Chapter 16

DC CIRCUITS

A.) Preamble:

1.) Consider a system designed to force a steady flow of water through a pipe. A schematic of the proposed system is shown in Figure 16.1. Note the following:

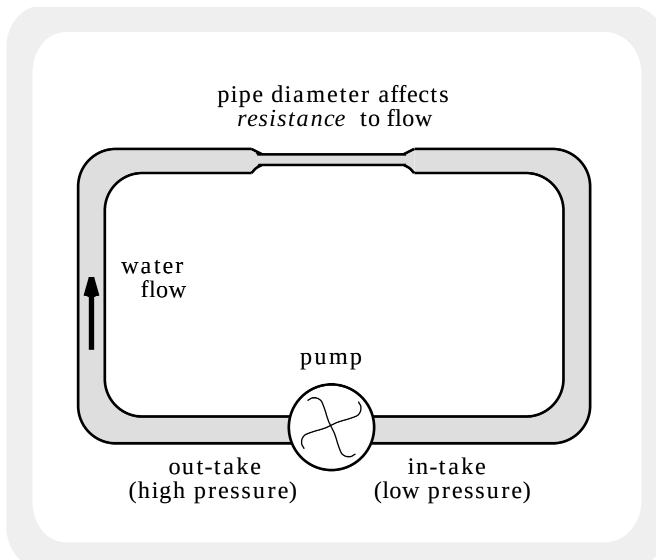


FIGURE 16.1

a.) Energy is provided to the system by a water pump. Water at low pressure enters the pump's intake port, has work done on it, and leaves the output port at higher pressure. All fluids move from high pressure to low pressure; the pressurized output water moves clockwise toward the low pressure side of the system.

b.) In some cases it is useful to know how much water passes a particular point in the circuit per unit time. This is called the flow-rate; its units are in gallons per second.

Note: Flow-rate should not be confused with rate of change quantities like acceleration. Flow-rate does not measure how far the flow travels per unit time (in my country we call that quantity velocity). Instead, it assumes a counter is sitting in the water's path counting the number of gallons of water that pass by per second.

Important Note: Assume, for the sake of argument, that someone at one point in the circuit records 12 gallons of water passing per second. If there is no place for the water to go except around the one closed circuit, how could someone else at another point record only 8 gallons passing by per second? Where would the lost 4 gallons have gone?

Bottom line: It does not matter whether a pipe's diameter is constant throughout or narrow in some spots and wide in others. The very nature of steady state water flow implies that THE AMOUNT OF WATER PASSING BY PER UNIT TIME AT ANY GIVEN POINT MUST BE THE SAME AT ALL POINTS IN THE CIRCUIT BRANCH.

Note to the Important Note: If the flow-rate is constant in a single-branch circuit, the quantity of water passing through narrow sections of pipe (per unit time) must be the same as the quantity of water passing through the broader sections. The only way the water can do this is by speeding up through the narrower sections.

That is exactly what happens. The water's speed changes with pipe diameter whereas the flow-rate stays the same throughout.

c.) There will undoubtedly be resistance to the water flow. This can be due to: 1.) the clinging of the water to the pipe, and 2.) restrictions on water-flow as a consequence of narrow piping.

For a given pump, the net resistance in a water circuit "limits the water flow" in the sense that high resistance (a narrow pipe) allows only a modest flow-rate whereas low resistance (a broader pipe) allows a substantial flow-rate.

2.) In some ways, an electrical system acts like a water system. Both require a power source; both experience the flow of some substance; both have a flow-rate of sorts; and both feel resistance to flow. But in many ways, there are also major differences. Specifically:

a.) Water flow through a pipe is affected by creating a pressure difference. Water at high pressure is motivated to move to lower pressure.

b.) In electrical circuits, electrical charges at the high voltage side of a power supply do not push neighboring charges to make them move. What creates charge motion is the electrical potential difference provided by the power supply. That potential difference creates an electric field which motivates all moveable charge carriers (i.e., the electrons) to move at the same time in response to the electric field (the field sets itself up at nearly the speed of light).

c.) Bottom line: Charge-motion in an electrical circuit is not a domino effect. It's more like a drill instructor ordering, "March!"

B.) Electrical Circuit Elements--Definitions and General Information:

1.) The electrical circuit in Figure 16.2 is a very simple example of the kind of circuits and circuit-elements you will be dealing with in this chapter. This section deals with the elements, their uses, and theoretical points-of-interest associated with each.

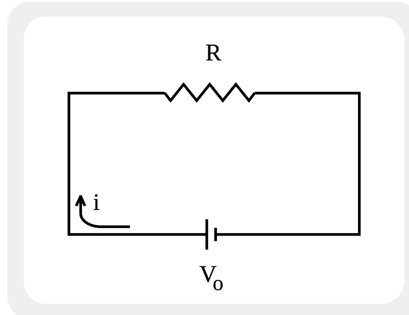


FIGURE 16.2

2.) The Power Supply:

a.) Power supplies do exactly as their name implies. They provide energy to a circuit through an electrical potential difference generated between their terminals.

b.) There are two kinds of power supplies: DC power sources and AC power sources. AC power supplies provide electric fields that alternate in direction. We will study AC circuits later.

DC sources provide an electric field that is always in the same direction. That means DC currents always flow in one direction only, hence the name Direct Current.

c.) The basic circuit representation for a DC power supply is shown in Figure 16.3a.

i.) The high voltage terminal, often referred to as the hot terminal, is usually labeled "+" on a circuit diagram. Whether labeled or not, the high voltage side is always the longer of two vertical lines in the schematic representation.

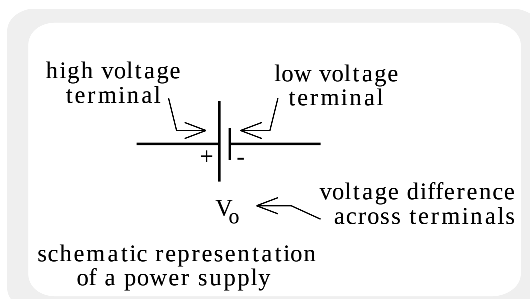


FIGURE 16.3a

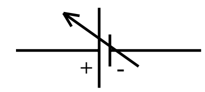
ii.) The low voltage terminal, often referred to as the ground or common terminal, is usually labeled "-". Whether labeled or not, the low voltage side is always the shorter of the two vertical lines in the schematic representation (it looks like a vertical negative sign).

Note: When actually confronted with a power supply in the lab, the low voltage terminal is normally color-coded black while the high voltage terminal is normally color-coded red.

d.) The representation for a variable DC power supply is shown in Figure 16.3b. The representation of a group of DC power supplies connected together in series is shown in Figure 16.3c.

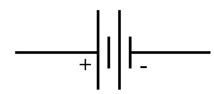
e.) When a battery is rated at, say, 12 volts, the voltage rating is the potential difference between the + and - terminals.

f.) The absolute electrical potential of the ground (-) terminal is always assumed to be ZERO.



schematic representation
of a variable power supply

FIGURE 16.3b



schematic representation
of more than one power supply
connected in series

FIGURE 16.3c

3.) Current:

a.) The flow of electrical charge through a circuit is called current. Its symbol is i and it is mathematically defined as:

$$i = q / t,$$

where q is the amount of charge that passes by a particular point in a circuit and t is the time required for the passing (it is also written as $i = dq/dt$ when a differential form is applicable).

Current is the analog to flow rate in a water circuit.

b.) The MKS units for current are coulombs per second. This is given a special name--the ampere--which is often shortened to amp.

In the MKS system, one amp is equal to one coulomb of charge passing by per second.

c.) Current is denoted in a circuit by an arrow (see Figure 16.4).

d.) Just like water-flow-rate, the current in a particular branch will be the same no matter where you are in the branch. In Figure 16.4, the current measured at Point A (i.e., the amount of charge that passed by Point A per unit time) must be the same as the current at Points B and C.

e.) MAJOR THEORETICAL FAUX PAS: Notice that the current in Figure 16.4 is directed from the higher to lower voltage terminals of the power supply

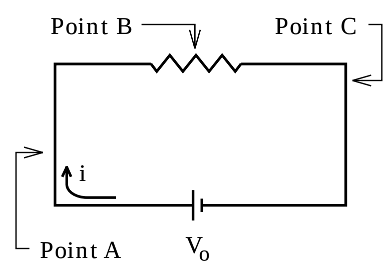


FIGURE 16.4

(i.e., in the direction that POSITIVE CHARGE would move if positive charge could move in wire). This is due to the fact that when this theory was being developed, scientists didn't know whether positive or negative charges moved through circuits. As all previous theory was based on the electrical response of positive charge, it was assumed that positive charge was the charge-type that circulated through electric circuits.

You and I and all of science now know that electrons move in metals, but because the "positive charge carriers" theory was so firmly rooted by the time the true nature of charge motion became evident, and because the model works as it stands, CURRENT DIRECTION IN CIRCUITS IS DEFINED AS THE DIRECTION POSITIVE CHARGES WOULD MOVE IF THEY COULD MOVE.

Be aware: when this or any other textbook refers to current, the assumption is that positive charges are in motion.

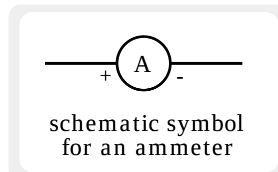


FIGURE 16.5

f.) The circuit device that measures current is called an ammeter. The circuit symbol for an ammeter is shown in Figure 16.5.

Ammeters have high voltage and low voltage terminals. These are usually marked on ammeters by + and - signs.

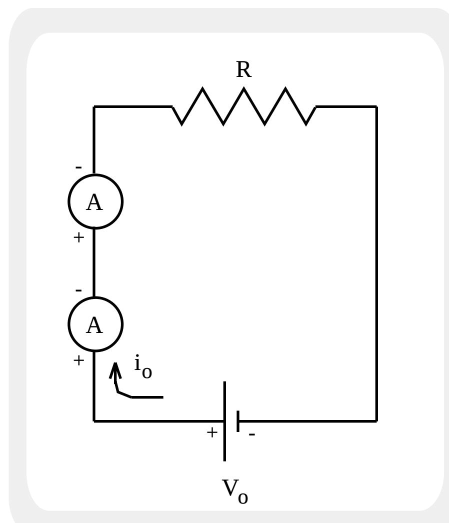


FIGURE 16.6

Note: If we put two ammeters in series with one another (Figure 16.6), notice that we will have the - terminal of the first meter hooked to the + terminal of the second meter. There is nothing wrong with this. The signs are <u>SPECIFIC TO EACH METER</u>. That is, a + sign simply designates the higher voltage side of the meter.

4.) Resistance:

a.) Under everyday circumstances, there will always be some resistance to the flow of charge through a circuit. The cause of resistance is summarized below:

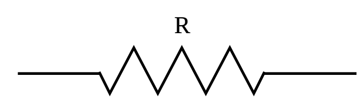
i.) Having felt the electric field set up by a power supply, free charge carriers in a wire will accelerate. Theoretically, they will move from higher electrical potential to lower electrical potential (we are assuming positive charges here) and in doing so they will turn their electrical <u>potential energy</u> into kinetic energy.

ii.) After moving some distance (this distance varies from material to material; for a given material, the average of this distance is called the mean free path), they inevitably collide with an atom in the atomic structure of the wire. When this happens, they bounce off the atoms, giving up their kinetic energy in the process.

iii.) Atoms absorb energy in one of two basic ways: they either begin to increase their vibrational motion relative to one another (this translates into what we call heat) or under the right conditions they throw one of their valence electrons into an upper atomic orbital. As the electron spontaneously cascades down to lower and lower orbitals on its way back to the ground level, it gives off energy as electromagnetic radiation. If the radiation's frequency is in the optical range, we see it as light.

iv.) Bottom line: No matter how the energy is dissipated, charge-carrier motion is impeded. Put another way, charge-carriers experience resistance to flow.

b.) There is an electrical element that is designed specifically to increase the resistance to charge flow in a circuit. It is called a resistor. The symbol for a resistor in a circuit diagram is shown in Figure 16.7. Resistor characteristics are listed below:



schematic for a resistor

FIGURE 16.7

i.) Resistors remove energy as charges pass through them. As such, the electrical potential (the available potential-energy per unit charge) on the charge-entry side of a resistor will always be higher than the electrical potential on the charge-exit side. In other words, charge carriers passing through resistors experience a voltage drop.

ii.) If the voltage drop across a resistor is V_R (note that this is really a change of voltage ΔV across the resistor, though no physics text writes it so--V_R is sloppy notation but unfortunately conventional), the current i_R through the resistor will be proportional to V_R. That is, doubling the voltage across the resistor will double the current through the resistor. To express this relationship, we write:

$$V_R = (i_R) R.$$

The proportionality constant R is defined as the resistance of the resistor.

iii.) The units for resistance--volts per amp--are given a special name--the ohm. In the MKS system, a one ohm resistance will allow a one volt potential difference to generate a one amp current through the resistor.

iv.) The symbol for the ohm is Ω. Resistance values run anywhere from a few thousandths of an ohm (the resistance of a short piece of wire) to millions of ohms. Abbreviations are often used, most commonly:

--1000 ohms $\Rightarrow$ 1 kΩ (the k stands for kilo, or thousand)

--1,000,000 ohms $\Rightarrow$ 1 MΩ (the M stands for mega, or million).

