CHAPTER 16 -- D.C. CIRCUITS

16.1) Consider the circuit to the right:

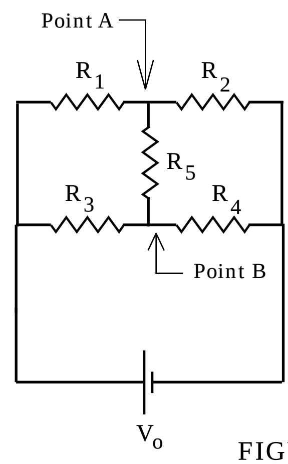


FIGURE I

a.) The voltage drop across R_5 must be zero if there is to be no current through it, which means the voltage of Points A and B on the sketch must be identical.

b.) Current flow is defined as the direction in which positive charge carriers travel (assume positive charges could move through a circuit). That means current flows from the higher absolute electrical potential (Point B at V_B = 5.25 volts) to the lower absolute electrical potential (Point A at V_A = 3.36 volts).

c.) Assuming $R_5 = 3\ \Omega$, the current through R_5 will be:

$$V_5 = i_5 R_5,$$

where V_5 is the voltage drop across R_5 (i.e., 5.25 volts - 3.36 volts = 1.89 volts).

NOTE: This voltage value should be properly called ΔV_5 as it denotes a voltage difference between the two sides of a resistor. It isn't denoted that way because physicists have become lazy with their notation. This book will go with the convention. You need to realize that when you see a V term in a circuit expression, the V is not denoting an electrical potential at a particular point but rather an electrical potential difference between two points.

Putting in the values yields:

$$V_5 = i_5 R_5$$
$$1.89 = i_5\,(3\ \Omega)$$
$$\Rightarrow \quad i_5 = .63 \text{ amps.}$$

16.2) The 12 Ω and 8 Ω resistors are in parallel. That means the voltage drop across each is the same. Call this V_p.

a.) As the voltage across any resistor is related to the current through the resistor by Ohm's Law ($V_R = iR$), we can write:

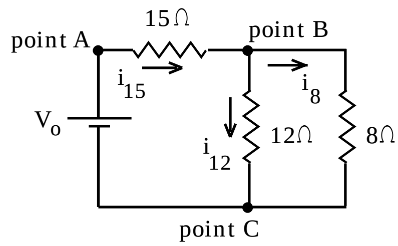


FIGURE II

$$V_{8\Omega} = V_{12\Omega}$$
$$i_8 \quad R_8 = \quad i_{12} \quad R_{12}$$
$$i_8\,(8\ \Omega) = (.5\ a)(12\ \Omega)$$
$$\Rightarrow \quad i_8 = .75 \text{ amps.}$$

b.) The voltage drop across the battery is the same as the sum of the voltage drops between Points A and B (i.e., the voltage drop across the 15 Ω resistor) and Points B and C (i.e., the voltage drop across either the 12 Ω resistor or the 8 Ω resistor--either will do as both have the same drop).

The voltage drop across the 15 Ω resistor is $i_{15}R_{15}$, where i_{15} is the total current being drawn from the battery. This will equal the total current passing through the parallel part of the circuit, or $i_{12} + i_8 = .5$ amps + .75 amps = 1.25 amps. Consequently:

$$\begin{aligned} V_{bat} &= V_{15} + V_{12} \\ &= i_{15}R_{15} + i_{12}R_{12} \\ &= (1.25\ a)(15\ \Omega) + (.5\ a)(12\ \Omega) \\ &= 24.75 \text{ volts.} \end{aligned}$$

16.3) If R_2 decreases:

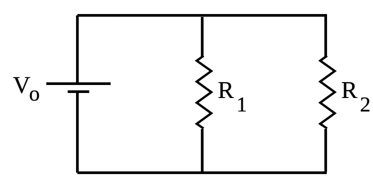


FIGURE III

a.) The voltage across R_2 is held constant by the battery. Decreasing the size of the resistor does nothing to the voltage across the resistor.

b.) Ohm's Law says the voltage across a resistor and the current through the resistor are related as $V_r = iR$. If V_r remains the same and R decreases, i must increase.

c.) The voltage across R_1 is held at V_o by the battery, therefore fooling around with R_2 will do nothing to R_1's voltage or current.

d.) The power dissipated by any resistor is equal to i^2R. If R decreases by, say, a factor of two (i.e., it halves), the current will go up by a factor of two (see Part b for the rationale). As power is a function of current squared while only being a linear function of resistance, halving the resistance while doubling the current will increase the power by a factor of two. Doing this mathematically, we get:

$$P_{old} = i^2R$$
$$P_{new} = (2i)^2(R/2)$$
$$= (4i^2)(R/2)$$
$$= 2i^2R = 2P_{old}.$$

16.4) This is a "use your head" problem. For the series circuit, we calculate the battery voltage as:

$$V_o = iR_{equ}$$
$$= i(R + R)$$
$$= (.4\text{ a})(2R)$$
$$= .8R.$$

For the parallel circuit, the battery voltage V_o (.8R as calculated above) is the same as the voltage across each resistor (resistors in parallel have the same voltage across them). Using Ohm's Law on each of those resistors yields:

$$V_o = i_1R$$
$$= i_1R.$$

Putting in the voltage V_o yields:

$$.8R = i_1R$$
$$\Rightarrow \quad i_1 = .8\text{ amps.}$$

As R is the same in both parallel branches, .8 amps is drawn through both branches making the total current drawn from the battery 1.6 amps.

