cos\phi) \\
&= \frac{\mu_o i}{4\pi}\int\frac{dl}{\left(y^2+R^2\right)}\left(\frac{R}{\left(y^2+R^2\right)^{1/2}}\right) \\
&= \frac{\mu_o i}{4\pi}\frac{R}{\left(y^2+R^2\right)^{3/2}}\oint dl \\
&= \frac{\mu_o i}{4\pi}\frac{R}{\left(y^2+R^2\right)^{3/2}}(2\pi R) \\
&= \frac{\mu_o i}{2}\frac{R^2}{\left(y^2+R^2\right)^{3/2}}.
\end{aligned}
$$

L.) The Force on a Charge Moving in a Magnetic Field:

Note: We have been examining the mathematics around the theoretical determination of magnetic field functions. For the next few sections, we will assume the availability of magnetic fields without considering their origin.

1.) As has already been stated, a charge moving in a magnetic field will feel a force under certain circumstances. The relationship between this magnetic force F_B, the magnetic field strength B, the velocity of the charge v, and the size of the charge q has been experimentally determined to be:

$$F_B = q\, v \times B.$$

a.) This cross product yields both the magnitude and direction of the force on a POSITIVE CHARGE moving in the magnetic field.

b.) <u>IMPORTANT</u>: If the charge is NEGATIVE, the magnitude of the force will be the same BUT THE DIRECTION OF THE FORCE WILL BE OPPOSITE that determined using the right-hand rule.

Note: From the MKS units for force, charge, and velocity, the units for the magnetic field vector B must be nt/[C·(m/s)], or kg/(c·s). This set of MKS units is given the special name "teslas."

2.) Examples: Determine the magnitude and direction of the force on a 4 coulomb charge moving with velocity 12 meters/second in a magnetic field whose strength is 5 teslas if the velocity and magnetic field vectors are as shown in Figures 18.18a through 18.18d.

Note: Vectors pointing perpendicularly into the page are depicted either by a group of circles with crosses in them or simply by crosses. Vectors pointing perpendicularly out of the page are depicted by a group of circles with points at their centers or simply by points.

a.) For Figure 18.18a: the magnitude of the force is

$$|F_B| = q|v||B| \sin\theta,$$

where θ is the angle between the line of v and the line of B (note that the sketch is a bit tricky --θ should be the angle between the line of v and the line of B--that is not the angle given in the figure. Putting in the numbers, we get:

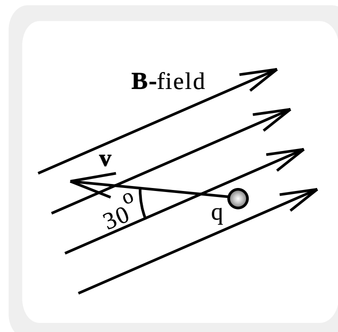

FIGURE 18.18a

$$|F_B| = (4\text{ C})(12\text{ m/s})(5\text{ T})(\sin 150^o)$$
$$= 120\text{ newtons.}$$

The direction is found using the right-hand rule for a cross product. The right hand moves in the direction of the line of the first vector (v); the fingers of the right hand curl in the direction of the line of the second vector (B). Doing so yields a force direction for this situation into the page.

You will not normally be asked to do so, but for the sake of completeness for this first try, this force can be written as a vector in unit vector notation as:

$$F_B = (120\text{ newtons})(-k).$$

b.) The magnitude of the force in Figure 18.18b is:

$$|F_B| = q|v||B| \sin\theta$$
$$= (4\text{ C})(12\text{ m/s})(5\text{ T})(\sin 90^o)$$
$$= 240\text{ newtons.}$$

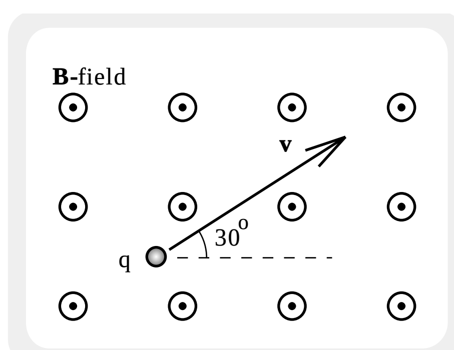

FIGURE 18.18b

The direction is found using the right-hand rule for a cross product. In this case, the direction will be toward the bottom of the page, perpendicular to v, and to the right. If asked a question like this on a test, you will not be asked to put the final force vector in a unit vector notation. You will be asked to draw in the force direction on the sketch in addition to determining the force magnitude.

c.) The magnitude of the force in Figure 18.18c is:

$$|F_B| = (4\ \text{C})(12\ \text{m/s})(5\ \text{T})(\sin 180^o)$$
$$= 0 \text{ newtons.}$$

There is no direction associated with zero force.

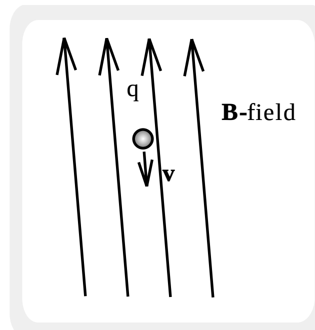

FIGURE 18.18c

Note: This should give you a bit of a hint as to the order of operations on a test problem. Determine magnitudes first before trying to determine direction--trying to get the right-hand rule to work on a cross product whose magnitude is zero can be enormously frustrating.

d.) For amusement, assume q is negative. The magnitude of the force in Figure 18.18d is:

$$|F_B| = (4\ \text{C})(12\ \text{m/s})(5\ \text{T})(\sin 90^o)$$
$$= 240 \text{ newtons.}$$

As the charge is negative, the direction is toward the bottom of the page.