Note: Even though a 43,500 ohm resistor will usually be written as 43.5 kΩ on circuit diagrams, all formulas using resistance values must be calculated in ohms. That is, you have to convert a resistance value like 43.5 kΩ to 43,500 ohms before you can use it in a problem.

v.) The resistance to charge flow in a wire is quite small. That means very little energy is lost when charge moves through wire. It also means the electrical potential at the high voltage terminal of the power supply shown in Figure 16.8 will, to a good approximation, be the same as the voltage just before the resistor (i.e., at Point A). On the other side of the resistor, the voltage between Point B and the ground terminal will be approximately the same. The consequences of this are simple:

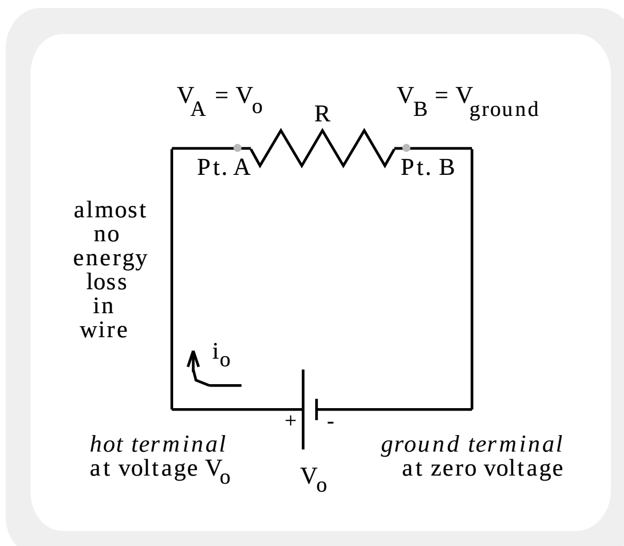


FIGURE 16.8

1.) It is legitimate to make the assumption that there is no voltage drop across wires linking circuit elements.

2.) In the case shown in Figure 16.8, the voltage difference across the power source V_o will be the same as the voltage difference across the resistor V_R.

vi.) Ammeters measure current--they tell you how much charge is passing through the meter per-unit-time. As such, they are placed directly into the flow of charge in the circuit. If they are to measure the current without changing the current characteristics of the system, ammeters must have very small resistances.

Voltmeters measure the voltage difference across a circuit element (i.e., the difference in the voltage between one side of an element and the other side of the element). If they are to do so without changing the current characteristics of the circuit, they must draw as little current as possible. As such, the resistance of a voltmeter is very large (usually at least 1000 times larger than the resistance of the element across which they are placed).

Note: If you inadvertently put an ammeter in place of a voltmeter, the ammeter's tiny resistance will allow large currents to be drawn from the power supply and you will most probably blow out both the power supply and the ammeter.

In other words, don't do that!

5.) Ohm's Law:

a.) If the relationship between the current through an element and the voltage across an element is proportional, the element is said to obey "Ohm's Law." The most commonly cited example of this situation is the resistor.

b.) From the relationship noted above, the voltage/current characteristic of a resistor is:

$$V_R = (i_R)R.$$

This relationship is often referred to as Ohm's Law (even though Ohm's Law technically covers a broader range of circuit elements).

6.) Nodes, Loops, and Branches:

a.) A loop in an electrical circuit is formally defined as any closed path within the circuit. The electrical diagrams so far presented have been very simple one-loop circuits. Most electrical systems are multiloop systems. The circuit in Figure 16.9a has three loops (they are shown in Figure 16.9b).

b.) A node is defined as a junction within the wiring of a circuit--a place where two or more wires meet. The current on one side of a node will never be the same as on the other side of the node. Figure 16.9c highlights the two nodes found in that circuit. Notice that Point E in that sketch is not a node (square corners don't automatically denote nodes).

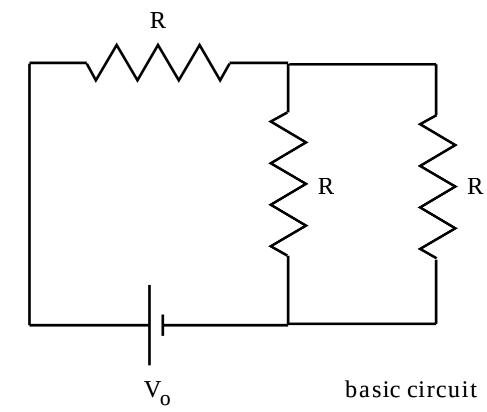
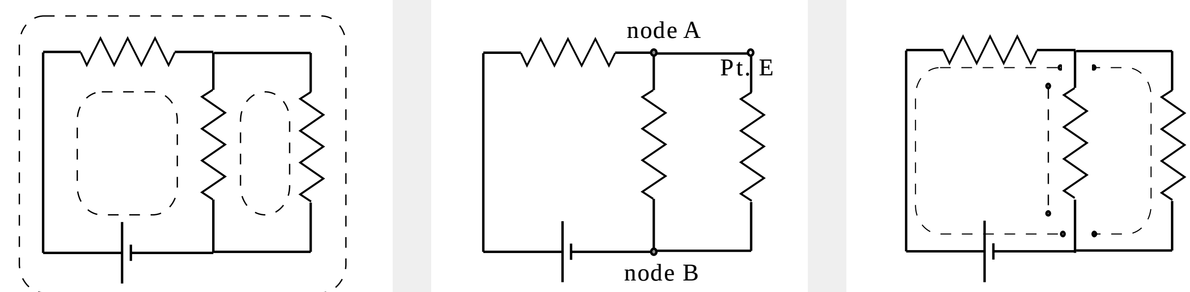


FIGURE 16.9a

c.) A section of circuitry between two nodes is called a branch. The current in a branch is the same throughout (there are no nodes in the middle of a branch where current can exit or enter). Figure 16.9d shows the three branches in our circuit.



the three loops in this circuit are depicted by dotted lines

FIGURE 16.9b

nodes are junctions: there are two in this circuit

FIGURE 16.9c

branches go from node to node: the three branches in this circuit are depicted with dotted lines

FIGURE 16.9d

7.) Voltmeters and Ammeters in a Circuit:

a.) In Figure 16.10 (next page), an ammeter and a voltmeter have been added to the above circuit.

b.) Meters are designed to measure electrical quantities within electrical circuits. Although all meters work by sensing the amount of current that flows through them, at least in theory their presence is not supposed to change the electrical nature (i.e., the current characteristics) of a circuit in which they are present.

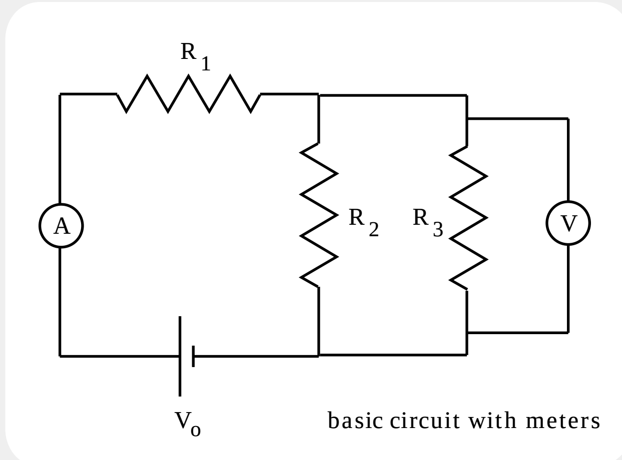


FIGURE 16.10

In other words, IN THEORY, AS FAR AS CHARGE-FLOW GOES, THERE IS NO DIFFERENCE BETWEEN THE CIRCUIT SHOWN IN FIGURE 16.9a AND THE CIRCUIT SHOWN IN 16.10.

c.) As stated above, ammeters are designed to measure currents. An ammeter is placed directly in the path of charge-flow. As such, the resistance of an ammeter must be very low (theoretically, zero).

d.) As stated above, voltmeters measure the voltage difference between two points. That is, hook a voltmeter up to a power supply (this is usually termed "putting a voltmeter across a power supply") and it will measure the power supply's voltage (i.e., the voltage difference between its terminals). By the same token, putting a voltmeter across a resistor will measure the voltage difference between the resistor's two sides.

Voltmeters are used in parallel to the element the-voltage-of-which-they-are-measuring. As voltmeters should not change the current characteristic of the circuit, they must sense the voltage by drawing just a tiny bit of current from the circuit (exactly how this process works will be discussed later). To keep from drawing more than an inconsequential trickle of current through the voltmeter, the resistance of a voltmeter must be enormous (theoretically, infinite).

e.) Bottom line: If you don't like seeing meters cluttering your circuits, TAKE THEM OUT (i.e., re-draw the circuit without the meters). If you do, though, remember that a question like, "What does the ammeter read?" is really asking, "How much current is in this branch?" Likewise, "What does the voltmeter read?" is the same as, "What is the voltage across this element?"

C.) Series and Parallel Combinations of Resistors:

1.) Figure 16.11 shows a series combination of resistors. Series combinations have three characteristics:

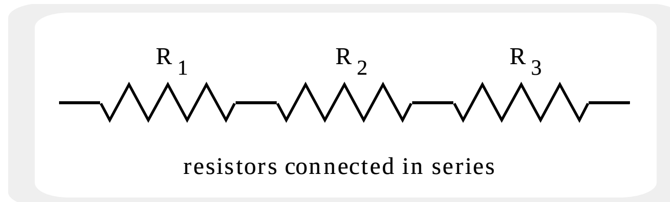


FIGURE 16.11

a.) Each resistance element in a series combination is linked to its neighbor at one place only;

b.) Although the voltage across each element may differ, the current through each element will always be the same. Put another way, CURRENT is common to all elements in a series combination;

c.) There can be no junctions between elements in a series combination (if there were, current would not be constant throughout).

2.) There will always be a single resistor that can take the place of the entire series combination in the circuit. The resistance of that resistor is called the equivalent resistance R_{eq} of the series combination. Replacing a series combination with its equivalent resistance does nothing to the circuit. The same current i_o will be drawn from a given power source V_o in both cases (see Figures 16.12 and 16.13).

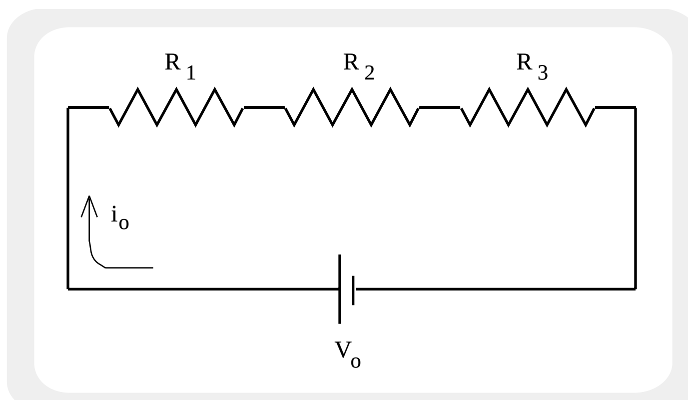


FIGURE 16.12

In the case of the series combination, the equivalent resistance is obvious--it is just the sum of the individual resistors in the combination. The proof of this follows.

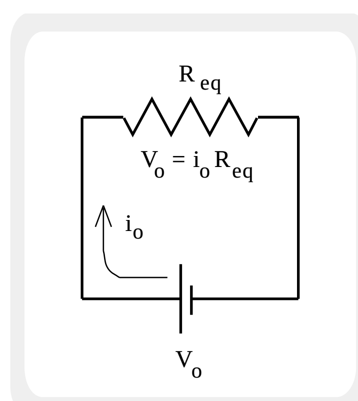


FIGURE 16.13

a.) The voltage drop across the resistor R_1 in the circuit shown in Figure 16.12 is $V_o = i_oR_1$. Similar expressions exist for the voltages across the other resistors (see Figure 16.14 for summary). If the total voltage across the entire series combination is the battery voltage V_o, we can write:

$$V_o = V_1 + V_2 + V_3 \ldots$$
$$= iR_1 + iR_2 + iR_3 \ldots$$

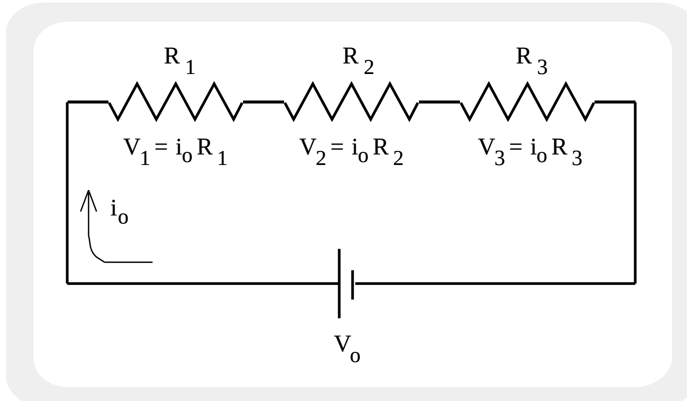


FIGURE 16.14

b.) When the equivalent resistor for our series combination R_{eq} is hooked up across the power supply V_o, the current drawn will BY DEFINITION be i_o--the same as that of our series combination. Figure 16.13 shows that hook-up along with the equivalent-resistance-related expression for V_o (i.e., $V_o = iR_{eq}$).

c.) Putting together the results from Parts a and b above, we can write:

$$V_o = V_1 + V_2 + V_3 \ldots$$
$$iR_{eq} = iR_1 + iR_2 + iR_3 \ldots$$

d.) Dividing out the current terms yields:

$$R_{eq} = R_1 + R_2 + R_3 \ldots$$

e.) Example: If the resistances in a series combination are 350 Ω, 125 Ω, 455 Ω, and 170 Ω, the equivalent resistance for the combination is:

$$R_{eq} = R_1 + R_2 + R_3 \ldots$$
$$= 350\ \Omega + 125\ \Omega + 455\ \Omega + 170\ \Omega$$
$$= 1100\ \Omega.$$

3.) Figure 16.15 (next page) shows a parallel combination of resistors. Parallel combinations have three characteristics:

a.) Each resistance element in a parallel combination is linked to its neighbor on both sides;

b.) There will be junctions between elements in a parallel combination;

c.) Although the current may differ, the voltage across each element in parallel will always be the same. Put another way, VOLTAGE is common to all elements in a parallel combination.