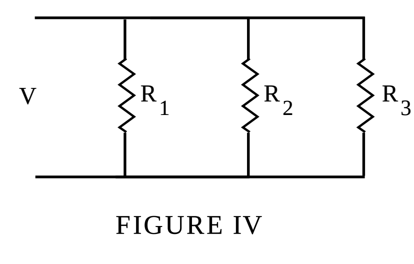


FIGURE IV

16.5) Knowing the current and resistance involved in one of the single-resistor branches allows us to determine the voltage across each branch (by definition, the voltage across any one branch of a parallel combination will equal the voltage across any other branch). As such:

$$V = i_2R_2$$
$$= (2\ a)(12\ \Omega)$$
$$= 24\ \text{volts.}$$

--For $R_1 = 10\ \Omega$:

$$V = i_1R_1$$
$$\Rightarrow \quad i_1 = V/R_1$$
$$= (24\ v)/(10\ \Omega)$$
$$= 2.4\ \text{amps.}$$

--For $R_3 = 16\ \Omega$:

$$V = i_3R_3$$
$$\Rightarrow \quad i_3 = V/R_3$$
$$= (24\ v)/(16\ \Omega)$$
$$= 1.5\ \text{amps.}$$

16.6) Whenever time is incorporated into a problem, there is a good chance you will either be working with power (work/unit time) or current (charge passing a point/unit time). In this case, it is both.

a.) The definition of current is q/t, where q is the total charge passing by a point in the circuit during a time interval t. Noting that time must be in seconds and using the current definition yields:

$$i = q/t$$
$$\Rightarrow \quad q = it$$
$$= (6\ a)[(30\ \text{min})(60\ \text{sec/min})]$$
$$= 10{,}800\ \text{coulombs.}$$

b.) In terms of electrical parameters, the electrical potential difference (i.e., $\Delta V = V$) between two points equals the work/charge available to any charge that moves between the points. As such:

$$W = qV.$$

As the voltage difference across a resistor is $V = ir$, we can write:

$$W = q(ir)$$
$$= (10{,}800\ C)[(6\ \text{amps})(45\ \Omega)]$$
$$= 2.9\text{x}10^6\ \text{joules.}$$

c.) Power is formally defined as the work per unit time done on or by a system. Using that definition yields:

$$P = W/t$$
$$= (2.9\text{x}10^6 \text{ joules})/[(30 \text{ min})(60 \text{ sec/min})]$$
$$= 1.6\text{x}10^3 \text{ watts.}$$

16.7) The power provided to the 18 Ω resistor is 125 watts. That means the resistor can (and does) dissipate 125 joules of energy per second (watts are joules/second).

a.) Using the definition of power, we get:

$$P = i^2R$$
$$\Rightarrow \quad i = (P/R)^{1/2}$$
$$= [(125 \text{ w})/(18\ \Omega)]^{1/2}$$
$$= 2.64 \text{ amps.}$$

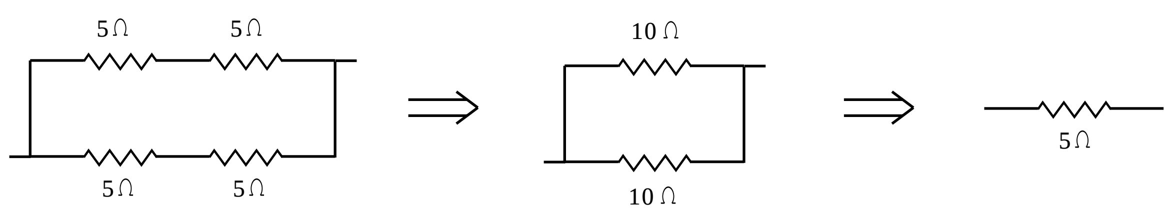
b.) The voltage across a resistor through which the current is known is determined using Ohm's Law. That is:

$$V = iR$$
$$= (2.64 \text{ a})(18\ \Omega)$$
$$= 47.52 \text{ volts.}$$

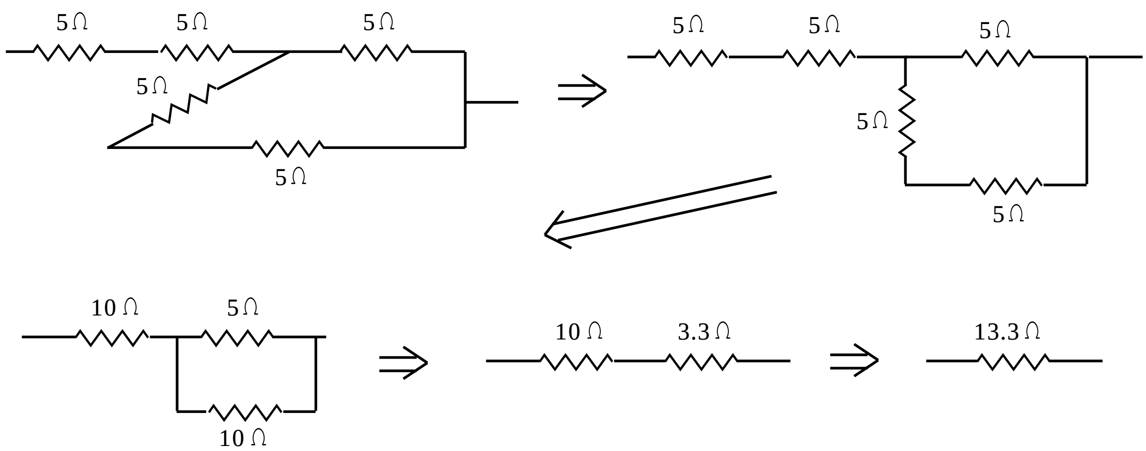
Note: $P = iV = (47.52 \text{ volts})(2.64 \text{ amps}) = 125.45 \text{ watts}$ . . . close enough!

