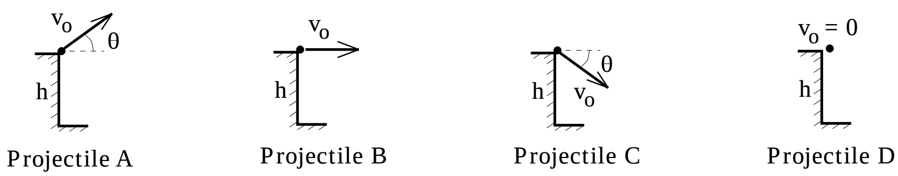
MULTIPLE CHOICE SOLUTIONS--MECHANICS

TEST II

Note:
--The following information pertains to Problems 1 through 5: Projectiles A, B, C, and D are fired at the same time from a height h meters above the ground. With the exception of Projectile D, which is dropped from rest, all the projectiles (i.e., Projectiles A, B, and C) have the same muzzle velocity v_o, (though each is fired at a different angle--see the sketches below and note that the angle defined as θ is the same in all cases). It takes t_1 seconds for Projectile A to get to the top of its flight. It takes t_2 seconds for Projectile D to reach the ground.



1.) At time t_1, Projectile A's:

a.) Velocity will be perpendicular to its acceleration. [At t_1, the projectile is at the top of its arc. At that point, its velocity vector is comprised of its x-component (that component stays the same throughout the motion as there are no x-direction forces and, hence, no x-direction acceleration acting to change it) but no y-direction velocity (it's at the top of its arc and, hence, will go no farther upward). That means the velocity direction at the top is in the horizontal. The acceleration vector throughout the motion is in the y-direction (i.e., gravity pointing down), so the velocity vector and the acceleration vectors are perpendicular to one another. This response is true, but are there others?]

b.) Velocity will be $v_o\cos\,\theta$. [At the top, the only velocity component that is non-zero is the horizontal component. As the x-component of the velocity will be constant throughout the motion, and as that component does, indeed, equal $v_o\cos\,\theta$, this response is true. Are there others?]

c.) X-component of acceleration will be twice what it was at t = 0. [If this were an A.P. test and you were thinking clearly, you would never have gotten to this question as you would have already determined that both a and b were true and, hence, d must be the answer. Nevertheless, this is an interesting and tricky question. The x-component of the acceleration is zero (see Response a for more discussion). Twice zero is still zero, so this statement is true.]

d.) All of the above responses are true. [Given all that has been said above, this is the one.]

2.) Projectile A's:

a.) Acceleration is greater on the way up than on the way down. [The only acceleration acting is gravity in the y-direction. It is a constant, so this statement is false.]

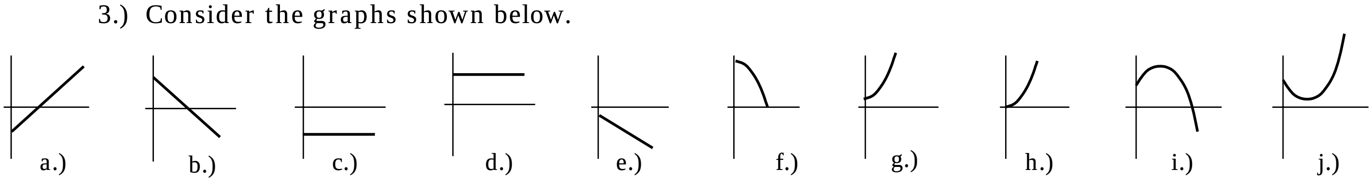
b.) Velocity changes at the same rate going up as going down. [This is the same as saying that the acceleration of the body is a constant, which it is. This statement is true.]

c.) Y-component acceleration sign is the same as its y-component velocity sign while going up. [The y-component of acceleration is that of gravity. Its sign is negative. The y-component of the velocity going upward is in the direction of motion, or positive. Clearly the two are not the same.]

d.) Velocity, when at h going upward, will be the same as its velocity when at h coming down. [This is tricky. Assuming there is no friction, the velocity <u>magnitude</u> when the body is at h going up will be the same as when going down, but the directions will be different. As we are dealing with vectors, this difference in direction makes the velocities different.]

e.) Both b and d. [Nope, but this was included to be tricky in case you hadn't realized that d was not a legitimate statement.]