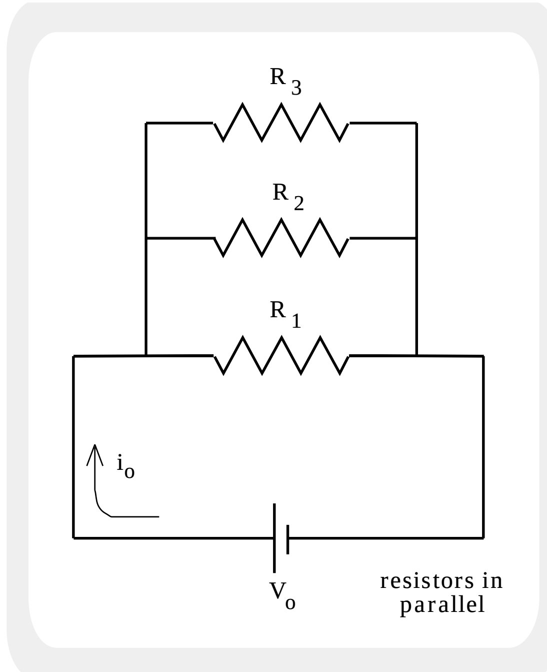


FIGURE 16.15

4.) As was the case with resistors in series, there is a single equivalent resistance R_{eq} that, when put across the power supply V_o, draws current i_o (exactly the same current as was drawn by the parallel combination). To determine the algebraic relationship that exists between R_{eq} and the parallel combination it mimics, consider the following:

a.) Although the voltage across each individual resistor will be the same in the parallel combination, the current through each resistor (i.e., i_1, i_2, and i_3) will be different.

b.) The sum of the currents through the various resistors ($i_1 + i_2 + i_3 + \ldots$) must be equal to the total current i_o being drawn from the power supply.

c.) By Ohm's Law, the current through a resistor R_1 is $i_1 = V_o/R_1$. Similar expressions exist for the currents through the other resistors (see Figure 16.16a). This means:

$$\begin{aligned} i_o &= i_1 + i_2 + i_3 \ldots \\ &= V_o/R_1 + V_o/R_2 + V_o/R_3 \ldots . \end{aligned}$$

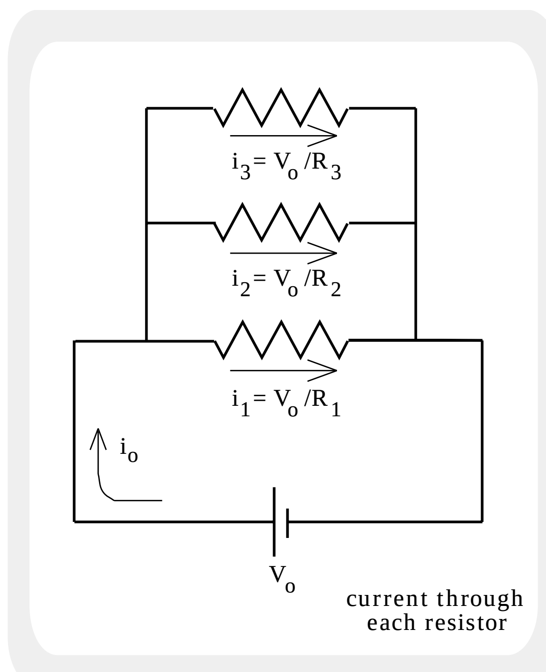


FIGURE 16.16a

d.) Figure 16.13 shows a typical R_{eq} circuit. As the voltage V_o across R_{eq} is the same as the voltages across each element in the parallel circuit, the current through R_{eq} will be $i_o = V_o/R_{eq}$.

e.) Putting together the results from the above steps, we get:

$$i_o = i_1 + i_2 + i_3 \ldots$$
$$V_o/R_{eq} = V_o/R_1 + V_o/R_2 + V_o/R_3 \ldots$$

f.) Dividing out the voltage terms yields the final R_{eq} relationship for a parallel combination of resistors:

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 \ldots$$

g.) Example: If the resistances in a parallel combination are 350 Ω, 125 Ω, 455 Ω, and 170 Ω, the equivalent resistance for the combination is:

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 \ldots$$
$$1/R_{eq} = 1/(350\ \Omega) + 1/(125\ \Omega) + 1/(455\ \Omega) + 1/(170\ \Omega)$$
$$= .01894\ (1/\Omega)$$
$$\Rightarrow \quad R_{eq} = 52.8\ \Omega.$$

5.) There are some interesting things to note about the equivalent resistance of a parallel circuit. To begin with, R_{eq} is SMALLER than the smallest resistance in the combination. That is the exact opposite of a series combination (the equivalent resistance of a series combination is always larger than the largest resistance in the combination). This actually makes sense if viewed from the power supply's perspective. Follow along:

a.) Consider the parallel circuit shown in Figure 16.16b. The resistors R_1 and R_2 have currents $i_1 = V_o/R_1$ and $i_2 = V_o/R_2$ flowing through them respectively.

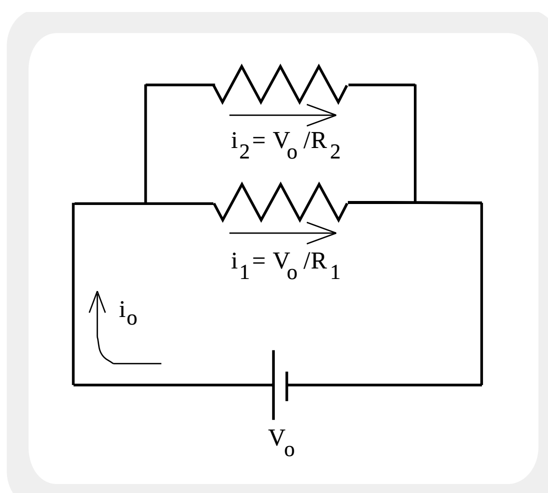


FIGURE 16.16b

b.) If we look at the circuit from the viewpoint of the power supply, we see a net, effective resistance "out there" to which we are required to supply current $i = i_1 + i_2$.

c.) A third resistor is added, as shown in the original circuit (Figure 16.16a). As a consequence:

i.) The power supply still supplies resistors R_1 and R_2 with currents i_1 and i_2 respectively (the voltage across R_1 and R_2 hasn't changed--it is still V_o--and the currents through those two resistors will still be V_o/R_1 and V_o/R_2 whether R_3 is in the circuit or not).

ii.) The power supply is now providing R_3 with current $i_3 = V_o/R_3$.

iii.) This means that from the battery's point of view, the current now being drawn from it is $i = i_1 + i_2 + i_3$.

d.) Remember that resistance limits the current flow (huge resistance across a given power supply will elicit a tiny current whereas a tiny resistance across a given power supply will elicit a huge current). If the current seems to have gone up due to the addition of the extra parallel resistor, then from the battery's point of view, the net effective resistance "out there" must have gone down.

e.) Bottom line: The more resistors there are added to a parallel circuit, the more current will be drawn from the power source and the more the equivalent resistance will diminish. The more resistors we take out of a parallel circuit, the less current is drawn from the power source and the more the equivalent resistance increases.

i.) Illustration: Two 1 ohm resistors in parallel yield an equivalent resistance of .5 ohms; three 1 ohm resistors in parallel yield an equivalent resistance of .333 ohms; four 1 ohm resistors in parallel yield an equivalent resistance of .25 ohms; etc.

D.) Parallel and Series Combinations in Combination:

1.) We want to determine the current i_o being drawn from the power supply V_o in the circuit shown in Figure 16.17 on the next page. One possible approach is to replace the mess of resistors with their net equivalent resistance R_{eq}, then use $i_o = V_o/R_{eq}$ to determine i_o. The trick is in the determination of R_{eq}.

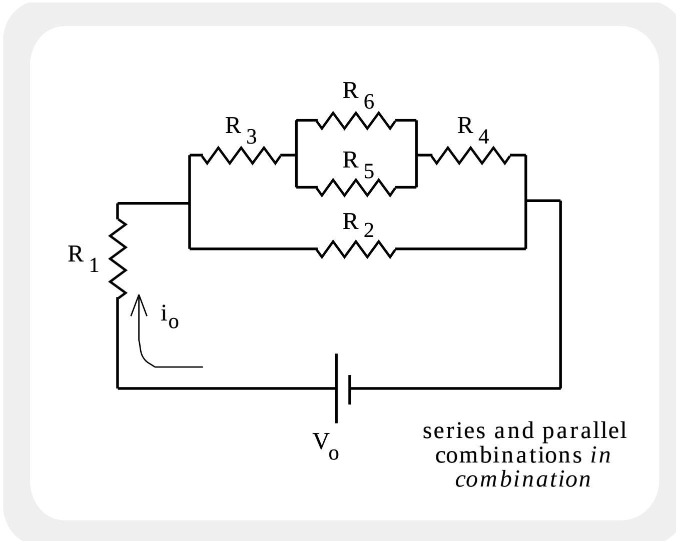


FIGURE 16.17

2.) Notice that the original circuit is a series combination--R_1 in series with a mess of resistors (mess #1 in Figure 16.18). The mess is actually a parallel combination--R_2 in parallel with mess #2. Mess #2 is really R_3 and R_4 in series with R_5 and R_6 in parallel. This information is all schematically represented in Figure 16.18. Starting from the inside going outward:

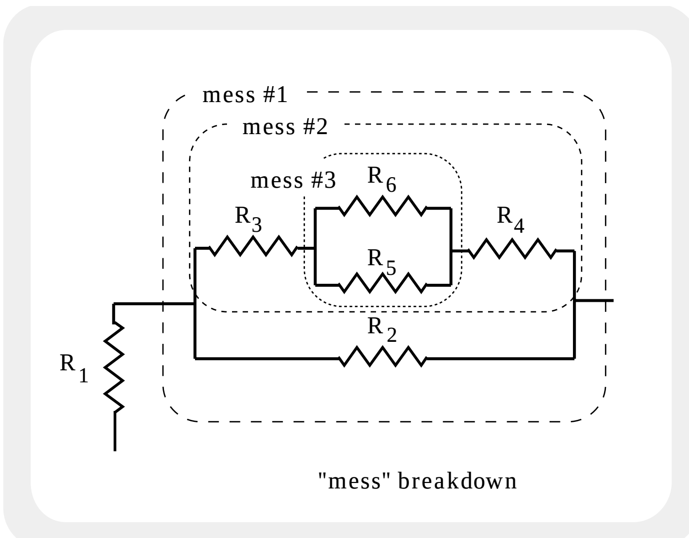


FIGURE 16.18

a.) Mess #3--R_5 and R_6 in parallel--has an equivalent resistance of:

$$R_{mess\ 3} = (1/R_5 + 1/R_6)^{-1}.$$

b.) The equivalent resistance for mess #2:

$$R_{mess\ 2} = R_3 + R_4 + R_{mess\ 3}.$$

c.) The equivalent resistance for mess #1 is:

$$R_{mess\ 1} = [1/R_2+1/(R_{mess\ 2})]^{-1}.$$

d.) The net equivalent resistance for the entire combination is, after the "mess" expressions are inserted:

$$R_{eq} = R_1 + R_{mess\ 1}$$
$$= R_1 + \left\{1/R_2 + 1/[R_3 + R_4 + (1/R_5 + 1/R_6)^{-1}]\right\}^{-1}.$$

e.) Assuming we have resistance values and a power-supply-voltage-reading, we can now solve for i_o.

Note: Questions like this are a lot easier if you use numbers from the beginning. The problem with this lies in the fact that once we begin combining numbers, quantities get absorbed into other quantities and it becomes very difficult for a second reader to follow one's reasoning.

Solution to the dilemma: Use sketches like those shown below in Figure 16.19.