16.8) Each successively simplified circuit is shown below:

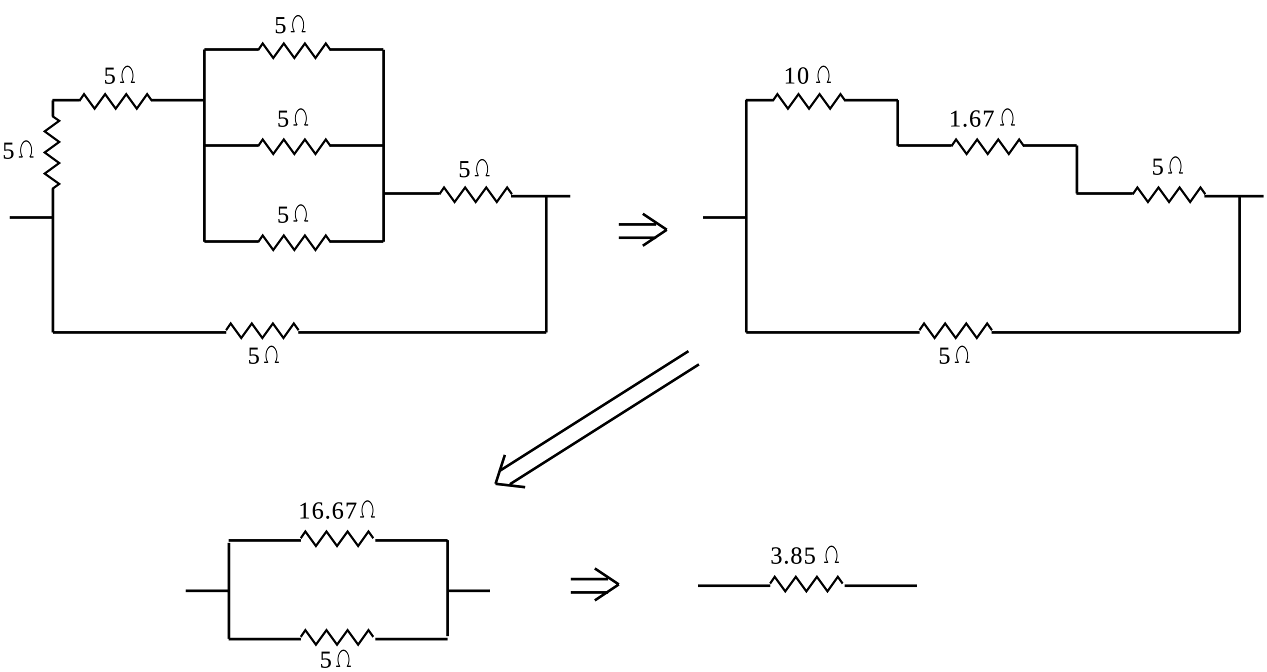
a.)



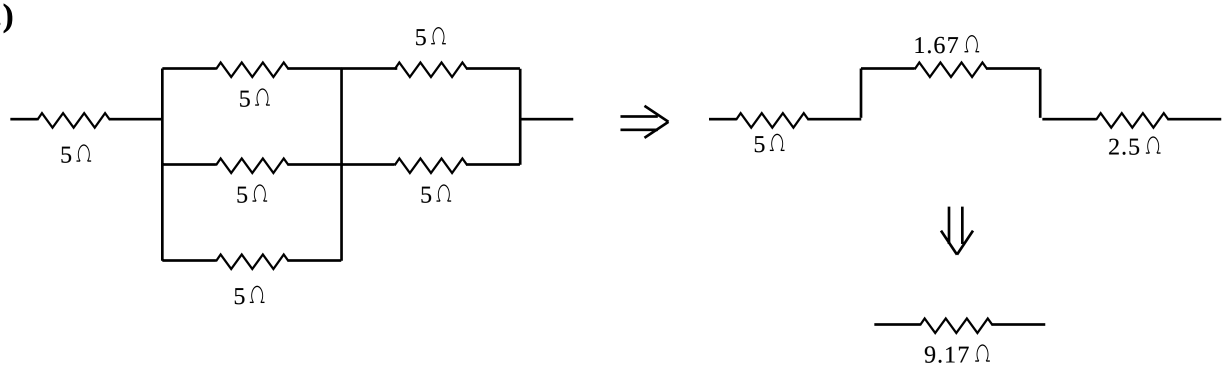
b.)