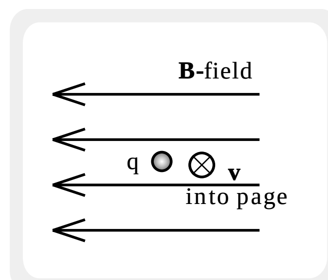

FIGURE 18.18d

3.) When electric and magnetic fields are present, the net possible electrical force acting on a charge is qvxB + qE. This is called Lorentz's equation.

M.) The Force on a Current-Carrying Wire in a Magnetic Field:

1.) Consider a current-carrying wire of length L situated in a magnetic field as shown in Figure 18.19 on the next page. Find the force on the current-carrying wire as the moving charge interacts with the magnetic field.

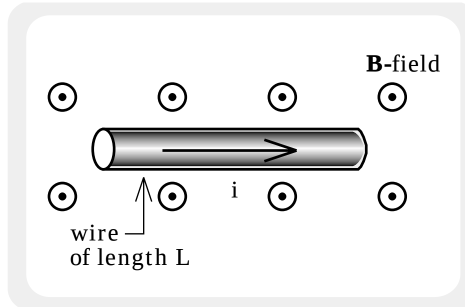

FIGURE 18.19

a.) Assume that all the free charges in the wire move at the same average velocity.

b.) If time t is the average amount of time required for one charge q to move the entire length L of the wire, the average velocity of that charge (and all the others) will be L/t, where the L is a vector whose magnitude is defined as the length of the wire and whose direction is defined as the direction of the charge's motion.

c.) We know that the force on a single charge q moving in a magnetic field is $F_{q,B} = qv \times B$. Substituting in $v = L/t$, we get:

$$F_{q,B} = q\,(L/t) \times B.$$

d.) The collective magnetic force F_B on all the charges moving in the wire (we'll call the total charge Q) yields a net force of:

$$\begin{aligned} F_B &= Q\,(L/t) \times B \\ &= (Q/t)\,L \times B. \end{aligned}$$

e.) The current i is defined as the ratio of the total charge Q passing a particular point over a period of time t (i.e., $i = Q/t$). With that in mind, we can write:

$$F_B = i\,L \times B.$$

f.) This relationship defines the net magnetic force felt by a current-carrying wire in a magnetic field.

Note: The direction of the cross product will yield the direction of the magnetic force on the wire; the magnitude of the cross product will yield the magnitude of that magnetic force.

2.) Examples: Determine the magnitude and direction of force on a .5 meter long current-carrying wire if the magnetic field intensity is 4 teslas, the current in the wire is 2 milliamps, and the vector directions are as shown in Figures 18.20a through 18.20d.

a.) The magnitude of the force in Figure 18.20a is the evaluation of a cross product, or:

$$|F_B| = i|L||B| \sin\theta,$$

where θ is the angle between the line of L and the line of B. Putting in the numbers, we get:

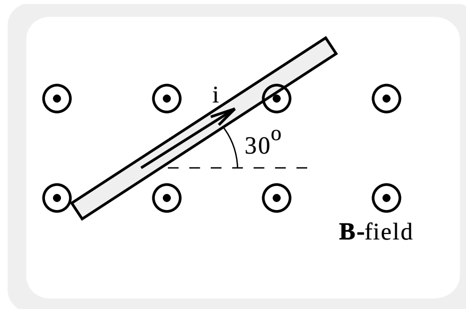

FIGURE 18.20a

$$|F_B| = (2\times10^{-3} \text{ amps})(.5 \text{ m})(4 \text{ T})(\sin 90^o)$$
$$= 4\times10^{-3} \text{ newtons.}$$

The direction is found using the right-hand rule for a cross product. The right hand moves in the direction of the line of the first vector (L); the fingers of the right hand curl in the direction of the line of the second vector (B). Doing so in this problem yields a force direction that is <u>perpendicular</u> to the wire and toward the bottom of the page.

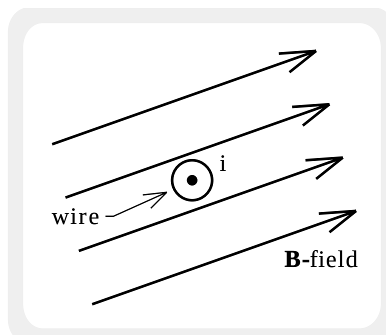

FIGURE 18.20b

b.) The magnitude of the force in Figure 18.20b is:

$$|F_B| = (2\times10^{-3} \text{ amps})(.5 \text{ m})(4 \text{ T})(\sin 90^o)$$
$$= 4\times10^{-3} \text{ newtons.}$$

The direction will be perpendicular to B and toward the top of the page.

c.) The magnitude of the force in Figure 18.20c is:

$$|F_B| = (2\times10^{-3} \text{ amps})(.5 \text{ m})(4 \text{ T})(\sin 120^o)$$
$$= 3.46\times10^{-3} \text{ newtons.}$$

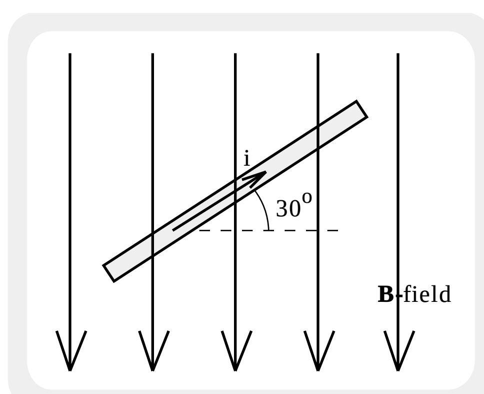

FIGURE 18.20c

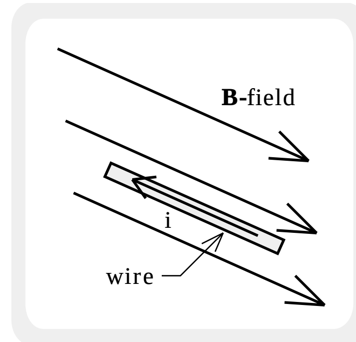

FIGURE 18.20d

The direction is perpendicularly into the page.

d.) The magnitude of the force in Figure 18.20d is:

$$|F_B| = (2\text{x}10^{-3} \text{ amps})\ (.5 \text{ m})\ (4 \text{ T})\ (\sin 180^o)$$
$$= 0 \text{ newtons.}$$

There is no direction for a zero-magnitude force.