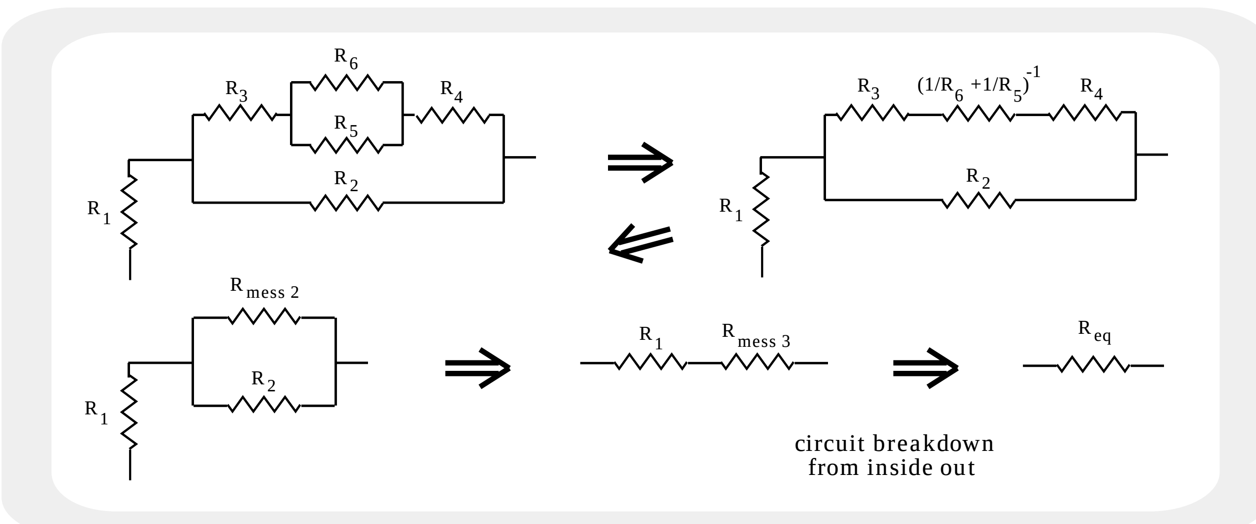


FIGURE 16.19

3.) The problem outlined in #2, but with numbers:

a.) Let resistors $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 18\ \Omega$, $R_4 = 12\ \Omega$, $R_5 = 28\ \Omega$, $R_6 = 20\Omega$. Determine the current drawn from a 12 volt battery as shown in Figure 16.17.

b.) The progression of equivalent resistances is shown in Figure 16.20 on the next page.

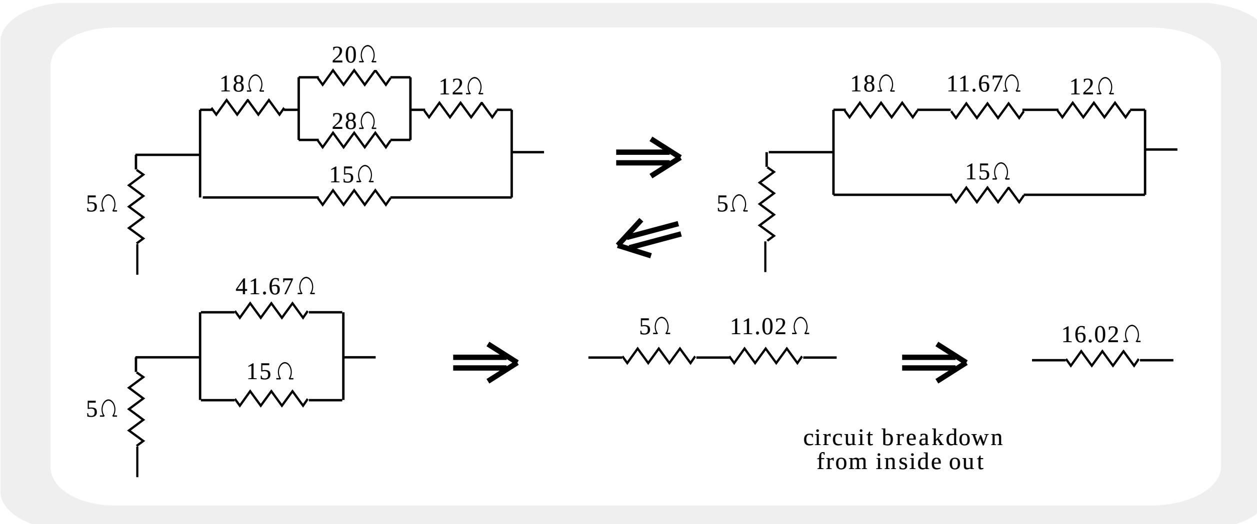


FIGURE 16.20

c.) Using the final circuit, we can write:

$$\begin{aligned} i_o &= V_o/R_{eq} \\ &= (12 \text{ volts})/(16.02\ \Omega) \\ &= .75 \text{ amps.} \end{aligned}$$

Note: The importance of this section is that you become able to determine the equivalent resistance of a mixed assortment of series and parallel combinations of resistors.

E.) Determining Branch Currents By the Seat of Your Pants:

1.) Note: There is a more orderly way of approaching circuitry problems outlined in the next section. This section has been included because it presents a fair amount of creative problem solving. You should intellectually understand what is being done here even if there is no bottom-line approach being presented.

2.) Consider the circuit shown in Figure 16.21 on the next page. Assume $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 23\ \Omega$, and $V_o = 15$ volts. What is the current in each branch of the circuit?

a.) Things to notice:

i.) There are three branches and two nodes in the circuit;

ii.) There is one simple parallel combination of resistors in the circuit (whether it is obvious or not, R_2 is in parallel with R_3--see "Big Note" below), and R_1 is in series with that parallel combination.

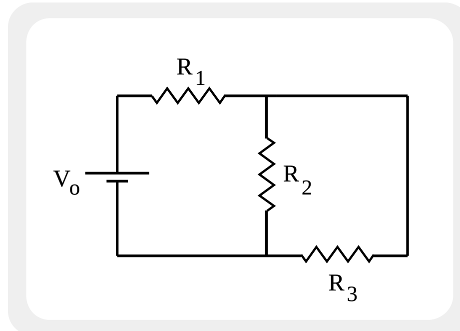


FIGURE 16.21

Big Note: Circuits are often drawn with the convenience of the designer overshadowing the convenience of all others. Following that tradition, notice that in Figure 16.21 R_3 is in parallel with R_2 even though the two do not look like a standard parallel set-up. This can be easily remedied by sliding R_3 around to the position shown in Figure 16.22.

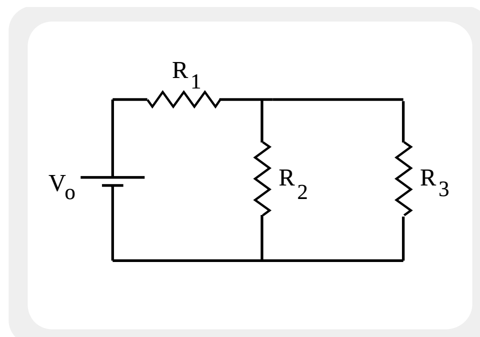


FIGURE 16.22

The moral: Look at your circuits closely. Be sure there are no oddball sections that can be simplified by using your head. Do not be bashful about re-drawing your circuit if you think the circuit creator is being tricky or obscure.

b.) At this point, we haven't much choice. Defining the branch currents as in Figure 16.23, we can determine R_{eq} for the circuit's resistors and from that, use $i_1 = V_o/R_{eq}$ to determine the battery current i_1. As for the other currents, we will have to see.

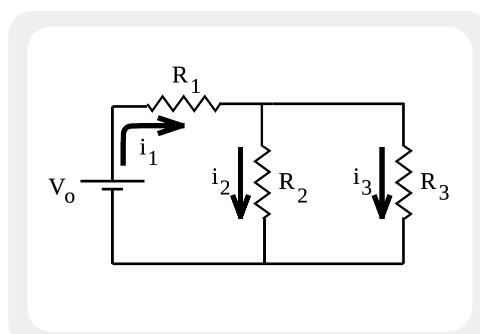


FIGURE 16.23

c.) In the circuit, R_1 is in series with the parallel combination of R_2 and R_3. Mathematically, the equivalent resistance for this combination is:

$$R_{eq} = R_1 + (1/R_2 + 1/R_3)^{-1}$$

$= (25\ \Omega) + [1/(18\ \Omega) + 1/(23\ \Omega)]^{-1}$
$= 35.1\ \Omega.$

d.) Using the R_{eq} and the known power supply voltage V_o, we get:

$$i_1 = V_o/R_{eq}$$
$$= (15 \text{ volts})/(35.1\ \Omega)$$
$$= .427 \text{ amps.}$$

e.) Knowing i_1, we can determine the voltage drop across R_1. It is:

$$V_1 = i_1 R_1$$
$$= (.427 \text{ amps}) (25\ \Omega)$$
$$= 10.675 \text{ volts.}$$

f.) We know the voltage across the entire resistor combination is the same as the voltage across the battery (V_o = 15 volts). If we take the voltage across R_1 to be V_1 (as already defined) and the voltage across the parallel combination to be V_2, then:

$$V_o = V_1 + V_2$$
$$\Rightarrow \quad V_2 = V_o - V_1$$
$$= (15 \text{ volts}) - (10.675 \text{ volts})$$
$$= 4.325 \text{ volts.}$$

g.) Because they are in parallel, V_2 is the voltage across both R_2 and R_3. Using that information and Ohm's Law, we get:

$$i_2 = V_2/R_2$$
$$= (4.325 \text{ volts})/(18\ \Omega)$$
$$= .24 \text{ amps.}$$

$$i_3 = V_2/R_3$$
$$= (4.325 \text{ volts}) / (23\ \Omega)$$
$$= .188 \text{ amps.}$$

Note: The current into the upper node is i_1 whereas the current out of the upper node is $i_2 + i_3$. It should not be surprising to find that to a very good approximation (giving round-off error, etc.):

$$i_1 = i_2 + i_3,$$

or

$$(.427 \text{ amps}) = (.24 \text{ amps}) + (.188 \text{ amps}).$$

3.) Reiterating this section's opening comment: Even though we have successfully analyzed this problem by using Ohm's Law, the idea of equivalent resistance, and a little bit of logic, there really was no rhyme or reason to the approach we employed. There is a more orderly way to approach such problems. That approach will be discussed shortly.

F.) Power:

1.) Power is defined as the amount of work per unit time done on an object. In the case of electrical systems, it is related to the time rate of work done on charge carriers moving through a circuit. As would be expected, the units of power are joules per second or the short-hand term watts.

a.) Example: A light bulb rated at 110 watts dissipates 110 joules of energy every second. The dissipated energy "leaves" as heat and light.

2.) How much power is provided to a circuit by a power-supply-voltage V when the current being drawn from the source is i? To determine that relationship, consider the following:

a.) A length of wire is hooked to a power supply whose voltage is V_o and whose current is i.

b.) Assume that it takes time Δt for charge to travel from the high voltage to low voltage terminals. Assume also that the distance between those points is d. If q's worth of charge passes by any point in the circuit during that time interval:

i.) The current in the circuit is $q/\Delta t$;

ii.) The relationship between the electric field E set up in the wire and the voltage difference ΔV across the power supply's terminals is $E \cdot d = -\Delta V$, where d is the distance charge moves in traveling from the higher to lower voltage terminal;

iii.) The relationship between electric force F on a charge q in an electric field E is $F = qE$;

c.) With this information, the net work per unit time done on the total charge q as they move during the time period Δt will be:

$$
\begin{aligned}
P &= W/\Delta t \\
&= |(F\cdot d)|/\Delta t \\
&= |[(qE)\cdot d]|/\Delta t \\
&= (q/\Delta t)|(E\cdot d)| \\
&= i|-\Delta V| \\
&= i\Delta V.
\end{aligned}
$$

Note: We have used absolute values because we are interested in the magnitude of the power rating only.

d.) Using the skimpy notation commonly used in electrical circuits (in this case, $\Delta V = V$), we find that the power supplied to a circuit by a power supply is equal to the product of the current drawn from the power supply and the voltage across the power supply. Mathematically, this is:

$$P = iV.$$

3.) Although this derivation came from a specific situation, the expression is generally true. The amount of power that any circuit element puts into the system or takes out of the system will always equal the voltage across the element multiplied by the current through the element.

4.) Consider the resistor's role in electrical circuits:

a.) A resistor characteristically limits current and additionally dissipates energy (usually as heat) in a circuit. According to the generalization made above, a resistor's rate of energy dissipation--its power rating--is equal to the voltage drop across the resistor V_R times the current through the resistor i_R. That is:

$$P_R = i_R V_R.$$

b.) According to Ohm's Law, the voltage across a resistor is:

$$V_R = i_R R.$$

c.) Combining these two relationships gives us the power dissipated by a resistor as:

$$P_R = i_R(i_R R) \\ = i_R{}^2R.$$

d.) There is one other way to write this: As $i_R = V_R/R$, we can substitute in for i_R and get:

$$P_R = (V_R/R)^2R \\ = V_R{}^2/R.$$

5.) Bottom line:

a.) Knowing the power rating of an element tells you how much energy per unit time the element can handle.

b.) If the element is a power supply, energy is supplied to the system and the power rating is iV, where V is the voltage across the terminals and i is the current drawn from the power source.

c.) If the element is a resistor, energy is removed from the system as heat or light and the power rating is iV, i^2R, or V^2/R depending upon which variables you know.