5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
10 Ω
5 Ω
10 Ω
10 Ω
3.3 Ω
13.3 Ω

c.)

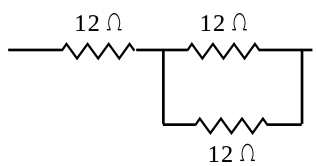

5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
5 Ω
10 Ω
1.67 Ω
5 Ω
5 Ω
16.67 Ω
5 Ω
3.85 Ω

d.)

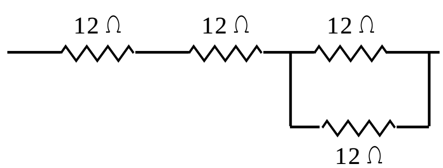


16.9) Doing problems like this, the best way to start is to use a series combination to get close to the required value, then use series and parallel combinations as need be to zero in on the actual value desired.

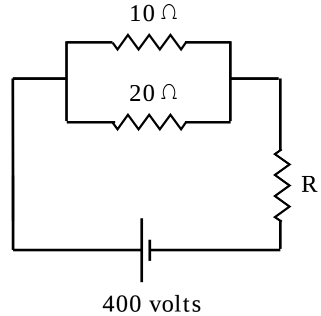
a.) For an 18 Ω resistance:



b.) For a 30 Ω resistance:



16.10) We can determine the total current drawn from the battery knowing that the power provided by the battery is 800 watts and the voltage of the battery is 400 volts. Doing so yields:



$$P_{bat} = i_{tot}V_{bat}$$
$$\Rightarrow \quad i_{tot} = P_{bat}/V_{bat}$$
$$= (800\ \text{w})/(400\ \text{v})$$
$$= 2\ \text{amps}.$$

The equivalent resistance of the parallel combination is:

$$1/R_{equ} = 1/(10\ \Omega) + 1/(20\ \Omega)$$
$$\Rightarrow \quad R_{equ} = 6.67\ \Omega.$$

The total equivalent resistance of the parallel resistor combination in series with R is:

$$R_{eq,tot} = R + 6.67\ \Omega.$$

The total voltage across all the resistors (equal to the battery's voltage) equals the total current through the systems times the total equivalent resistance of the system ($R_{eq,tot}$), or:

$$V_{bat} = i_{tot} R_{eq,tot}$$
$$(400\ v) = (2\ a)(R + 6.67\ \Omega)$$
$$\Rightarrow \quad R = 193.3\ \Omega.$$

16.11) Note that the nodes and currents are identified and/or defined in Figure I to the right and the loops are defined on the Figures below and on the next page.

a.) There are four nodes in this circuit (see Figure I to the right).

b.) There are seven loops in this circuit. Each has been highlighted in the composite Figures shown below and on the next page. Note that although you don't

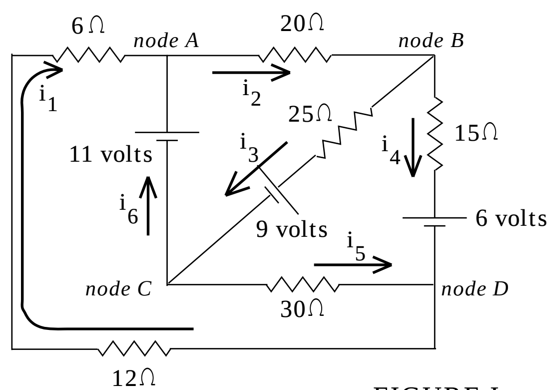
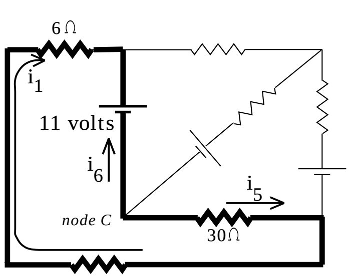
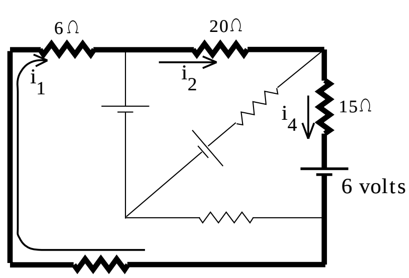