3.) Example--Newton's Second Law: A metallic bar of mass m is placed on two supports a distance L = a units apart that form an incline whose angle is ϕ with the horizontal (see Figure 18.21). A battery is attached across the supports and a constant, downward magnetic field B permeates the setup (see Figure 18.22). If the net resistance in the circuit is R, what voltage V_o is required to ensure that the bar does not accelerate down the incline?

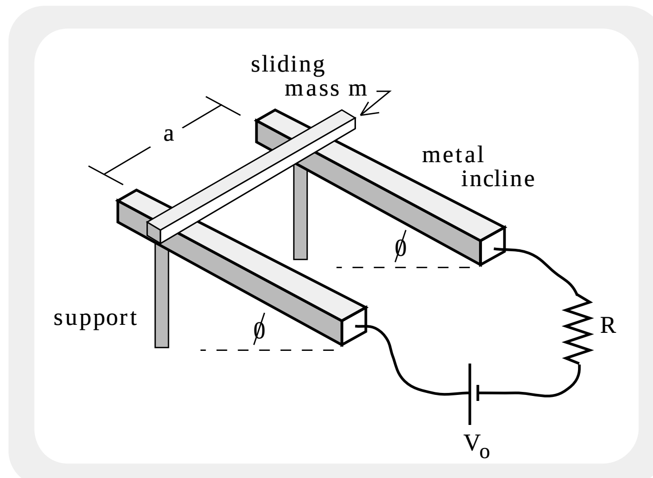

FIGURE 18.21

a.) We know $V_o = iR$. If we can determine the current i needed to suspend the bar, we can determine V_o.

b.) There are three forces acting on the bar. The first two are gravity and a normal force. The third is a magnetic force due to the fact that the current passing through the bar is in a B-field. Knowing this, we should be able to exploit Newton's Second Law to derive an equation that will be helpful.

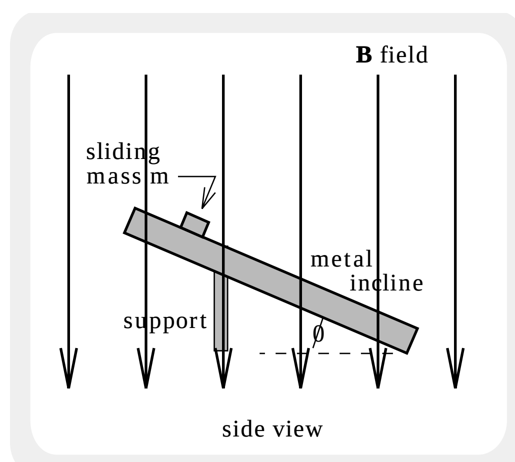

FIGURE 18.22

c.) The magnitude of the magnetic force is $F = i\,|L\text{x}B|$ where L's magnitude is a and the angle between L and B is 90^o (L is into the page while B is in the plane of the page). The free body diagram for the situation is found in Figure 18.23 to the right with components highlighted. Using Newton's Second Law:

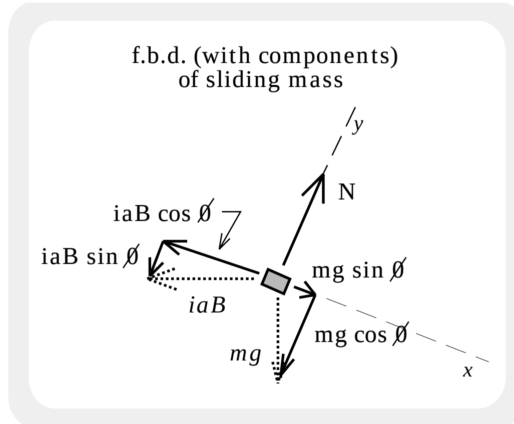

FIGURE 18.23

$\underline{\Sigma F_x}$:

$$mg(\sin\phi) - (iaB \sin 90^o)(\cos\phi) = ma_x = 0$$

$$\Rightarrow \quad i = mg(\tan\phi)/aB.$$

d.) As V = iR, we get:

$$V_o = [mg(\tan\phi)/aB]\, R.$$

e.) Assuming $\phi = 30^o$, m=.15 kg, B = 3 T, a = .2 m, and R = 5 Ω, the voltage is found to be:

$$V_o = [mg\,(\tan\phi)/aB]R$$
$$= \big[[(.15\text{ kg})(9.8\text{ m/s}^2)\tan 30^o]/[(.2\text{ m})(3\text{ T})]\big]\,(5\,\Omega)$$
$$= 7.07 \text{ volts.}$$

N.) The Use of Magnetic Fields in Building Meters:

1.) Preliminary observations: Consider a pinned, single-looped rectangular coil whose side-lengths are equal to a and whose top and bottom lengths are equal to b. If a current i passes through the wire while in a magnetic field (see Figure 18.24a), the moving charges will feel a force according to $F_B = i\,L \times B$.

Note 1: The magnetic field lines generated by bar magnets are not constant (see Figure 18.24b). Nevertheless, we will assume a constant B-field for simplicity.