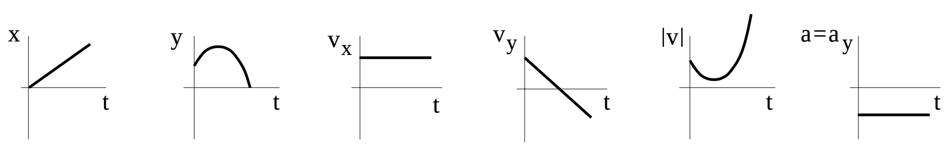
3.) Consider the graphs shown below.



Projectile A's:

a.) Y-component of Position vs. Time graph looks like graph a.
b.) X-component of Position vs. Time graph looks like graph j.
c.) Y-component of Velocity vs. Time graph looks like graph b.
d.) X-component of Velocity vs. Time graph looks like graph c.
e.) Y-component of Acceleration vs. Time graph looks like graph d.

[Commentary: A graph for each of the major parameters for this situation is shown below. This is something you should have been able to both visualize and sketch on your own. If you think you wouldn't have been able to do that, use the graphs provided as a stimulus to do the visualization part.]



4.) The time t_2:

a.) Depends only on h and constant(s). [Using $x_2 = x_1 + v_1 t + .5at^2$ with $a = -g$, $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = .5(-g)t^2$. This selection is evidently true, but are there other true statements?]

b.) Is the same time it takes Projectile B to hit the ground. [The time it takes to hit the ground is a y-motion related question. As the initial velocity in the y-direction for both cases is the same (it's zero), and as the gravitational acceleration is the same in both cases, the two projectiles should hit the ground in the same amount of time. This statement is true. Other truths?]

c.) Is more than the time it takes Projectile C to hit the ground, but less than the time it takes Projectile A to hit. [Because Projectile C had a downward initial velocity in the y-direction, it will take less time to hit the ground than does Projectile D which had no initial velocity in the y-direction. And as Projectile A had an upward y-component of its velocity, it will take more time to reach the ground. This statement is true.]

d.) Both a and b. [Nope.]

e.) All of the above except d. [This is the one.]

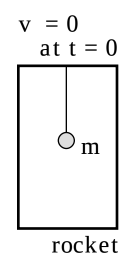
5.) If h were doubled, Projectile D's:

a.) Time to touch down will double. [Using $x_2 = x_1 + v_1t + .5at^2$ with $a = -g$, $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = .5(-g)t^2$. This means that $t = (2h/g)^{1/2}$. If h is doubled, t goes up by a factor of $(2)^{1/2}$, not by a factor of 2. This response is false.]

b.) Velocity just before touch down will double. [Using $v_2 = v_1 + at$ with v_2 being the velocity just before hitting the ground, $v_1 = 0$, and $a = -g$, we get $v_2 = -gt$. We have already determined that doubling h does not mean t doubles, so this statement is false.]

c.) Acceleration just before touch down will double. [Acceleration in these cases is always constant in both the x and y-direction. False.]

d.) None of the above. [This must be the one.]



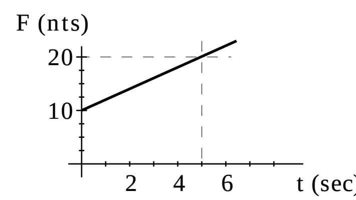
6.) A mass m is attached to a string hung from the ceiling of a rocket. The rocket accelerates from rest upward with an acceleration equal to a_1. Just an instant after t = 0:

a.) The tension in the line is mg. [If the rope were stationary, or if the rocket were moving with constant velocity (i.e., no acceleration) either up or down, this statement would be true. But in this case, summing the force and using N.S.L. on m yields T - mg = ma. T does not equal mg. Response false.]

b.) The tension in the line is $m(g + a_1)$. [From above, this is true.]

c.) The tension in the line is $m(g - a_1)$. [Nope.]

d.) None of the above. [Nope.]