Note: I would not suggest you memorize all of these. If you remember P = iV and Ohm's Law, you can easily derive the rest.

G.) Kirchoff's Laws--Preliminary Definitions and Discussion:

1.) The approach used on the problem in Section E above can best be described as "by guess and by God." It doesn't hurt to be able to think through such a problem--the ease or difficulty involved should tell you a lot about how much you understand circuits--but there was no real technique to what was done.

Fortunately, there is a more orderly way of dealing with circuit problems using what are called Kirchoff's Laws.

2.) Kirchoff's First Law states that the net (total) current into a node must equal the net current out of a node.

a.) Example: If wires are connected at the node shown in Figure 16.24, we can write:

$$i_{in} = i_{out}$$

$$\Rightarrow \quad i_1 + i_2 + i_4 = i_3 + i_5.$$

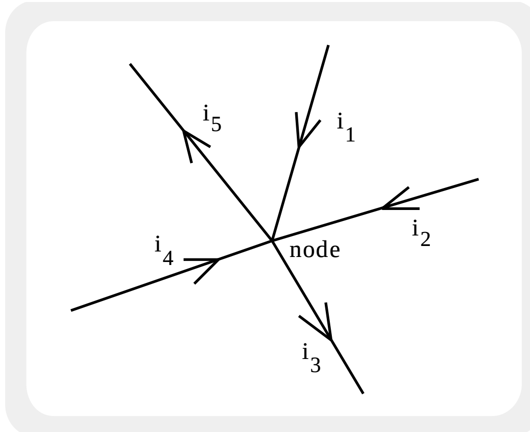


FIGURE 16.24

Note: The problem executed in Section E numerically substantiates this claim. The calculated value (rounded) for the current entering the top node in that problem was i_1 = .43 amps; the calculated values for the currents leaving that node were i_2 = .24 amps and i_3 = .19 amps. The sum of the currents in equals the sum of the currents out.

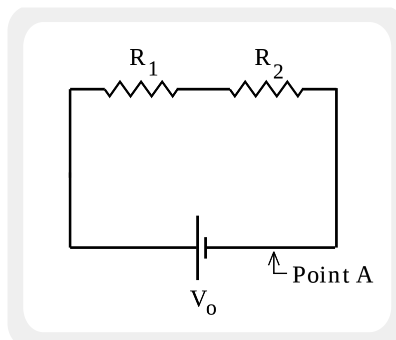


FIGURE 16.25

3.) Kirchoff's Second Law states that the sum of the voltage differences around a closed loop must equal zero.

a.) The easiest way to see Kirchoff's Second Law is to graphically track the electrical potential differences around a single loop circuit (see Figure 16.25).

To make the plotting easier, we will unfold the circuit (see Figure 16.26). The beauty of the unfolding is that it allows us to easily plot the voltage versus position at various points around the circuit.

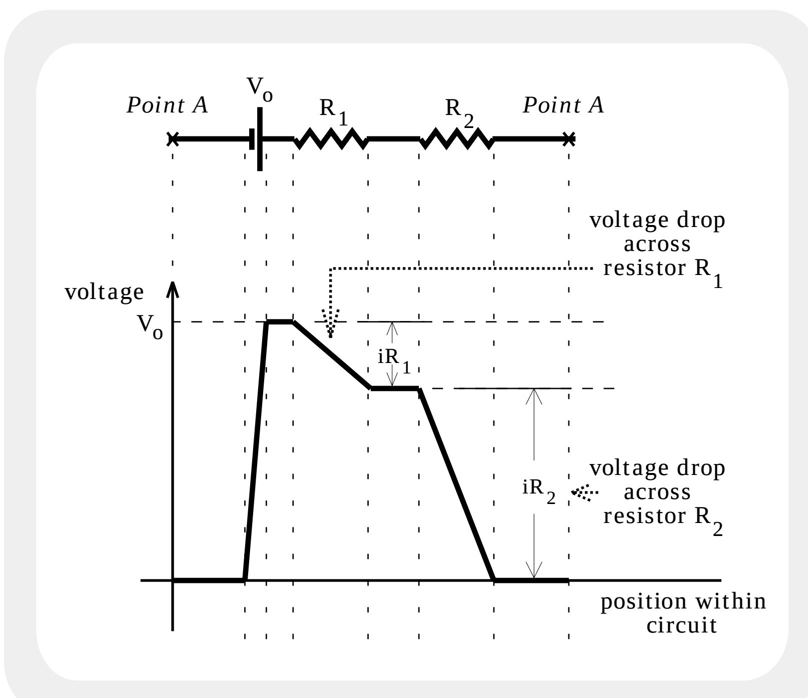


FIGURE 16.26

b.) In the circuit, the electrical potential of the power supply's ground terminal has been assumed to equal zero while the high voltage terminal is set to V_o. Beginning at Point A on the unfolded circuit, we move toward the battery. As we do, notice that:

i.) The voltage is the same at Point A as it is at the battery's ground terminal (at least to a very good approximation), which means Point A's voltage is zero;

ii.) Once at the power supply, the voltage changes from zero at the ground terminal to V_o at the high voltage terminal. The difference between those two terminals is a voltage increase;

iii.) The voltage at every point between the high voltage terminal and R_1 is the same, or V_o;

iv.) We know that current flows from higher voltage to lower voltage. As the current flows from left to right through the resistor, the left side of resistor R_1 must be at higher electrical potential than the right side. That means there must be a voltage drop across the resistor. The magnitude of this voltage drop will be iR_1;

v.) Between the resistor R_1 and R_2, the voltage will be the same;

vi.) There will be a voltage drop across resistor R_2--a drop that must bring us back to the voltage at Point A, or zero.

vii.) NOTICE THAT THE NET CHANGE AS WE ADD UP ALL THE VOLTAGE INCREASES AND DECREASES AROUND THE CLOSED LOOP IS ZERO. That is what Kirchoff's Second Law states.

c.) It should always be remembered that Kirchoff's Laws are useful only as a technique for generating equations in which current variables are present and from which specific current values for specific circuit designs can be calculated.

4.) Kirchoff's Laws--The Technique: There is a specific technique to using Kirchoff's Laws. Simply stated, that technique is described below (it will be applied to a problem in the next section).

a.) Begin by defining and appropriately labeling a current variable <u>for every branch</u> (a branch starts and ends at a junction).

Note 1: Students often assume one has to "psyche out" a circuit problem before actually doing it. That is, you might think it was important to somehow determine the direction of the current in a particular branch before assigning a current direction and variable to that branch. The beauty of this technique is that it does not matter whether current direction is obvious or not. As long as you are consistent throughout the problem, the mathematics will take care of any incorrectly assigned current directions. It will do so by generating a negative sign in front of each calculated current value whose direction was incorrectly assumed.

Note 2: If this is not clear, don't worry. The current defined as i_3 in the problem presented in the next section has intentionally been defined in the wrong direction. When you get there, watch to see how the mathematics takes care of the oversight.

b.) Write out Node Equations: Pick a node and apply Kirchoff's First Law for that junction. Do this for as many nodes as you can find, assuming you aren't duplicating equations (you will see just such a duplication situation in the next section's problem).

c.) Write out Loop Equations: Choose a closed loop and apply Kirchoff's Second Law to that closed path. Do so for as many loops as are needed to accommodate the number of unknowns you have.

d.) Solve the Loop and Node Equations simultaneously for the currents in the circuit.

Note 3: There is a technique for solving selected currents using matrix analysis. That technique is discussed further on in this chapter. You will be expected to know that technique.

H.) Kirchoff's Laws in Action--Two Examples:

1.) Consider the circuit in Figure 16.27. Determine the voltmeter and ammeter readings when $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 13\ \Omega$, $R_4 = 10\ \Omega$, and $V_1 = 15$ volts. Note that this is essentially the same problem you did

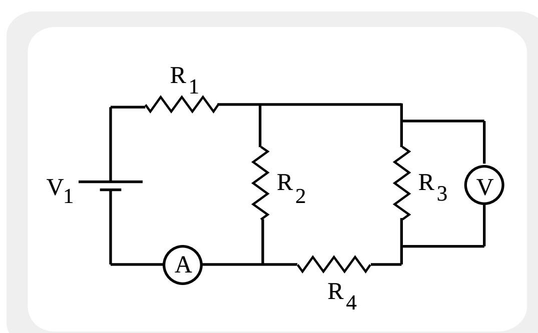
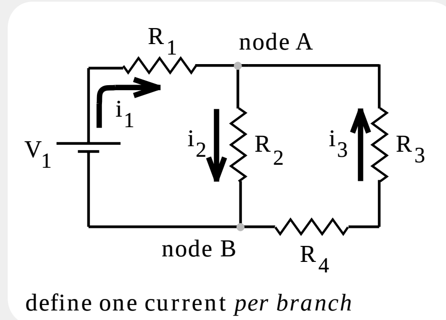


FIGURE 16.27

"by the seat of your pants" in Section E. The only change has been to break $R_3 = 23\ \Omega$ into a new $R_3 = 13\ \Omega$ in series with an $R_4 = 10\ \Omega$ (the change has been made to make the problem more general in appearance).

By the numbers:

a.) Removing the meters for the sake of simplicity, we define current variables for every branch and identify all nodes in the circuit (see Figure 16.28).



define one current *per branch*

FIGURE 16.28

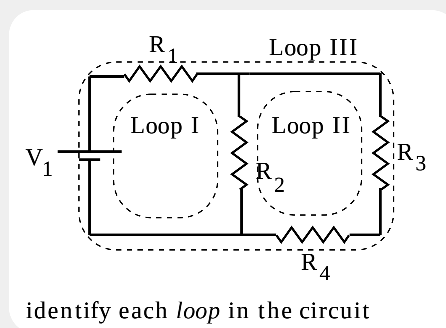
Note: The current i_3 in this problem has intentionally been defined in the wrong direction. Watch how the mathematics takes care of the error.

b.) Kirchoff's First Law written for Node A:

$$i_{in} = i_{out}$$
$$\Rightarrow \quad i_1 + i_3 = i_2.$$

Note: Satisfy yourself that the node equation written for Node A will give you the same information as the node equation written for Node B (this second equation is the above promised duplicate expression).

c.) Loop Equations: The loops are defined in Figure 16.29. We will need only two (there are three unknowns and we already have one equation courtesy of the node equation above).



identify each *loop* in the circuit

FIGURE 16.29

Note: The voltage DROPS (i.e., ΔV is negative) when one traverses through a resistor IN THE DIRECTION OF CURRENT FLOW. The voltage INCREASES (i.e., ΔV is positive) when traversing through a resistor IN THE DIRECTION OPPOSITE CURRENT FLOW.

Loop I: (traversing clockwise from Node B):

$$V_1 - i_1R_1 - i_2R_2 = 0$$
$$\Rightarrow \quad i_1 R_1 + i_2 R_2 = V_1$$
$$\Rightarrow \quad 25\, i_1 + 18\, i_2 = 15.$$

Loop II: (traversing counterclockwise from Node B):

$$- i_3R_4 - i_3R_3 - i_2R_2 = 0$$
$$\Rightarrow \quad i_2R_2 + i_3R_3 + i_3R_4 = 0$$
$$\Rightarrow \quad i_2R_2 + i_3(R_3 + R_4) = 0$$
$$\Rightarrow \quad 18\, i_2 + 23i_3 = 0.$$

Note 1: Notice that if we had traversed Loop I counterclockwise, the Loop Equation would have read:

$$- V_1 + i_1R_1 + i_2R_2 = 0$$
$$\Rightarrow \quad i_1R_1 + i_2R_2 = V_1.$$

This is the same equation as was acquired by traversing clockwise.

Bottom line: You can traverse in any direction you wish. I usually try to move through batteries from ground to high voltage terminal because that gives positive voltage values for power supplies, but it really does not matter which way you do it as long as you keep your signs consistent.

Note 2: There are three loops in this circuit--the two used above and the one that moves around the outside of the circuit (Loop III). Although we don't need it, the third Loop Equation would have been:

$$V_1 - i_1R_1 + i_3\, R_3 + i_3R_4 = 0.$$

Note 3: Just below we will be solving three equations simultaneously--two loop equations and one node equation. The temptation might be to forget the node equation and try to solve the three loop equations. PLEASE NOTE: The equation from Loop I added to the equation from Loop II gives us the equation from Loop III. That is, even though we have executed Kirchoff's Second Law on three loops, we have only two INDEPENDENT equations.