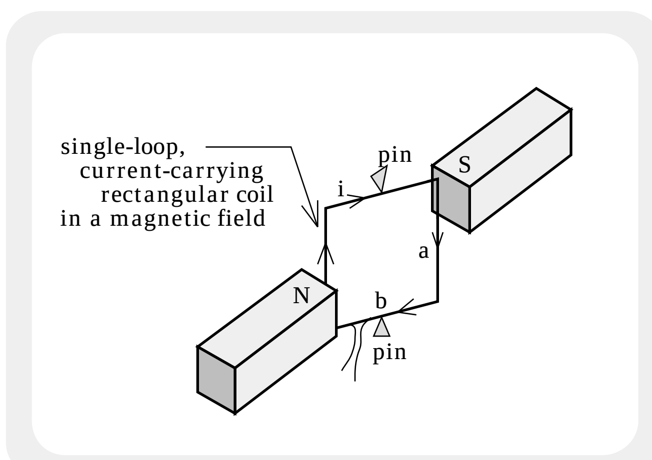

FIGURE 18.24a

Note 2: The direction of the force will depend upon the current's direction relative to the magnetic field vector. Looking at the wire and charge flow from above (see Figure 18.24b), we can make the following observations:

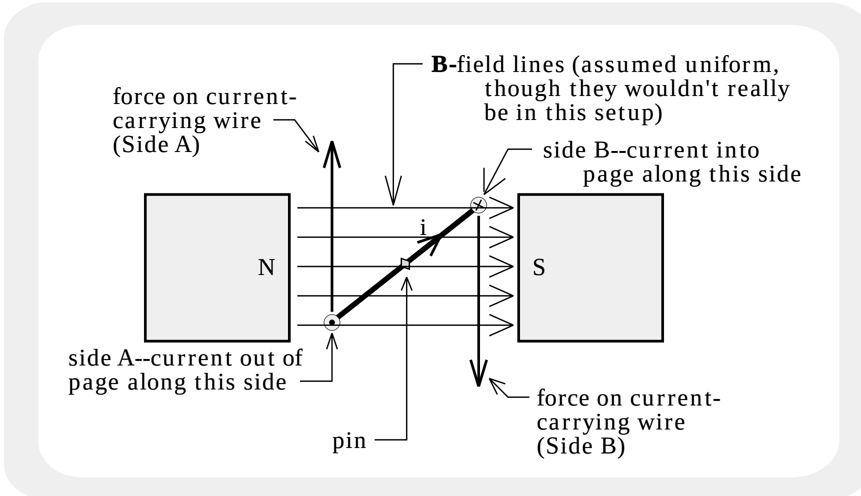

FIGURE 18.24b

a.) The section of wire with current moving out of the page (side A in Figure 18.24b) will feel a force whose direction is toward the top of the page and whose magnitude is iaB.

b.) The section of wire with current moving into the page (side B in Figure 18.24b) will feel a force whose direction is toward the bottom of the page and whose magnitude is iaB.

c.) Each of these forces will produce a torque on the coil about the pin. With r = b/2, we can write:

$$|r \times F| = (b/2)\,(iaB)\sin\phi,$$

where ϕ is the angle between r and F.

d.) As there are two such wires, the total torque on the coil about the pin will be:

$$|\Gamma_{net}| = 2\,[(b/2)\,(iaB)\sin\phi]$$
$$= AiB\sin\phi$$

(A is defined as the area--length a times width b--of the square loop's face).

e.) If there are N winds in the coil (our original coil had only one loop), the net torque becomes:

$$\left|\Gamma_{net}\right| = NAiB \sin \phi.$$

f.) Defining a vector $\boldsymbol{\mu}_m$ called the magnetic moment whose direction is perpendicular to the face of the coil (oriented so that the thumb of the right hand is in the direction of this vector when the fingers of the right hand curl in the direction of current flow--see Figure 18.25) and whose magnitude is equal to NAi, we get the relationship:

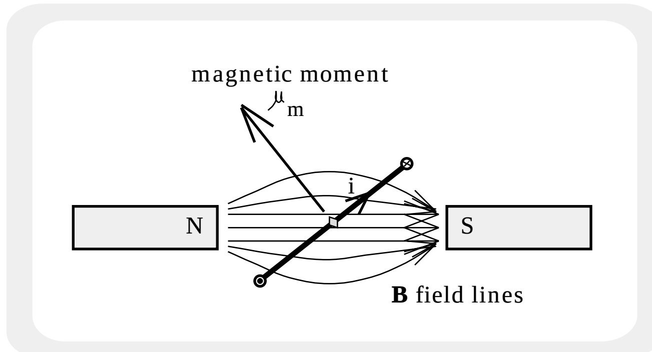

FIGURE 18.25

$$\boldsymbol{\Gamma}_{net} = \boldsymbol{\mu}_m \times B,$$

where the angle between $\boldsymbol{\mu}_m$ and B is ϕ.

Note: The above expression is analogous to the torque expression for an electric dipole in an electric field. Continuing with the analogy, the amount of potential energy wrapped up in a current-carrying coil is:

$$U = -\boldsymbol{\mu}_m \cdot B.$$

g.) Noting that current-carrying coils in magnetic fields can have torques applied to them, consider the magnetically engulfed, current-carrying coil shown in Figure 18.26. A spring attached to the bottom of the coil produces a counter-torque if the coil rotates. When rotation occurs, a needle attached to the coil also rotates. If that needle is placed over a visible, calibrated scale, we end up with the prototypical current-sensing meter.