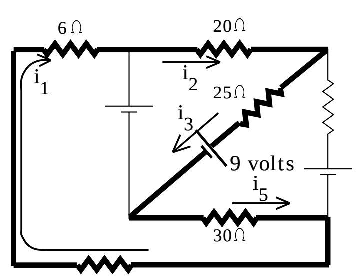
FIGURE I



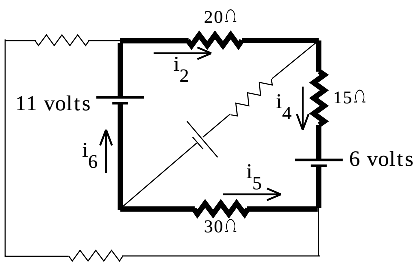
Loop 1



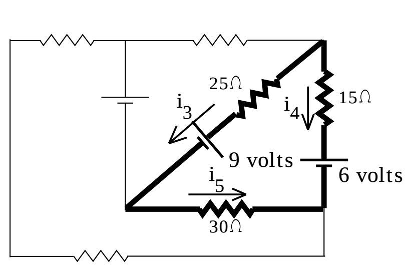
Loop 2



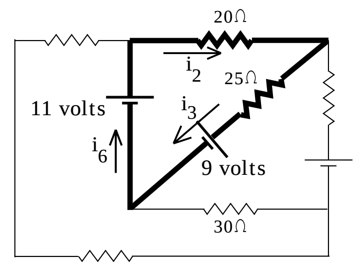
Loop 3



Loop 4

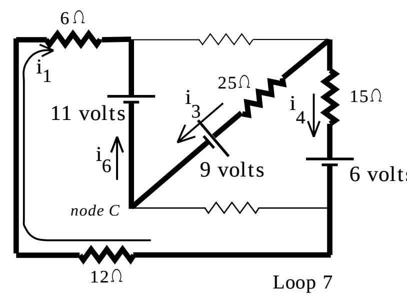


Loop 5



Loop 6

have to include current directions when defining loops, the current information is needed when writing out loop equations. As such, relevant currents have been included in the diagram while extraneous information has been deleted.



Loop 7

c.) The possible node equations are (see Figure I):

Node A: $i_1 + i_6 = i_2$;
Node B: $i_2 = i_3 + i_4$;
Node C: $i_3 = i_6 + i_5$;
Node D: $i_4 + i_5 = i_1$;

d.) The possible loop equations are shown below. Note that in all cases, I have arbitrarily chosen to traverse the loop in a CLOCKWISE direction. Note also that I have included the units for the resistors--something you would probably not bother to do on a test.

Loop 1:
$$-(6\ \Omega)i_1 - (11\text{ volts}) - (30\ \Omega)i_5 - (12\ \Omega)i_1 = 0$$

Loop 2:
$$-(6\ \Omega)i_1 - (20\ \Omega)i_2 - (15\ \Omega)i_4 - (6\text{ volts}) - (12\ \Omega)i_1 = 0$$

Loop 3:
$$-(6\ \Omega)i_1 - (20\ \Omega)i_2 - (25\ \Omega)i_3 - (9\text{ volts}) - (30\ \Omega)i_5 - (12\ \Omega)i_1 = 0$$

Loop 4:
$$-(20\ \Omega)i_2 - (15\ \Omega)i_4 - (6\text{ volts}) + (30\ \Omega)i_5 + (11\text{ volts}) = 0$$

Loop 5:

$(9 \text{ volts}) + (25\ \Omega)i_3 - (15\ \Omega)i_4 - (6 \text{ volts}) + (30\ \Omega)i_5 = 0$

Loop 6:

$(11 \text{ volts}) - (20\ \Omega)i_2 - (25\ \Omega)i_3 - (9 \text{ volts}) = 0$

Loop 7:

$-(6\ \Omega)i_1 - (11 \text{ volts}) + (9 \text{ volts}) + (25\ \Omega)i_3 - (15\ \Omega)i_4 - (6 \text{ volts}) - (12\ \Omega)i_1 = 0.$

16.12) By rewriting and rearranging the equations as shown below, the matrix manipulation will be easier. The equations are:

$$13i_1 - 9i_2 + 4i_3 = -6$$
$$-4i_1 + 0i_2 - 7i_3 = 0$$
$$-5i_1 + 3i_2 + 0i_3 = 0.$$

These equations can be written as a three-by-three matrix called a DETERMINATE D equal to a one-by-three matrix, or:

$$\begin{vmatrix} 13 & -9 & 4 \\ -4 & 0 & -7 \\ -5 & 3 & 0 \end{vmatrix} = \begin{vmatrix} -6 \\ 0 \\ 0 \end{vmatrix}$$

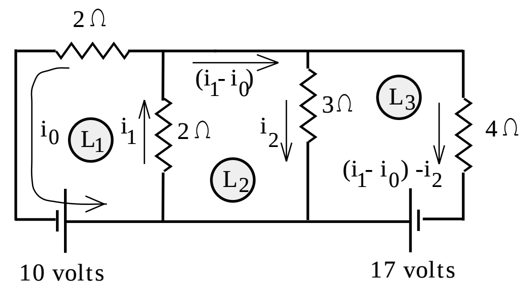
Solving for i_2 requires dividing the evaluated determinate into a second matrix defined by replacing the determinate's i_2 column by the one-by-three matrix to the right of the equal sign. Doing so yields:

$$i_2 = \frac{D_{mod,i_2}}{D} = \frac{\begin{vmatrix} 13 & -6 & 4 \\ -4 & 0 & -7 \\ -5 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 13 & -9 & 4 \\ -4 & 0 & -7 \\ -5 & 3 & 0 \end{vmatrix}}$$

OR

$$i_2 = \frac{[(13)[0 - (-7)(0)] + (-6)[(-7)(-5) - (-4)(0)] + (4)[(-4)(0)-0(-5)]]}{[(13)[0 - (-7)(3)] + (-9)[(-7)(-5) - (-4)(0)] + (4)[(-4)(3) - 0(-5)]]}$$

$= 2.33$ amps.