Bottom line: There will always be one more node and one more loop than there are independent node equations and/or independent loop equations. We will always have to use at least some node and some loop equations in solving circuit problems via simultaneous equations.

d.) Solve simultaneously:

$$i_1 + i_3 = i_2 \qquad \text{(Equation 1)}$$
$$25i_1 + 18i_2 = 15 \qquad \text{(Equation 2)}$$
$$18i_2 + 23i_3 = 0 \qquad \text{(Equation 3).}$$

Manipulating Equation 1 yields:

$$i_1 = i_2 - i_3 \qquad \text{(Equation 4).}$$

Substituting Equation 4 into Equation 2 yields:

$$25i_1 + 18i_2 = 15$$
$$25(i_2 - i_3) + 18i_2 = 15 \qquad \text{(Equation 5).}$$

Manipulating Equation 5:

$$43i_2 - 25i_3 = 15$$
$$\Rightarrow \quad i_2 = (25i_3 + 15)/43$$
$$= .58i_3 + .35 \qquad \text{(Equation 6).}$$

Substituting Equation 6 into Equation 3 yields:

$$18i_2 + 23i_3 = 0$$
$$18(.58i_3 + .35) + 23\ i_3 = 0 \qquad \text{(Equation 7).}$$

Manipulating Equation 7:

$$10.44i_3 + 6.3 + 23i_3 = 0$$
$$\Rightarrow \quad 33.4i_3 = -6.3$$
$$\Rightarrow \quad i_3 = -.189 \text{ amps.}$$

Note 1: The negative sign simply points out that the direction of the current i_3 was originally defined in the wrong direction. There is no need to change anything now--THE NEGATIVE SIGN SPEAKS FOR ITSELF.

Substituting back into Equation 6 yields:

$$i_2 = .58i_3 + .35$$

$$\Rightarrow \quad i_2 = .58\,(-.189) + .35$$
$$= .24 \text{ amps.}$$

Substituting back into Equation 1:

$$i_1 = i_2 - i_3$$
$$= .24 - (-.189)$$
$$= .43 \text{ amps.}$$

Note 2: These solutions are exactly the same as we determined using the less methodical "seat of your pants" technique outlined in Section E.

e.) The solutions to the problem: a.) The ammeter will read the current i_1 which equals .43 amps. b.) The voltmeter will read the voltage across R_3 which is i_3R_3 = (.189 amps)(13 ohms) = 2.46 volts.

Minor Technical Note: Voltmeters read magnitudes. If our voltmeter had been hooked up on the assumption that the current in R_3's branch ran counterclockwise (that is what we assumed in the problem), the voltmeter would have been hooked up backwards and the meter's pointer would have been forced to move to the left off-scale instead of to the right on-scale.

Bottom line: In theory, the mathematics will take care of any mis-assumptions you may make about current directions. In lab, things are not so forgiving. That is why all meters used in lab must initially be set on their highest, least sensitive scale and power must be increased slowly.

2.) Consider the circuit in Figure 16.30. Determine the branch currents when V_1 = 12 volts, V_2 = 15 volts, $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, $R_3 = 13\ \Omega$, and $R_4 = 7\ \Omega$. (Note that there are neither series nor parallel combinations in this circuit).

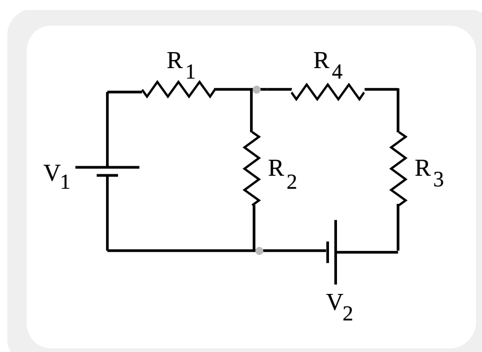


FIGURE 16.30

By the numbers:

a.) Define: current variables for every branch; circuit nodes; and circuit loops (see Figure 16.31 on the next page).

b.) Node Equations written for Node A:

$i_{in} = i_{out}$

$\Rightarrow \quad i_2 + i_3 = i_1.$

c.) Loop Equations:

Loop I: (traversing clockwise from node A):

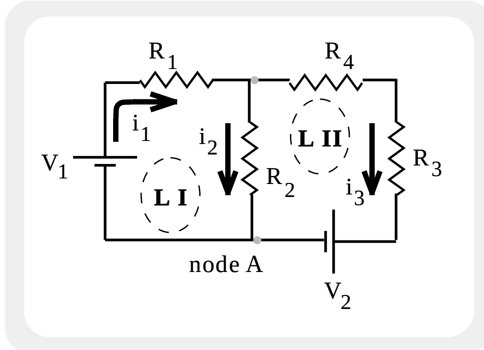


FIGURE 16.31

$$V_1 - i_1R_1 - i_2R_2 = 0$$
$$\Rightarrow \quad i_1R_1 + i_2R_2 = V_1$$
$$\Rightarrow \quad 25i_1 + 18i_2 = 12.$$

Loop II: (traversing counterclockwise from A):

$$V_2 + i_3R_3 + i_3R_4 - i_2R_2 = 0$$
$$\Rightarrow \quad i_2R_2 - i_3(R_3 + R_4) = V_2$$
$$\Rightarrow \quad 18i_2 - (13 + 7)i_3 = 15$$
$$\Rightarrow \quad 18\ i_2 - 20\ i_3 = 15.$$

d.) Solve simultaneously:

$i_1 = i_2 + i_3$ (Equation 1)
$25i_1 + 18i_2 = 12$ (Equation 2)
$18i_2 - 20i_3 = 15$ (Equation 3).

Substituting Equation 1 into Equation 2:

$$25i_1 + 18i_2 = 12$$
$\Rightarrow \quad 25(i_2 + i_3) + 18i_2 = 12$ (Equation 4).

Manipulating Equation 4:

$$43i_2 + 25i_3 = 12$$
$$\Rightarrow \quad i_2 = (-25i_3 + 12)/43$$
$= -.58i_3 + .28$ (Equation 5).

Substituting Equation 5 into Equation 3 yields:

$$18i_2 - 20\, i_3 = 15$$

$$\Rightarrow \quad 18(-.58\, i_3 + .28) - 20i_3 = 15 \qquad \text{(Equation 6).}$$

Manipulating Equation 6:

$$- 10.44i_3 + 5.04 - 20i_3 = 15$$
$$\Rightarrow \quad -30.44i_3 = 9.96$$
$$\Rightarrow \quad i_3 = -.327 \text{ amps.}$$

Substituting back into Equation 5 yields:

$$i_2 = -.58i_3 + .28$$
$$\Rightarrow \quad i_2 = -.58\,(-.327) + .28$$
$$= .47 \text{ amps.}$$

Substituting back into Equation 1:

$$i_1 = i_2 + i_3$$
$$= .476 + (-.327)$$
$$= .14 \text{ amps.}$$

I.) Matrix Approach to Analyzing Simultaneous Equations

1.) With three or more variables, there is a much easier way to solve simultaneous equations. It requires the manipulation of matrices, and although that might sound horrifying, it is not that difficult. This section lays out the technique (without proofs).

2.) A matrix can be used to write out simultaneous equations in a shorthand way. Assume you have the equations:

$$ax + by + cz = m_1,$$
$$dx + ey + fz = m_2,$$
$$gx + hy + iz = m_3,$$

where x, y, and z terms are variables; the a through i terms are coefficients (positive or negative); and the m terms are constants. For those who would like an example, try: $3x + (-4)y + 2z = -16$.

If we want to solve for the variable z without solving for the other variables in the process, the following approach will do the job.

a.) Notice that we can display the coefficients as shown below.

a	b	c	m_1
d	e	f	m_2
g	h	i	m_3

b.) There are a number of things to note about this situation:

i.) Row #1 holds the coefficients of the first equation $ax + by + cz = m_1$, while;

ii.) Column #1 holds the x coefficients for each of the equations. Likewise, column #2 holds the y coefficients, column #3 holds the z coefficients, and column #4 (this is actually a 1x3 matrix unto itself) holds the constants.

c.) It may not be obvious at first glance, but this presentation of the variable coefficients is a natural matrix. Put in classical matrix notation, we end up with:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

d.) The matrix on the left is called the determinate matrix D.

3.) <u>Side note</u>: The evaluation of any matrix yields a number. The technique for doing such an evaluation has been or will be discussed in class. If the following explanation doesn't make perfect sense at first reading, don't panic. It will make sense sooner or later.

To evaluate a 3x3 matrix, one must be able to evaluate a 2x2 matrix first. That is where we will begin.

a.) The evaluation of a 2x2 matrix follows:

$$\begin{bmatrix} o & p \\ q & s \end{bmatrix} = (os - pq),$$

where the elements o, p, q, and s can be either positive or negative.

That is, it is executed by subtracting the product of the upper right and bottom left entries from the product of the upper left and bottom right entries.

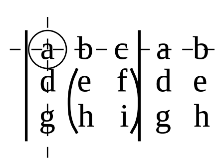
b.) Example of a 2x2 matrix evaluation: Evaluating the matrix presented below, we get:

$$\begin{bmatrix} -4 & 7 \\ -12 & 3 \end{bmatrix} = ((-4)(3) - (7)(-12)) = 72$$

4.) Consider now the 3x3 matrix shown below (it will become obvious shortly why the first and second columns are repeated to the right of the matrix). The evaluation of that matrix requires the sum of the evaluations of three 2x2 matrices. The technique follows:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

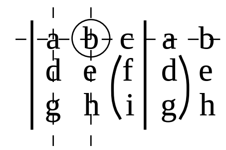
a.) Begin by drawing a line across the top row and down the first column of the 3x3 matrix. This will leave the top-left entry with a double-line through it:



b.) From the double-crossed entry (the a term in this case), there is a 2x2 matrix that starts in the row just below and the column just to the right of that entry (i.e., in the second row, second column). The first part of our 3x3 evaluation is the product of a times the evaluation of that 2x2 matrix, or:

a(ei - fh).

c.) Next, leaving the top row penciled out, draw a line down through the second column. This will leave the middle-top entry with a double-line through it (the b term in this case). From that double-crossed entry, there is a 2x2 matrix that starts in the row just below and the column just to the right of that entry (i.e., in the second row, third column).



The second part of our 3x3 evaluation is the product of b times the evaluation of that 2x2 matrix, or:

$$b(fg - di).$$

Note: It should now be obvious why the first column was repeated to the right of the matrix.

d.) Following a similar pattern, the third member of our 3x3 matrix evaluation is evaluated as:

$$c(dh - eg).$$

e.) The final evaluation is the sum of the three parts determined in Sections b, c, and d above. Mathematically, that is

$$a(ei - fh) + b(fg - di) + c(dh - eg).$$

5.) Let's assume we want to determine the z variable that satisfies our three equations but we do not care about the solutions for the variables x and y. There is a matrix technique that allows us to selectively solve for z while virtually ignoring x and y.

The technique maintains that the value of the z variable is equal to the ratio of the evaluation of two matrices--the determinate matrix D and a modified version of the determinate. Specifically:

$$z = D_{mod,z} / D.$$

a.) We have already defined the determinate matrix D. The modified determinate matrix is the determinate matrix <u>with one change</u>. The column associated with the variable for which we are solving (in this case, the z column) is replaced by the constants column from the original configuration (i.e., the 1x3 matrix that holds the constants in our equations). The modified matrix is shown below:

$$D_{mod,z} = \begin{bmatrix} a & b & m_1 \\ d & e & m_2 \\ g & h & m_3 \end{bmatrix}$$

b.) Mathematically, our z variable is equal to:

$$z = D_{mod,z} / D \, .$$

6.) An example with numbers. Assume:

$$\begin{aligned} x + 3y - z &= -4, \\ 3x - 4y + 2z &= 12, \\ 2x + 5y + 0z &= 9. \end{aligned}$$

Determine z.

a.) The original matrix configuration for this set of equations is:

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & -4 & 2 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \\ 9 \end{bmatrix}$$

b.) According to our technique:

$$z = \frac{D_{mod,z}}{D} = \frac{\begin{bmatrix} 1 & 3 & -4 \\ 3 & -4 & 12 \\ 2 & 5 & 9 \end{bmatrix}}{\begin{bmatrix} 1 & 3 & -1 \\ 3 & -4 & 2 \\ 2 & 5 & 0 \end{bmatrix}}$$

c.) The evaluation of these two matrices is:

$$z = \frac{(1)[(-4)(9) - (12)(5)] + (3)[(12)(2) - (3)(9)] + (-4)[(3)(5) - (-4)(2)]}{(1)[(-4)(0) - (2)(5)] + (3)[(2)(2) - (3)(0)] + (-1)[(3)(5) - (-4)(2)]}$$

$$= \frac{(1)[-96] + (3)[-3] + (-4)[23]}{(1)[-10] + (3)[4] + (-1)[23]}$$

$$= (-197)/(-21)$$

$$= 9.38.$$

7.) With this in mind, reconsider the two-battery circuit presented in the example in Section H-2 of this chapter. The circuit is shown in Figure 16.30; the equations generated through Kirchoff's Laws were:

$$i_1 - i_2 - i_3 = 0 \qquad \text{(Equation 1)}$$
$$25\, i_1 + 18\, i_2 + 0 i_3 = 12 \qquad \text{(Equation 2)}$$
$$0 i_1 + 18\, i_2 - 20\, i_3 = 15 \qquad \text{(Equation 3).}$$

Note 1: The equations have been put in order in the sense that all the i_1 coefficients are in the first column, all the i_2 coefficients in the second column, etc.