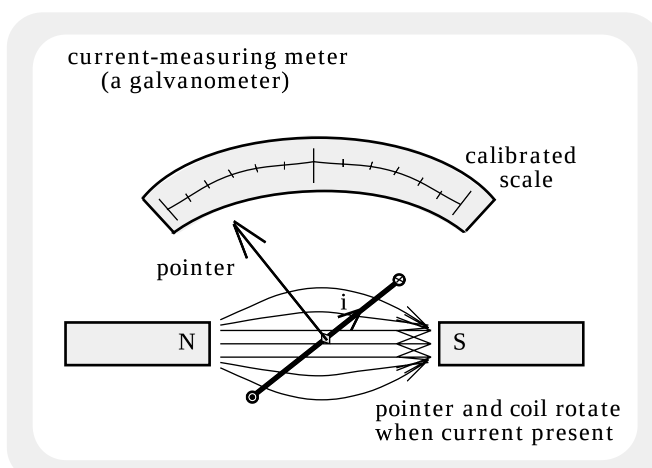

FIGURE 18.26

2.) The most basic version of a current-sensing meter is called a galvanometer (the sketch shown in Figure 18.26 is actually that of a galvanometer). A coil in a known magnetic field has attached to it a spring that is just taut enough to allow the needle to rotate full-deflection (i.e., to the end of the scale) when 5×10^{-4} amps flow through it. In that way, if an unknown current flows through the galvanometer and the needle fixes at half deflection, the user knows that the current is half of 5×10^{-4} amps, or 2.5×10^{-4} amps.

ALL GALVANOMETERS ARE MADE TO SWING FULL DEFLECTION WHEN 5×10^{-4} AMPS FLOW THROUGH THEM. This uniformity is the reason galvanometer scales are labeled 1 through 5 without any other hint as to the meaning of the numbers. It is assumed that if you know enough to be using a galvanometer, you know that its units are "$\times10^{-4}$ amps" (quite a conceit if you think about it).

As all galvanometers are made to the same specifications throughout the industry, they are the cornerstone in the production of all other meters, voltmeters and large-current ammeters alike.

Note: Although it is not evident in Figure 18.26, a galvanometer's needle always points toward the center of the scale when no current is passing through the meter. In that way, the needle can deflect either to the right or the left, depending upon which meter-terminal the high voltage is connected to. Galvanometers are the only meters that have this "center-zero" setup. All other meters have their zero to the left, swinging to the right when current passes through them. That means they depend upon you, the user, to hook the high voltage leads to the correct terminal.

3.) The <u>Ammeter</u>: The sketch in Figure 18.27 shows the circuit for a 12 amp ammeter (the sketch is general to all ammeters; I have arbitrarily chosen 12 amps for the sake of a number example). Notice the design requires a galvanometer (designated by the resistance R_g) and a second resistor R_s. Assume the resistance of the galvanometer is 5 ohms. The rationale behind the design is as follows:

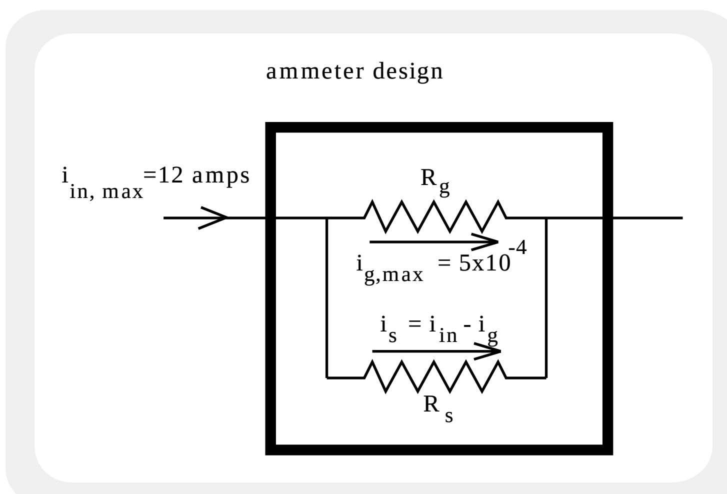

FIGURE 18.27

a.) We want the galvanometer's pointer to swing full-deflection when twelve amps of current passes through the ammeter. In other words, when twelve amps flow into the meter, we want $5x10^{-4}$ amps to flow through the galvanometer.

b.) The parallel design allows current passing through the meter to split up. If we pick just the right size resistor R_s (this is called a shunt resistor because it shunts off current from passing through the galvanometer), all but $5x10^{-4}$ amps will flow through that resistor whenever twelve amps flow into the device. The trick is in finding the proper value for the shunt resistor. To do so:

i.) Noticing that the voltage across R_g is the same as the voltage across the R_s (the two resistors are in parallel), we can write:

$$i_{g,max} R_g = i_{s,max} R_s.$$

ii.) We know that $i_{g,max}$ will be $5x10^{-4}$ amps when 12 amps flow into the circuit, so the amount of current passing through R_s must be whatever is left over, or:

$$i_{s,max} = (12 \text{ amps}) - (.0005 \text{ amps}) = 11.9995 \text{ amps}.$$

iii) Putting it all together, we get:

$$i_{g,max} R_g = i_{s,max} R_s$$
$$(5x10^{-4} \text{ amps})(5\ \Omega) = (11.9995 \text{ amps}) R_s$$
$$\Rightarrow \qquad R_s = 2.08x10^{-4}\ \Omega.$$

c.) A short piece of wire will have resistance in this range. In other words, a typical ammeter is nothing more than a galvanometer with a measured piece of wire hooked in parallel across its terminals.