16.13) All meters can be removed from a circuit as long as we remember what we are looking for (i.e., the voltage across a particular circuit element or whatever). The re-wired circuit is shown to the right, complete with loops and nodes.

a.) We must determine all the currents, so it really doesn't matter which we call what as long as we get it down to three unknowns. Because we have already used our node equations in defining our currents, our last three equations must come from loops. The ones that have been chosen have been chosen for ease of presentation. There would not have been anything wrong with making one of the loops the outside loop, or the loop that includes the outside 2Ω resistor, the inside 3Ω resistor, and the 10 volt battery. I'm not going to define the direction I'm traversing in each loop. You should be able to tell by looking to see if the voltage difference across a resistor is positive or negative (if it is negative, we are traversing IN THE DIRECTION OF CURRENT FLOW; if positive, it's vice versa).

Loop I:

$$(10 \text{ volts}) - (2\ \Omega)i_1 - (2\ \Omega)i_o = 0$$

$$\Rightarrow \quad 2i_o + 2i_1 = 10 \qquad \text{(Equation A).}$$

Loop II:

$$-(2\ \Omega)i_1 - (3\ \Omega)i_2 = 0$$

$$\Rightarrow \quad 2i_1 + 3i_2 = 0 \qquad \text{(Equation B).}$$

Loop III:

$$(17 \text{ volts}) + (3\ \Omega)i_2 - (4\ \Omega)(i_1 - i_o - i_2) = 0$$

$$\Rightarrow \quad 4i_0 - 4i_1 + 7i_2 = -17 \qquad \text{(Equation C).}$$

Putting these in DETERMINATE matrix form yields:

$$\begin{vmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 4 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 10 \\ 0 \\ -17 \end{vmatrix}$$

Solving for i_o yields:

$$i_o = \frac{D_{mod.i_o}}{D} = \frac{\begin{vmatrix} 10 & 2 & 0 \\ 0 & 2 & 3 \\ -17 & -4 & 7 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 4 & -4 & 7 \end{vmatrix}}$$

$$i_o = \frac{\left[(10)[14-(-12)]\right] + (2)\left[(-51)-(0)\right] + (0)\left[(0)-(-34)\right]}{\left[(2)[14-(-12)] + (2)[(12)-(0)] + (0)[(0)-(-8)]\right]}$$
$$= 2.08 \text{ amps.}$$

Solving for i_1 yields:

$$i_o = \frac{D_{mod.i_o}}{D} = \frac{\begin{vmatrix} 2 & 10 & 0 \\ 0 & 0 & 3 \\ 4 & -17 & 7 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 4 & -4 & 7 \end{vmatrix}}$$

$$i_o = \frac{\left[(2)[(0)-(-51)]\right] + (10)\left[(12)-(0)\right] + (0)\left[(0)-(0)\right]}{\left[(2)[14-(-12)] + (2)[(12)-(0)] + (0)[(0)-(-8)]\right]}$$
$$= 2.92 \text{ amps.}$$

Solving for i_2 yields:

$$i_o = \frac{D_{mod.i_o}}{D} = \frac{\begin{vmatrix} 2 & 2 & 10 \\ 0 & 2 & 0 \\ 4 & -4 & -17 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 4 & -4 & 7 \end{vmatrix}}$$

$$i_2 = \frac{[(2)[(-34) - 0] + (2)[(0) - 0] + (10)[(0) - (8)]]}{[(2)[14 - (-12)] + (2)[(12) - 0] + (0)[(0) - (8)]]}$$
$= -1.95$ amps.

NOTE: The negative sign simply means we have assumed the wrong direction for the current i_2. This isn't a big deal. The current through the 4Ω resistor, for instance, is just as stated: $(i_1 - i_o - i_2) = (2.92 - 2.08 - (-1.95)) = +2.79$ amps. The plus sign here means we have assumed the correct current direction through that resistor.

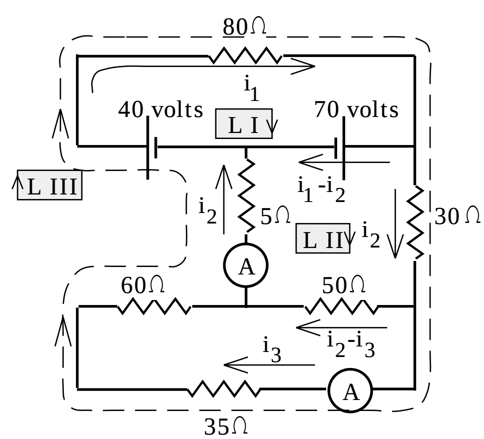
This means:

--The current through ammeter A_1 is $i_1 - i_o = 2.92 - 2.08 = .84$ amps;

--The current through ammeter A_2 is $i_2 = -1.95$ amps; and

--The voltage across the 2Ω resistor is $i_oR_{2\Omega} = (2.08\text{ a})(2\ \Omega) = 4.16$ volts.

b.) The circuit, loop and currents are shown to the right. The branches through which we need currents have been defined with single-variable currents (in this case they are i_2 or i_3). Notice that i_2 pops up twice in the circuit. This is a consequence of the way the currents have been defined (follow through from branch to branch and you will find that the current through the 30Ω resistor must equal the current through the 5Ω resistor and ammeter).