Note 2: Even if there were, say, no i_2 coefficient in a particular equation, we would still need to acknowledge that spot in the matrix by placing a zero in the appropriate spot.

a.) Presenting and solving:

$$\begin{bmatrix} 1 & -1 & -1 \\ 25 & 18 & 0 \\ 0 & 18 & -20 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 15 \end{bmatrix}$$

so that

$$i_1 = \frac{D_{\text{mod},1}}{D} = \frac{\begin{bmatrix} 0 & -1 & -1 \\ 12 & 18 & 0 \\ 15 & 18 & -20 \end{bmatrix}}{\begin{bmatrix} 1 & -1 & -1 \\ 25 & 18 & 0 \\ 0 & 18 & -20 \end{bmatrix}}$$

b.) The evaluation of these two matrices is:

$$i_1 = \frac{(0)[(18)(-20) - (0)(18)] + (-1)[(0)(15) - (12)(-20)] + (-1)[(12)(18) - (18)(15)]}{(1)[(18)(-20) - (0)(18)] + (-1)[(0)(0) - (25)(-20)] + (-1)[(25)(18) - (18)(0)]}$$

$$= \frac{0 + (-1)[240] + (-1)[-54]}{1[-360] + (-1)[500] + (-1)[450]}$$

$= (186)/(1310)$

$= .14$ amps.

Note that this is the same value we calculated using the much more complicated substitution method.

8.) Bottom line: Kirchoff's Laws, in conjunction with the matrix approach we have been examining, are a very powerful tool for analyzing circuits in which only resistors and power supplies reside.

9.) MAJOR TECHNICAL NOTE: For those of you who have a calculator comparable to the TI-82 (or better), there is a tricky way you can solve for all three unknown currents at once (or, for that matter, as many unknown currents as you'd like). I'll prove the assertion below, but simply stated, it maintains the following:

a.) Enter the matrix (call it A) into your calculator.

b.) Have the calculator determine the inverse matrix A^{-1}.

c.) Multiply the inverse matrix A^{-1} by the constants matrix (this will be the single-column matrix in which the VOLTAGE terms have been placed).

d.) Your calculator will give you a single-column matrix. The value of the first entry in that matrix will numerically equal i_1, the second entry will numerically equal i_2, etc.

e.) Proof: Technically, our 3x3 matrix should be written as $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} x \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$, where we can shorthand the 3x3 matrix by calling it matrix A (that is, $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix}$ so that $Ax \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$). If we multiply both sides of our matrix equation by A^{-1}, we get: $(A^{-1}xA)x \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = A^{-1}x \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$. Noting that $(A^{-1}xA) = 1$ (or, at least, a unit matrix), we get the expression $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = A^{-1} x \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$. Going back to our original assertion, if you multiply the inverse matrix A^{-1} by the single-column constants matrix (i.e., the matrix in which the voltages are found), you will end up with a matrix whose first term is numerically equal to the current associated with the first column of A, etc.

J.) Exotica--Real, Live Light Bulbs:

1.) There are a couple of fine points about electrical circuits that are brought out nicely when we examine real, working light bulbs. We will do this through two seperate scenarios.

2.) Scenario I: The resistor in the circuit shown in Figure 16.32a is a 40 watt household light bulb (note that household light bulbs require the equivalent of 120 volts DC to operate properly). We will assume that the power supply in the circuit is a variable DC source.

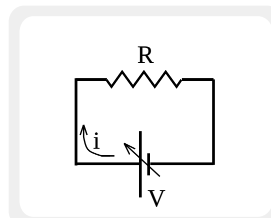


FIGURE 16.32a

a.) To begin with: Assuming we are using a couple of those mythical ideal meters physicists are so fond of utilizing (i.e., an ammeter that has no resistance and a voltmeter whose resistance is infinite), there is absolutely no difference in the circuit shown Figure 16.32a and the circuit shown in Figure 16.32b.

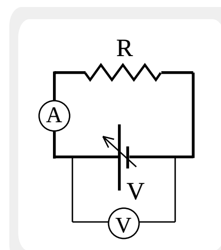


FIGURE 16.32b

Note: This is a minor side point, but it never hurts to reinforce the fact that meters are theoretically assumed to do NOTHING in a circuit except sense whatever they are designed to measure. That's why I suggest that students redraw diagrams without the meters included when confronted with a circuit they are expected to evaluate.

b.) With the observation from Part J-2a made, consider the following: Set the voltage across the variable power supply (hence the voltage across the light bulb) at 40 volts. Once done, examine your ammeter. Let's say it reads .20 amps.

c.) What should happen when you double the voltage to 80 volts?

i.) According to Ohm's Law, if you double the voltage across a resistor, the current through the resistor should double. The light bulb is acting like a resistor (or so we assume), so doubling its voltage to 80 volts should generate a doubled current of .4 amps.

ii.) In fact, if you try this in real life, you will find that the current goes up to only around .27 amps.

d.) So what's going on? It turns out that the resistance of a light bulb is dependent upon the temperature of the light bulb's filament.

i.) At low temperatures, there is very little vibratory motion of the atomic lattice through which charge carriers must flow. As a consequence, the carriers can travel fairly far (relatively speaking) before running into something.

ii.) At a macroscopic level, this long mean free path translates into little resistance to current flow.

iii.) At high temperature, the atomic lattice through which charge carriers must flow experiences a lot of vibrational motion. As a consequence, charge carriers can't go very far (relatively speaking) without running into something.

iv.) At a macroscopic level, this short mean free path translates into high resistance to current flow.

Side Note: Although it is complicated, this partially explains why near-death light bulbs blow when they are being turned on, versus blowing after they've been on for a while. As was deduced above, a light bulb that is off has a very low filament temperature and, hence, low resistance to charge flow. When turned on, 120 volts is placed across the initially low resistance filament and a high current surges through the filament. This high current is what snaps the wire, sort of. Actually, this is a simplification. There's more to the story.

In fact, there are three reasons why bulbs blow. First, the filament physically thins down with time as atoms slough off the hot wire when at operating temperature. This thinning down makes the filament more vulnerable to breakage. Second, manufacturers who are not willing to wait for the natural demise of a thinning filament have dallied with planned obscelescence by putting a little bit of water vapor inside the bulb to corrode the filament with time. This also makes the filament more vulnerable to breakage. And third, the filament flexes as it expands due to heating when turned on and contracts when cooling after being turned off. With time, this expansion-contraction-expansion cycle fatigues the wire in the same way that the metal of a paper clip is fatigued when bent back and forth. At some point, the sudden expansion caused by the big initial current surge through a cold, low resistance, already fatigued and frail filament is what snaps the wire and kills the bulb.

e.) Back at the ranch, standard household light bulbs operate at the DC equivalent of 120 volts. So at 40 volts, the filament temperature is relatively low. When you double the voltage, the current and filament temperature will go up. As a consequence of the increase in temperature, the bulb's net resistance will also go up.

f.) In short, increasing the voltage will increase the current, but because the resistance has also gone up, the current won't go up as much as we might otherwise have expected. For that reason, in our example, the current went up from .2 amps to .27 amps instead of from .2 amps to .4 amps.

g.) Bottom line: Because a light bulb's resistance is temperature dependent, and because the filament temperature changes at different voltages, the current and voltage assocated with a lit bulb are not linearly related. This is an example of a non-Ohmic device. Most electrical devices, from capacitors to inductors to transistors, are non-Ohmic devices.

Note: On AP tests, unless stated otherwise, a light bulb is assumed to be an Ohmic device with a constant resistance value regardless of the current.

3.) Scenario II: Consider the parallel circuit shown in Figure 16.33a. Assume the DC power supply is variable, and assume the resistors are two, identical, 40 watt light bulbs.

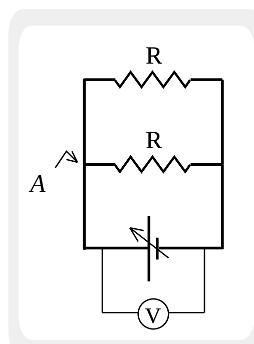


FIGURE 16.33a

a.) To begin with, a standard, trick AP question is to present the circuit in Figure 16.33a and ask what would happen to the current through the top resistor if the middle resistor were removed from the circuit.

i.) Because the resistors are identical, most students observe that the current being drawn from the power supply (let's assume it is .6 amps) will split at the junction labeled A with half going through the middle resistor and the other half going through the top resistor. That's .3 amps through each resistor.

ii.) What a lot of students think is that when the middle resistor is removed, that full .6 amps will then flow through the top resistor. NOT SO!

iii.) What determines the amount of current that flows through a resistor is the size of the resistance and the voltage across the resistor. By taking the middle resistor out, you haven't changed the voltage or resistance of the top resistor. As a consequence, it will continue to draw the same amount of current it always drew--.3 amps in this case.

iv.) In other words, what changes in the circuit due to the removal of one of the parallel resistors is not the voltage or current through the remaining resistor. What changes is the amount of current being drawn from the battery. There is now one less path requiring current, so the battery current goes down.

b.) Having made the observation in Part J-3a, consider the following: Set the voltage across the variable power supply in Figure 16.33a (hence the voltage across the two light bulbs) at 50 volts. Once accomplished, what will happen if you unscrew the middle light bulb?

i.) What you'd expect, according to our theory as presented, is that the upper light bulb would continue to burn with the same brightness as before. After all, nothing has changed in that branch of the circuit.

ii.) In fact, what happens if you try this is that the upper light bulb becomes MORE BRIGHT.

iii.) Furthermore, you will also find that the voltage across the power supply, as measured by the voltmeter in the circuit, will have increased. (When I tried this with my equipment at school, it went all the way up to 70 volts.)

d.) So what's going on? The key is in the fact that the voltage across the power supply seems to have gone up on its own. On the surface, this is very strange. Yet if you understand how power supplies really work, it makes sense. Follow along.

e.) Until now, all we have known about any power supply we've used has been what our voltmeter has told us. What voltmeters measure is called the terminal voltage of the source. That is, they measure the electrical potential difference between the terminals of the supply. This quantity is usually characterized as V, though for clarity, I will refer to it here as either V_{term} or $V_{terminal}$.

f.) What people usually ignored, at least if they treat their power supplies as "ideal," is that real power supplies have two relevant, measureable qualities that need to be considered if we are to have an accurate reflection of what the power supply actually, fully does within a circuit.

g.) The first of these qualities is the ability to supply energy to the circuit thereby motivating charge to flow. Power supplies do this internally by creating an electric field via an electrical potential difference across the supply's terminals.

i.) A true measure of the actual, energy-supplying, charge motivating aspect of the power supply is technically called the supply's electromotive force, or EMF. That quantity is usually characterized by an ε.

Note: This name happens to be a misnomer. The quality we are talking about here is not, as the name suggests, a force. It is a measure of the actual electrical potential difference internally available within the power supply. Its units are volts (note that these are the same units as the power supply's terminal voltage), not newtons.

h.) What has so-far been ignored is the second current-affecting quality associated with a power supply. That has to do with the power supply's internal resistance $r_{internal}$. (This will sometimes be characterized as r_i.) This internal resistance also creates a potential difference which is associated with the energy that is lost as current flows through the power source.

i.) This means that when we use a voltmeter to measure a power supply's terminal voltage, we are really measuring the energy-providing EMF (in volts) minus the energy-removing voltage drop due to the internal resistance (see Figure 16.33b). This is formally expressed as:

$$V_{terminal} = \varepsilon - ir_{internal}.$$

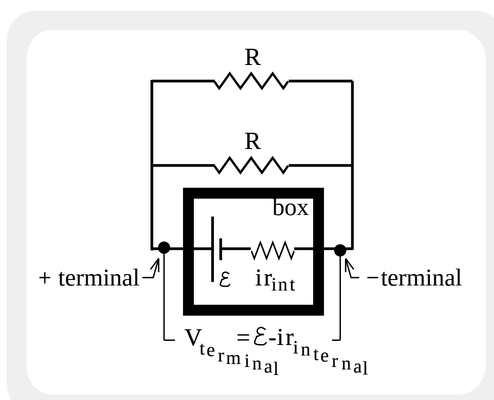


FIGURE 16.33b

j.) With all of this in mind, what happened with our parallel circuit?

i.) Once the power supply's EMF was initially set, it remained constant and doesn't change.

ii.) There was a certain amount of net resistance wrapped up in the parallel combination of resistors coupled with the internal resistance of the power supply. The power supply generated the appropriate current for that net resistance and all was well.

iii.) One resistor was then removed.

iv.) With one less resistor drawing current, the current from the power supply went down.

v.) The EMF didn't change, but less current meant a smaller $ir_{internal}$ drop (if i goes down, ir goes down). That meant the terminal voltage $V_{terminal}$ went UP (look at the expression--if ε stays the same and $ir_{internal}$ goes down, then $V_{terminal}$ goes up).

vi.) This was why the terminal voltage in our scenario went from 50 volts to 70 volts.

vii.) Continuing, an increase in the terminal voltage increased the voltage across the remaining light bulb. That elicited a corresponding increase in current through that light bulb. That's why the bulb got brighter.