4.) <u>The Voltmeter</u>: The sketch in Figure 18.28 shows the circuit for a 12 volt voltmeter (the sketch is general to all voltmeters; I have arbitrarily chosen 12 volts

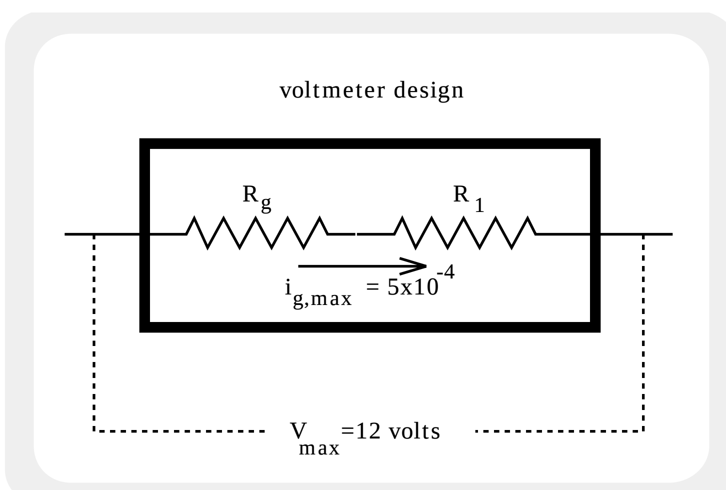

FIGURE 18.28

for the sake of a number example). Notice the design requires a galvanometer (designated by the resistance R_g) and a second resistor R_1. Assuming the galvanometer's resistance is 5 ohms, the rationale behind the design is as follows:

a.) We want the galvanometer's pointer to swing full-deflection when twelve volts are placed across the voltmeter (i.e., when the voltmeter is hooked across an electrical potential difference of twelve volts). In other words, when 12 volts are placed across the meter, we want $5\text{x}10^{-4}$ amps of current to flow through the galvanometer.

b.) The series design requires that voltage across the voltmeter be split up between the two series resistors (i.e., the voltage drop across the galvanometer plus the voltage drop across the second resistor must sum to 12 volts). If we pick just the right size resistor R_1, a current of $5\text{x}10^{-4}$ amps will flow through both resistors whenever twelve volts are placed across the meter. The trick is in finding the proper value for the second resistor. To do so:

i.) When the total voltage across the meter is 12 volts, the galvanometer's voltage must be $i_{g,max}R_g$ while the second resistor's voltage must be $i_{g,max}R_1$ (the two resistors are in series, hence the current is common to both). As such we can write:

$$V_o = (i_{g,max} R_g) + (i_{g,max} R_1)$$

$$12 \text{ volts} = (5\text{x}10^{-4} \text{ amps})(5\ \Omega) + (5\text{x}10^{-4} \text{ amps})(R_1)$$

$$\Rightarrow \quad R_1 = 2.3995\text{x}10^{4}\ \Omega.$$

c.) In short, a typical voltmeter is nothing more than a galvanometer hooked in series to a large resistor. As would be expected, they draw very little current when hooked across an element in a circuit.

5.) Bottom line: All analog meters (i.e., meters that are not digital) are based on the galvanometer, and all galvanometers are based on the proposition that current moving through an appropriately pinned coil in a magnetic field will feel a torque-producing force which is proportional to the amount of current passing through the coil.

QUESTIONS

18.1) Copper wire is electrically neutral whether there is current flowing through it or not. Does this make sense in light of Einstein's theory regarding length contraction? That is, if there are electrons in motion in a wire (i.e., if there is a current), shouldn't the electrons bunch up due to length contraction, creating an electric field that would affect even stationary charge next to the wire? What do you think Einstein's response would be?

Note: Don't think about this too hard--I'd say fifteen seconds should do nicely. You won't be tested on any of the Relativity material. This question is more of a teaser for the chapter on Relativity at the end of the book than anything else (a quick and dirty answer is supplied in the Solutions).

18.2) A series of parallel, current-carrying wires are shown in Figure I. What is the direction of the net magnetic field at:

a.) Point A on the sketch;
b.) Point E on the sketch;
c.) Point C on the sketch;
d.) Point D on the sketch.
e.) Ignoring gravity, if an electron is placed at Point E, what force will it feel?

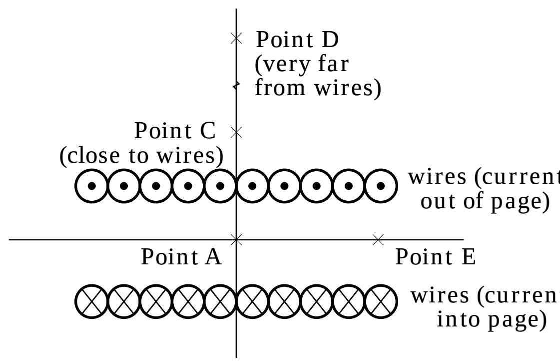

FIGURE I

18.3) Six particles with the same mass move through a magnetic field directed into the page (Figure II).

a.) Identify the positively charged, negatively charged, and electrically neutral masses. (Hint: How would you expect a positively charged particle to move when traveling through a B-field directed into the page?)

b.) Assuming all the particles have the same charge-magnitude, which one is moving the fastest? (Hint: For a fixed charge, how is charge velocity and radius of motion related? Think!)

c.) Assuming all the particles have the same velocity, which one has the greatest charge? (Same hint as above, but reversed.)

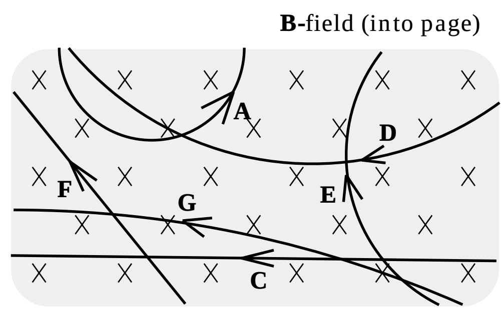

FIGURE II

18.4) At what angle will a -5×10^{-10} coulomb charge moving with velocity magnitude 3×10^{2} m/s have to enter a magnetic field B = (.138 tesla)i if the force magnitude it feels is to be 1.7×10^{-8} newtons?