(b)

Loop I:

$$(40\text{ volts}) - (80\ \Omega)i_1 - (70\text{ volts}) = 0$$
$$\Rightarrow \quad i_1 = -.375\text{ a} \qquad \text{(Equation A).}$$

We have just eliminated one unknown. To solve for the others we could use a matrix approach or straight algebra. I'll do it both ways. In either case, we will need the other two loop equations:

Loop II:

$$(70 \text{ volts}) - (30\ \Omega)i_2 - (50\ \Omega)(i_2 - i_3) - (5\ \Omega)i_2 = 0$$

$$\Rightarrow \quad 85i_2 - 50i_3 = 70 \qquad \text{(Equation B).}$$

Loop III:

$$(40 \text{ volts}) - (80\ \Omega)i_1 - (30\ \Omega)i_2 - (35\ \Omega)i_3 - (60\ \Omega)i_3 - (5\ \Omega)i_2 = 0$$

$$40 - (80\ \Omega)(-.375 \text{ a}) - (30\ \Omega)i_2 - (35\ \Omega)i_3 - (60\ \Omega)i_3 - (5\ \Omega)i_2 = 0$$

$$\Rightarrow \quad -35i_2 - 95i_3 = -70$$

$$\Rightarrow \quad 35i_2 + 95i_3 = 70 \qquad \text{(Equation C).}$$

SOLVING EQUATIONS B and C ALGEBRAICALLY:

$$85i_2 - 50i_3 = 70$$

$$\Rightarrow \quad i_2 = (70 + 50i_3)/85$$

$$= .824 + (.588)i_3.$$

Substituting into $35i_2 + 95i_3 = 70$ yields:

$$35i_2 \qquad + 95i_3 = 70$$

$$35[.824 + .588i_3] + 95i_3 = 70$$

$$28.84 + 20.56i_3 + 95i_3 = 70$$

$$\Rightarrow \quad i_3 = .356 \text{ amps.}$$

To determine i_2:

$$85i_2 - \quad 50i_3 \quad = 70$$

$$85i_2 - 50(.356) = 70$$

$$\Rightarrow \quad i_2 = 1.033 \text{ amps.}$$

Using a matrix form (I will do it only for i_2--you can try it on your own to see if i_3 checks out), we start by putting Equations B and C in matrix form:

$$85i_2 - 50i_3 = 70$$
$$35i_2 + 95i_3 = 70$$

becomes:

$$\begin{vmatrix} 85 & -50 \\ 35 & 95 \end{vmatrix} = \begin{vmatrix} 70 \\ 70 \end{vmatrix}$$

Solving for i_2 (i.e., the "first-column" variable) yields:

$$i_2 = \frac{D_{mod,i_2}}{D} = \frac{\begin{vmatrix} 70 & -50 \\ 70 & 95 \end{vmatrix}}{\begin{vmatrix} 85 & -50 \\ 35 & 95 \end{vmatrix}}$$

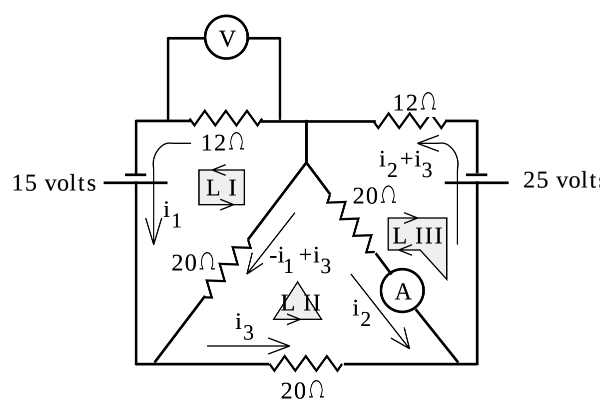
or

$$i_2 = \frac{[(70)(95) - (-50)(70)]}{[(85)(95) - (-50)(35)]}$$

$$= 1.03 \text{ amps.}$$

Similarly solving for i_3 yields i_3 = .36 amps.

c.) The circuit along with loops and currents is shown to the right. Notice that the currents we need are defined as i_1 and i_2. The loop equations follow.



(c)

Loop I:

$$(15 \text{ volts}) + (20\ \Omega)(-i_1 + i_3) - (12\ \Omega)i_1 = 0$$

$$\Rightarrow \quad 32i_1 - 20i_3 = 15 \qquad \text{(Equation A).}$$

Loop II:

$$-(20\ \Omega)(-i_1 + i_3) - (20\ \Omega)i_3 + 20i_2 = 0$$

$$\Rightarrow \quad 20i_1 + 20i_2 - 40i_3 = 0 \qquad \text{(Equation B).}$$

Loop III:

$$(25 \text{ volts}) + (20\ \Omega)i_2 + (12\ \Omega)(i_2 + i_3) = 0$$

$$\Rightarrow \quad 32i_2 + 12i_3 = -25 \qquad \text{(Equation C).}$$

These are put in DETERMINATE matrix form as shown to the right:

$$\begin{vmatrix} 32 & 0 & -20 \\ 20 & 20 & -40 \\ 0 & 32 & 12 \end{vmatrix} = \begin{vmatrix} 15 \\ 0 \\ -25 \end{vmatrix}$$

Solving for i_1 yields:

$$i_1 = \frac{D_{mod,i_1}}{D} = \frac{\begin{vmatrix} 15 & 0 & -20 \\ 0 & 20 & -40 \\ -25 & 32 & 12 \end{vmatrix}}{\begin{vmatrix} 32 & 0 & -20 \\ 20 & 20 & -40 \\ 0 & 32 & 12 \end{vmatrix}}$$

$$i_1 = \frac{\big[(15)[240 - (-1280)] + [0] + (-20)[(0) - (-500)]\big]}{\big[(32)[240 - (-1280)] + [0] + (-20)[(640) - (0)]\big]}$$

$= .357$ amps.