7.) A 2.5 kg mass moves one-dimensionally over a frictionless surface. A single, external force F = 2t + 10 is applied (see graph to the right). The body's velocity changes from 0 m/s to 30 m/s during the first 5 seconds:

Important Note: When taking the multiple choice part of an AP test, it is important to do the easy problems immediately, leaving the doable but more difficult problems for later. This problem is neither. If I were taking this as a test, I'd skip it!

a.) The net work done by the force is 75 joules (the area under the line). [The area under a force vs time graph has the units of newton·seconds. The units of work are newton·meters, or joules. Just from a consideration of the units, this statement must be false.]

b.) The net work done by the force is 1125 joules. [To begin with, as both the force and direction of motion are positive (a body going from 0 to 30 m/s is moving in the positive direction), the work must be positive. That excludes Responses c and e below. As far as the numerical answer to the work question goes, there is the hard (but educational) way and the easy way. First, the hard way.

We know that $W = \int F \bullet dr$, that F = 2t + 10, and that as F = ma, a = F/m = (2t+10)/m. Knowing a as a function of time, we can also use Calculus to derive an expression for the velocity as a function of time (in this case, noting that the initial velocity is zero, that expression is $v = (t^2 + 10t)/m$).]

Note: Why might we need that last bit of information? F is a function of time, which means we must express dr as a function of time before we can deal with the integral. Using the Chain Rule, that relationship is dr = (dr/dt)dt (notice how the dt's could cancel leaving dr if one were interested in doing such a thing), or dr = v dt. This is why we need v(t).

With all of this, we can write:

$$
\begin{aligned}
W &= \int F \bullet dr \\
&= \int (F)(dr)\cos 0^o \\
&= \int (F)\left(\frac{dr}{dt}dt\right)(1) \\
&= \int (F)((v)dt) \\
&= \int (2t+10)\left(\frac{t^2+10t}{m}\right)dt \\
&= \frac{1}{m}\int_{t=0}^{5}(2t^3+20t^2+10t^2+100t)dt \\
&= \frac{1}{m}\int_{t=0}^{5}(2t^3+30t^2+100t)dt \\
&= \frac{1}{2.5}\left(\frac{1}{2}t^4+10t^3+50t^2\right)_{t=0}^{t=5} \\
&= 1125 \text{ joules.}
\end{aligned}
$$

Now, the easy way. We know from the Work/Energy theorem that $W_{net} = KE_2 - KE_1$. That means that $W_{net} = (.5)(2.5)(30)^2 - (.5)(2.5)(0)^2 = 1125$ joules. In either case, this response is true.]

c.) The net work done by the force is -1125 joules. [Wrong sign.]

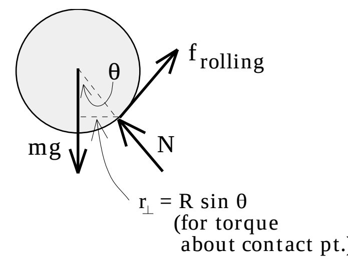
d.) The net work done by the force is 2813 joules. [Got this by forgetting to divide by the mass m. This response is false.]

e.) The net work done by the force is -2813 joules. [Nope.]

8.) A 20 centimeter diameter ball whose moment of inertia is $(2/5)mR^2$ accelerates down a 30^o incline plane. After traveling .2 meters down the incline, its angular velocity is 14 rad/sec. Assume the magnitude of g is 10 m/s^2.

a.) The velocity of the ball's center of mass at the .2 meter point is 140 m/s. [The relationship between the velocity of a ball's center of mass and its angular velocity at some point in its motion is $v_{cm} = R\omega$, where R is the number of meters per radian there are associated with the body's motion. Putting in the numbers yields: v_{cm} = (.1 m/rad)(14 rad/sec) = 1.4 m/s. If you correctly halved the diameter to get the radius but didn't convert from centimeters to meters, you got a numerical value for the velocity of 140. Unfortunately, its units are cm/s instead of m/s, so this response is false.]

b.) After traveling an additional .2 meters, the ball's angular velocity will be 28 rad/sec. [A constant angular acceleration situation, which this is, means that the angular velocity will change linearly in time. That is, the body will pick up twice the angular velocity in twice the time (not twice the distance). As the second .2 meter stretch of the trip will take considerably less time than did the first .2 meter leg, the angular velocity will not double during the second leg, and this response is false.]