18.5) An intrepid student decides she wants to build a circular nuclear accelerator. Her design is simple. A voltage difference of V_{plates} = 5,000 volts is to be placed across two metal plates that are L = .5 meters apart. A proton (mass 6.67×10^{-27} kg, charge 1.6×10^{-19} coulombs) accelerates from the positive plate to the negative plate (i.e., through the voltage difference), picking up energy and velocity in the process. Each plate has a hole in its center through which the proton is to travel, and each plate is shielded in such a way as to allow the proton to pass beyond the plate and be free once through (that is, assume the proton is not attracted back toward the plate it has just passed through). Upon leaving one such set of plates, it enters the field of a second set of identical plates, accelerates as in the previous situation, then passes into another section, etc. Figure III shows the setup.

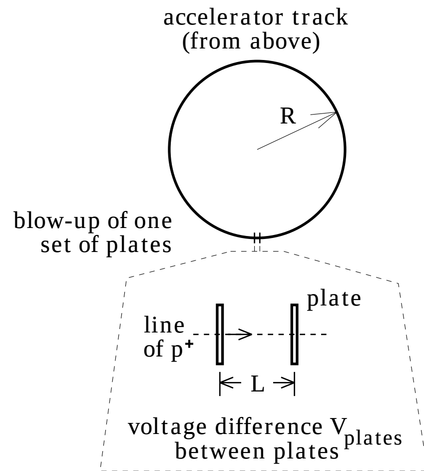

FIGURE III

The force field she wants to use to keep the proton moving in its circular path is to be provided by a time-varying magnetic field (more force will be needed as the proton picks up speed). The circular track is to have a radius of R = 100 meters (real accelerators have radii of around 1000 meters).

Before beginning the project, our young scientist decides to make some calculations to see if her design will work. Following in her footsteps:

a.) Ignoring relativistic effects (i.e., electron mass increasing at speeds close to the speed of light), how large must her B-field be to keep the proton moving in the appropriate circular path when its velocity is .95c? Note that c is the symbol for the speed of light, or 3×10^{8} m/s.

b.) Ignoring relativistic effects, derive an expression for the magnetic field AS A FUNCTION OF TIME required to keep a proton moving along the circular track. (Hint: The idea behind Lorentz's Equation should come in handy here). Your result should be in terms of V_{plates}, the plate distance L, and the radius R of the track.

c.) According to Relativity, the mass of a particle increases as the particle's speed increases. The relationship is:

$$m_{moving} = m_{rest} / [1- (v/c)^2]^{1/2},$$

where m_{rest} is the mass of the object when at rest, v is the object's speed, and c is the speed of light. With this information in mind, re-do Part a taking into account relativistic effects.

18.6) A positive charge $q = 4x10^{-9}$ coulombs and mass $m = 5x10^{-16}$ kilograms accelerates from rest through a potential difference of $V_o = 2000$ volts. Once accelerated, it enters a known magnetic field whose magnitude is B = 1.8 teslas.

a.) On the sketch in Figure IV, draw in an approximate representation of the charge's path.

b.) We would like to know the velocity of the charge just as it enters the B-field. Use conservation of energy and your knowledge about the electrical potentials to determine the charge's velocity at the end of the acceleration (yes, this is a review-type question).

c.) Determine the particle's radius of motion once in the B-field.

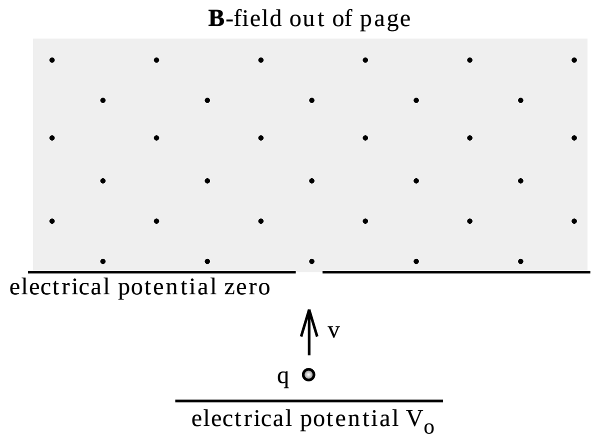

FIGURE IV

18.7) At t = 0 seconds, a -2 coulomb charge finds itself with velocity v = 4 j m/s in an unknown B-field and a known electric field of E = 25 i nts/C. Determine B (as a vector) if the particle's acceleration at that time is zero.

18.8) A wire carries 8 amps. The earth's magnetic field is approximately $6x10^{-5}$ teslas.

a.) How far from the wire will the earth's magnetic field and the wire's magnetic field exactly cancel one another?

b.) How must the wire be oriented (i.e., north/south, or south-east/north-west, or what?) to effect the situation outlined in Part a? (Assume there is no "dip" in the earth's B-field)?

18.9) Three long wires all have 15 amps flowing through them (see Figure V). If the wires are positioned on a .25 meter square:

a.) Determine the magnetic field (as a vector) at Point P.

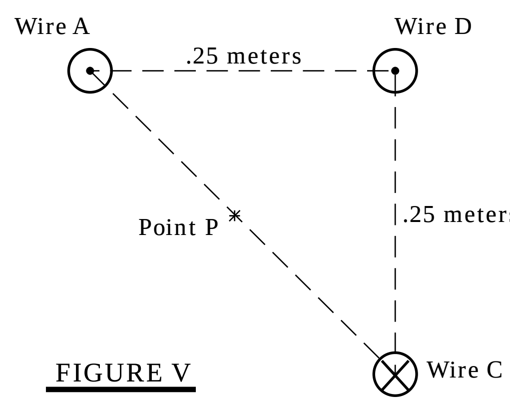

FIGURE V

b.) A charge $q = -7x10^{-12}$ coulombs moving at 3200 m/s passes through Point P moving out of the page. Determine the magnetic force (direction and magnitude) on the charge.

c.) Re-do Part b assuming the charge is moving along a line from wire A to wire C.

d.) Re-do Part b assuming the charge is moving along a line between Point P and wire A.

e.) Determine the force per unit length on wire D.