Solving for i_2 yields:

$$i_2 = \frac{D_{mod,i_2}}{D} = \frac{\begin{vmatrix} 32 & 15 & -20 \\ 20 & 0 & -40 \\ 0 & -25 & 12 \end{vmatrix}}{\begin{vmatrix} 32 & 0 & -20 \\ 20 & 20 & -40 \\ 0 & 32 & 12 \end{vmatrix}}$$

$$i_2 = \frac{\left[(32)[0 - (1000)] + (15)[(0) - (240)] + (-20)[(-500) - (0)]\right]}{\left[(32)[240 - (-1280)] + [0] + (-20)[(640) - (0)]\right]}$$

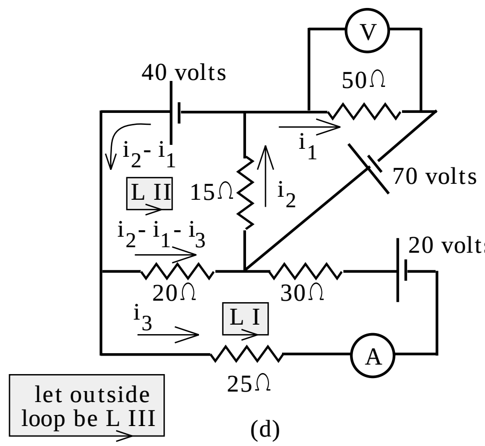
$= -.714$ amps.

We now know:

--the current through the ammeter A is $i_2 = -.714$ amps;

--the voltmeter will read $i_1 R_{12\Omega} =$ $(.357\ a)(12\ \Omega) = 4.29$ volts.

d.) The circuit along with loops and currents is shown to the right. Notice that the currents we need are defined as i_1 and i_3. The loop equations follow.



(d)

Loop I:

$$(20\text{ volts}) - (30\ \Omega)i_3 + (20\ \Omega)(i_2 - i_1 - i_3) - (25\ \Omega)i_3 = 0$$

$$\Rightarrow \quad 20i_1 - 20i_2 + 75i_3 = 20 \qquad \text{(Equation A).}$$

Loop II:

$$(40\text{ volts}) - (20\ \Omega)(i_2 - i_1 - i_3) - (15\ \Omega)i_2 = 0$$

$$\Rightarrow \quad -(20\ \Omega)i_1 + (35\ \Omega)i_2 - 20i_3 = 40 \qquad \text{(Equation B).}$$

Loop III:

$$-(25\ \Omega)i_3 + (20\text{ volts}) - (30\ \Omega)i_3 - (70\text{ volts}) + (50\ \Omega)(i_1) + (40\text{ volts}) = 0$$

$$\Rightarrow \quad 50i_1 - 55i_3 = 10 \qquad \text{(Equation C).}$$

These are put in DETERMINATE matrix form as shown below:

$$\begin{vmatrix} 20 & -20 & 75 \\ -20 & 35 & -20 \\ 50 & 0 & -55 \end{vmatrix} = \begin{vmatrix} 20 \\ 40 \\ 10 \end{vmatrix}$$

Solving for i_1 yields:

$$i_1 = \frac{D_{mod,i_1}}{D} = \frac{\begin{vmatrix} 20 & -20 & 75 \\ 40 & 35 & -20 \\ 10 & 0 & -55 \end{vmatrix}}{\begin{vmatrix} 20 & -20 & 75 \\ -20 & 35 & -20 \\ 50 & 0 & -55 \end{vmatrix}}$$

$$i_1 = \frac{\left[(20)[(-1925) - (0)] + (-20)[(-200) - (-2200)] + (75)[(0) - (350)]\right]}{\left[(20)[(-1925) - (0)] + (-20)[(-1000) - (1100)] + (75)[(0) - (1750)]\right]}$$

$= .82$ amps.

Solving for i_3 yields:

$$i_3 = \frac{D_{mod,i_3}}{D} = \frac{\begin{vmatrix} 20 & -20 & 20 \\ -20 & 35 & 40 \\ 50 & 0 & 10 \end{vmatrix}}{\begin{vmatrix} 20 & -20 & 75 \\ -20 & 35 & -20 \\ 50 & 0 & -55 \end{vmatrix}}$$

$$i_3 = \frac{\left[(20)[(350) - (0)] + (-20)[(2000) - (-200)] + (20)[(0) - (1750)]\right]}{\left[(20)[(-1925) - (0)] + (-20)[(-1000) - (1100)] + (75)[(0) - (1750)]\right]}$$

$= .56$ amps.

We now know:

--The current through the ammeter A is $i_3 = .564$ amps;

--The voltage across the 50 Ω resistor is $i_1 R_{50\Omega} = (.82\ a)(50\ \Omega) = 41$ volts.