c.) The ball's angular acceleration is 35.7 rad/sec^2. [Before summing up the torques acting about the contact point between the ball and the incline, we need the moment of inertia about the contact point. Using the parallel axis theorem, we can write: $I_{contact} = I_{cm} + md^2$, where d is the distance between the two parallel axes (in this case, that distance is the radius R of the ball). Doing the math yields $I_{contact}$ = $(7/5)mR^2$. Summing the torques about the contact point yields:

$mg(R\sin\theta) = I_{contact}\alpha = [(7/5)mR^2]\alpha$, or $\alpha = \frac{5g\sin\theta}{7R} = \frac{5(10m/s^2)(\sin 30^o)}{7(.1m)}$. This equals 35.7 rad/sec^2, which means that this response is true.]

d.) Both a and c are true statements. [As Response a is incorrect, this is false.]

9.) A .5 kg ball moving at v = (20 m/s)i strikes a massive wall over a .02 second period and, reversing itself, leaves with a velocity magnitude equal to 16 m/s.

a.) The impulse applied to the ball during the collision is (2 nt·sec)i, and the magnitude of the average applied force is 100 nts. [The body's change of momentum vector and the impulse vector are equal to one another. If the incoming velocity is positive, the outgoing velocity will be negative. If you were NOT careful about the sign of each velocity, you might have taken the difference between .5(16) - (.5)(20) and ended up with 2 nt·sec. This is wrong. With the signs taken into account, we can write: impulse = $mv_{after} - mv_{before}$ = (.5 kg)(-16 m/s) - (.5 kg)(20 m/s) = -18 nt·sec. Although the i unit vector should technically be included, the fact that the motion is one dimensional means that only the sign of the impulse is important. This response is false.]

b.) The impulse applied to the ball during the collision is (18 nt·sec)i, and the magnitude of the average applied force is 900 nts. [Nope. Wrong sign for the impulse.]

c.) The impulse applied to the ball during the collision is (-2 nt·sec)i, and the magnitude of the average applied force is 100 nts. [Nope.]

d.) The impulse applied to the ball during the collision is (-18 nt·sec)i, and the magnitude of the average applied force is 900 nts. [From the calculation in Response a, Δp = -18 nt·sec. With the collision occurring over .02 seconds and the expression $F_{avg}\Delta t = \Delta p$, we can write that F_{avg} (.02 sec) = (-18 nt.·sec), or F_{avg} = 900 nts. In short, this is the one.]

e.) None of the above. [Nope.]

10.) A satellite 3R units from the surface of a planet of radius R has potential energy U_1. Its gravitational potential energy 6R units from the surface (call this potential energy U_2) will be:

a.) Larger and equal to $(1/2)U_1$. [This is tricky if you over think the problem. In general, a body's gravitational potential energy gets smaller the closer you get to a field producing planet. This makes sense even though the gravitational potential energy function associated with $F_{grav} = -Gm_1m_2/r^2$ is inherently negative (at infinity, U = 0; in closer, it is negative Gm_1m/r--the biggest the potential energy ever gets is zero!). That means that the farther you are out from a planet, the larger the potential energy value will be (this eliminates Response d). As for algebraic values: The distance between the center of the two bodies is 4R (not 3R--you want the center to center distance, not the center to surface distance). Using our gravitational potential energy expression, we get $U_1 = -Gm_sm_p/(4R)$. At the second position, $U_2 = -Gm_sm_p/(7R)$. Writing U_2 in terms of U_1, we get $U_2 = U_1(4/7)$, which means that this option/response is not true.]

b.) Larger and equal to $2U_1$. [From above, this option is false.]

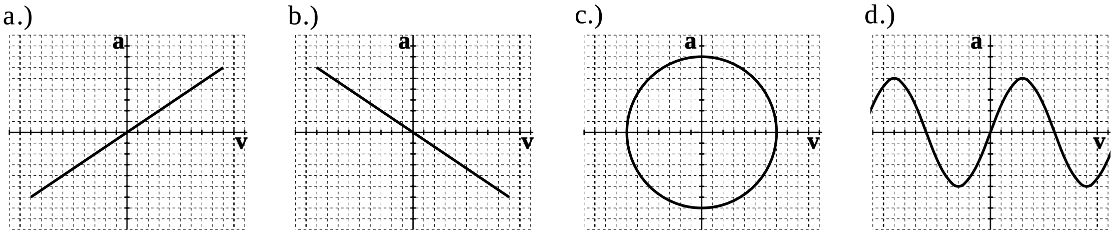
c.) Larger and equal to $(4/7)U_1$. [This is the one.]

d.) Smaller and equal to $(4/7)U_1$. [From above, this option is false.]