18.10) The Hall Effect was an experiment designed to determine the kind of charge that flows through circuits (electrons were suspected but there was no proof). The device is shown in Figure VI. It consists of a battery attached to a broad, thin plate that is bathed in a constant magnetic field. Using the device, how might you determine the kind of charge carriers that move in electrical circuits?

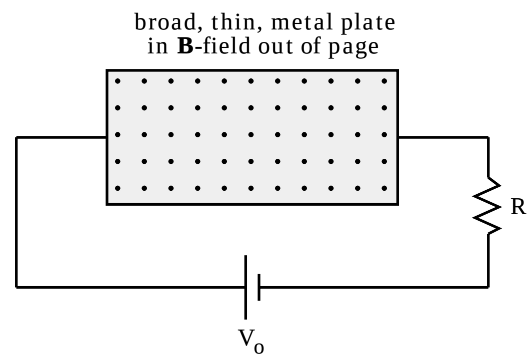

FIGURE VI

Step #1: Assume electrons flow in the circuit. What path, on the average, will those negative charges take as they pass through the plate in the magnetic field? Which side of the plate will be the high voltage side?

Step #2: Do the same exercise as suggested in Step #1 assuming positive charge flow.

Culmination: If you didn't know whether the situation depicted in Step 1 or Step 2 was the real situation, how could the use of a voltmeter help?

18.11) Assuming the resistance of a galvanometer is 12 ohms, draw the circuit design for and determine all pertinent data required to build:

a.) A 300 volt voltmeter;

b.) A .25 amp ammeter.

18.12) An oddly constructed coaxial cable has a normal, thin wire down its axis. Around the wire is insulation, then a thick metal tube on the outside (the tube's inner radius is r_1 and its outer radius is r_2--see Figure VII). The inside wire carries a current equal to i_o. The outer tube

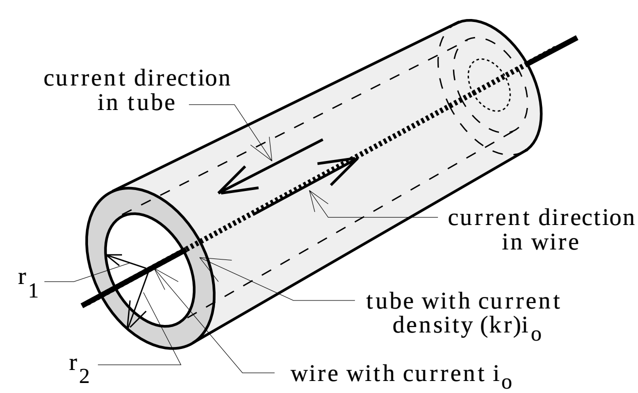

FIGURE VII

has a current density $j = (kr)i_o$ amps per unit area passing through it in a direction opposite to the current flowing in the inside wire (note that r is a distance out from the inside wire and k is simply included to make the units correct). With all this information, derive an expression for the magnetic field (direction and all):

a.) For $r < r_1$;

b.) For $r_1 < r < r_2$;

c.) For $r_2 < r$;

d.) For the amusement of it, what are the units of k?

18.13) A net current of i_o passes through a thin sheet of metal whose thickness is h and whose width is w. Derive an expression for the net magnetic field produced at a point y units above the central axis of the sheet (see Figure VIII for an above view and Figure IX for an on-line view).

Hint: Start by breaking the sheet into a series of differentially thick wires. Once done, determine the B-field due to one wire, then determine the field for all the wires. Be careful of direction.

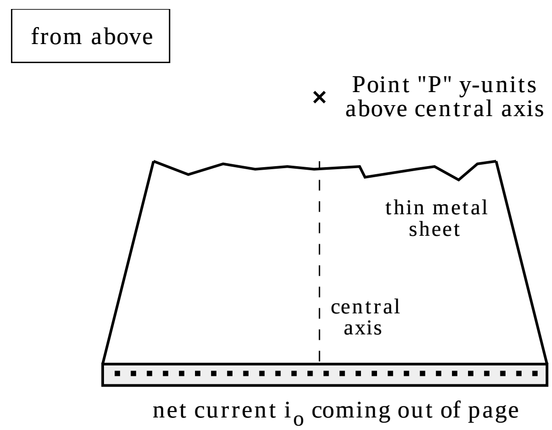

FIGURE VIII

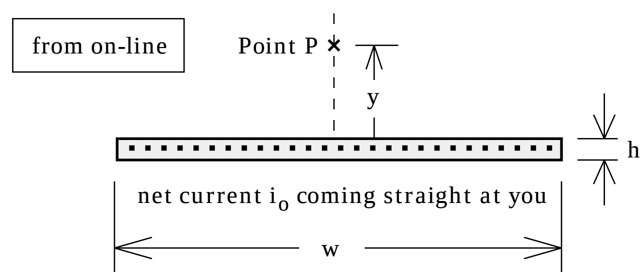

FIGURE IX

18.14.) Use Biot Savart to derive an expression for the magnetic field due to an infinitely long current-carrying wire (assume the wire's current is i_o). It might be useful to note that:

$$\int \frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\left(x^2 + a^2\right)^{1/2}}.$$