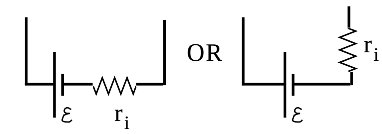
Note: Yes, when the current in the circuit goes up, there will be an increase in ir_i drop which will, in turn, decrease the terminal voltage some . . . but not enough to counteract the terminal voltage increase due to the removal of the light bulb. It all works out in the end.

k.) So why is all of this being thrown at you?

i.) Most books talk about EMFs and internal resistance, but it usually seems like hair-splitting as they don't give concrete examples as to why you might want to care. Now you know how you can get messed up in at least one circumstance if you don't understand what is really going on inside a power source.

ii.) Standardized tests like the AP test will often give you a terminal voltage and nothing else. In that case, what they are telling you is that the power supply's internal resistance isn't going to play a part in the problem and you can forget it.

iii.) If, for whatever reason, the AP folks want you to consider the power supply's internal resistance, they will use the ε symbol for the battery's voltage and will include a resistor labeled r_i in series with the EMF value. Figure 16.34 depicts the two general ways this is done. It isn't a big deal. It simply means you have to treat that additional resistance as you would any other in the circuit.



two ways internal resistance is characterized in a circuit

FIGURE 16.34

l.) So much for amusement.

QUESTIONS

16.1) Considering Figure I:

a.) If no current is to flow through R_5, what must be true?

b.) If V_A = 3.36 volts and V_B = 5.25 volts (Points A and B are defined in the sketch to the right), in which direction will current flow through R_5?

c.) Given the situation outlined in Part b and assuming R_5 = 3 ohms, what is the current through R_5?

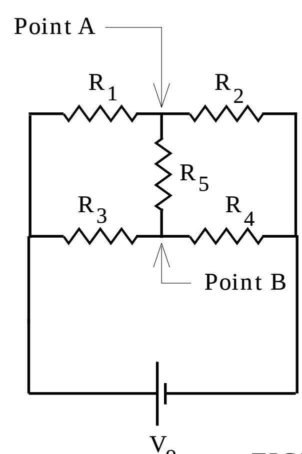


FIGURE I

16.2) In the circuit in Figure II, the current through the 12 Ω resistor is .5 amps.

a.) What is the current through the 8 Ω resistor?

b.) What is the power supply's voltage?

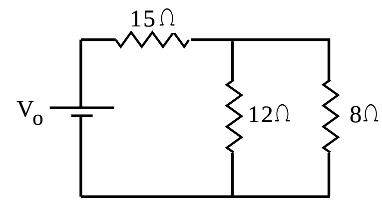


FIGURE II

16.3) In Figure III, R_2 is decreased. What happens to:

a.) R_2's voltage;

b.) R_2's current;

c.) R_1's voltage;

d.) the power dissipated by R_2?

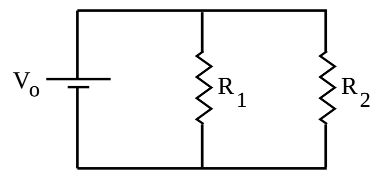


FIGURE III

16.4) In Figure IV, all the resistors are the same and all the power sources are the same. If the current in the series circuit is .4 amps, what is the current drawn from V_o in the parallel circuit?

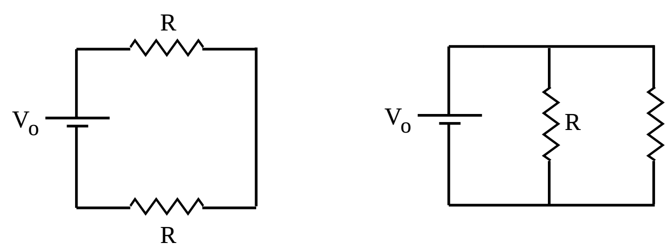


FIGURE IV

16.5) Resistors $R_1 = 10\Omega$, $R_2 = 12\Omega$, and $R_3 = 16\ \Omega$ are connected in parallel. If the current through the 12 Ω resistor is 2 amps, determine the currents through the other two resistors.

16.6) A battery charger delivers 6 amps of current to a 45 Ω resistor for 30 minutes.

a.) How much charge passes through the resistor?

b.) How much work does the charger do?

c.) How much power does the charger deliver?

16.7) A power supply provides 125 watts to an 18 Ω resistor. Determine:

a.) The current through the resistor; and

b.) The voltage across the resistor.

16.8) Assuming all resistors available are 5Ω, determine the equivalent resistance of each of the four independent circuits found in Figure V.

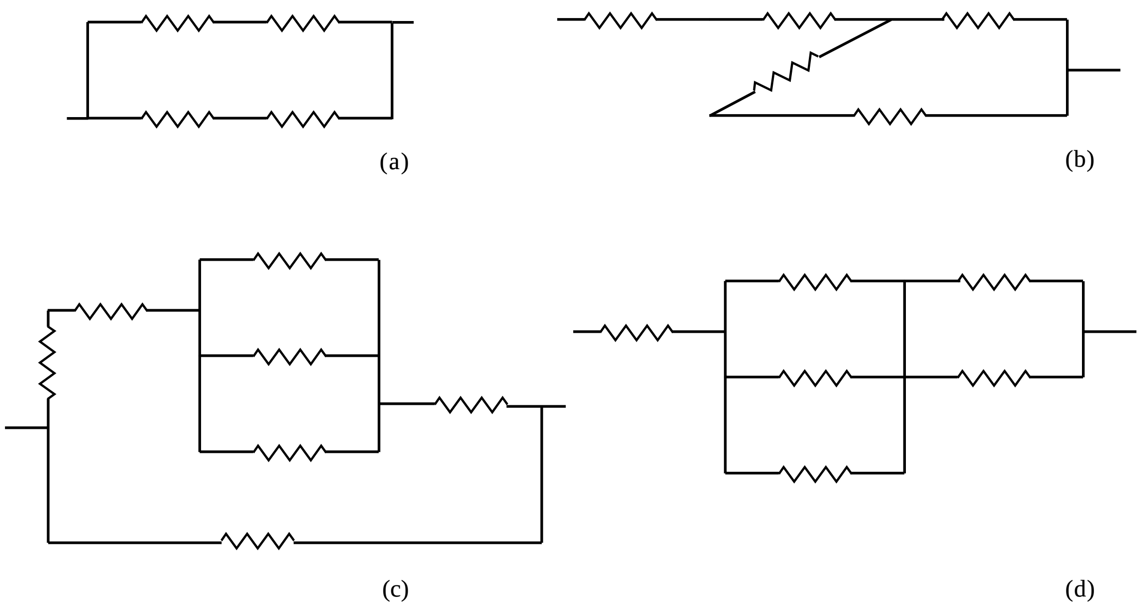


FIGURE V

16.9) Using as many 12 Ω resistors as you need, produce a resistor circuit whose equivalent resistance is:

a.) 18 Ω; and

b.) 30 Ω.

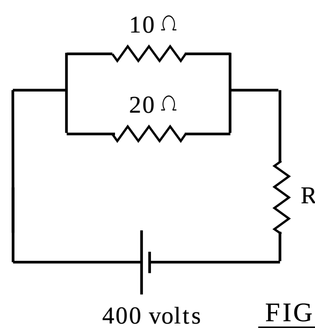


FIGURE VI

16.10) The power dissipated by the circuit in Figure VI is 800 watts. What is R?

16.11) Examine Figure VII:

a.) How many nodes are there in the circuit?

b.) How many loops?

c.) Write out any three node equations using the information provided in the circuit.

d.) Write out any three loop equations.

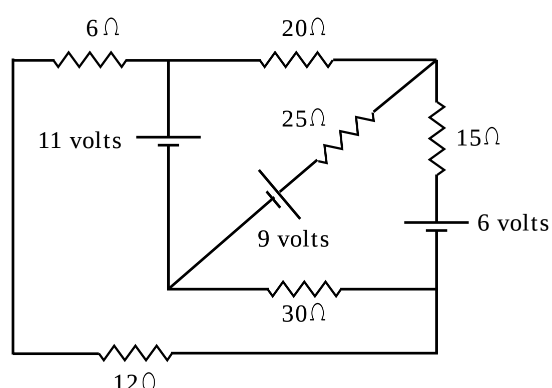


FIGURE VII

16.12) Using a matrix approach, solve the three equations presented below for i_2. (Note: The equations have not been presented in any particular order; you may find it useful (read this "necessary") to re-write the equations in a re-ordered form).

$$13i_1 - 9\,i_2 = -\,4i_3 - 6$$
$$-7i_3 = 4i_1$$
$$-5i_1 = -3i_2.$$

Note: The only expression with a constant (-6 volts) in it has been placed first in the listing. I've done this because it will make the matrix manipulation easier should you do it longhand. Whenever you generate equations like this, put the expressions with non-zero voltage terms first.

16.13) Use Kirchoff's Laws to determine the meter readings in the four circuits shown in Figure VIII (see notes on next page).

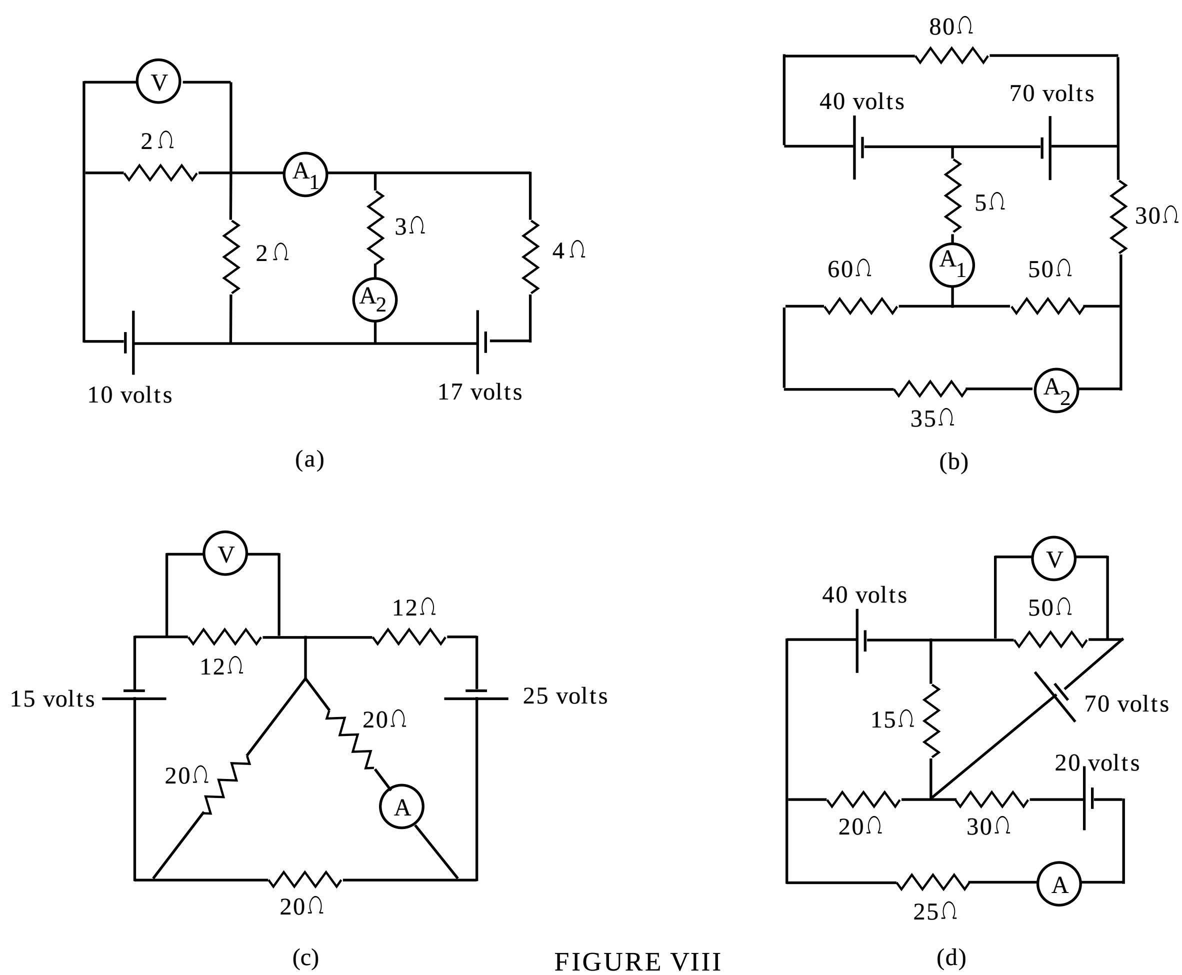


FIGURE VIII

Note 1: There are four primary branches in circuit "a" (primary branches do not include voltmeter branches). That means you are either going to have to analyze a 4x4 matrix or be clever in the way you define your currents. My suggestion: be clever. Begin by defining a current or two, then use your node equations to define all the other currents in terms of the first few. In doing so, you should be able to whittle the number of variables down considerably (or, at least, by one).

Note 2: You may have to use the trick mentioned in Note 1 more than once when doing these circuits!