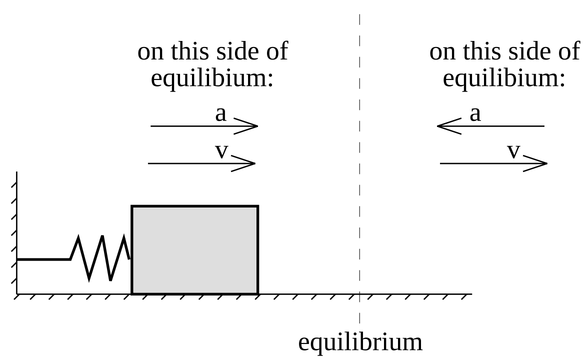
e.) None of the above. [Nope.]

11.) A graph of the acceleration vs. velocity of a body oscillating in simple harmonic motion looks like:

a.) b.) c.) d.) e.) None of these.



[Commentary: If you will remember, the position function for this kind of motion is characterized by a sine wave whereas the velocity function is a cosine and the acceleration function is a negative sine. Put a little differently, the velocity and acceleration are out of phase with one another by a quarter of a cycle. What does this mean? Consider the sketch to the right. If the body's velocity at the point shown is, say, +2 m/s (note that the body is picking up speed because the acceleration is in the direction of motion), there will be a point on the other side of x = 0 where the body's velocity will again be +2 m/s (in that case, the body will be slowing down because the acceleration will be opposite the direction of motion). That means there are two accelerations to be assigned to +2 m/s. How does one graph such a function? With a circle! In short, graph c does the trick.]



12.) A 30 newton force acting at 30^o below the horizontal acts over a 12 meter distance. The work done by F:

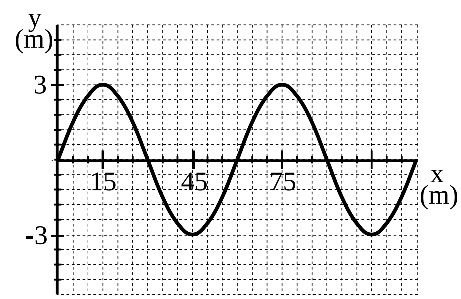
a.) Is -360 joules. [In the first place, you don't know if the direction of motion is the same as the direction of the force, which means you don't know whether the angle is 30^o or 150^o. If you assume the displacement and force are in the same direction (something you haven't the right to do), the work done by the force would be W = Fd cos 30^o = (30 nt)(12 m) cos 30^o = +312 joules. The answer in this response was obviously obtained by making the displacement/force assumption coupled with forgetting to multiply by the cosine term in the dot product. In short, this answer is false for all sorts of reasons.]

b.) Is +360 joules. [For the same reasons outlined in Response a, this statement is false.]

c.) Would be the same if the angle had been above the horizontal. [This response makes no assumptions about the displacement and force direction. It simply observes that if the angle is the same, it doesn't matter whether the force vector is above or below the line of the displacement. The amount of work will be the same. This statement is true.]

d.) Both a and c. [Nope.]

e.) Both b and c. [Nope.]



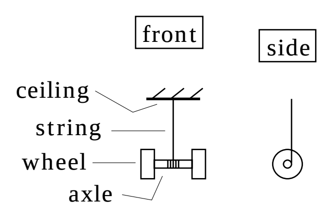
13.) At a particular point in time during the motion of a wave, the displacement vs. position function is as graphed to the right.

a.) The amplitude of the wave is 3 meters, its wavelength is 60 meters, and its frequency is .5 Hz. [From inspection, the amplitude of this wave is 3 meters. The wavelength can be determined by measuring the distance between two crests or two troughs. In this case, that value is 60 meters. The frequency is a time related quantity, and as this graph tells us the displacement of the wave at various points in space AS THEY EXIST AT ONE TIME AND ONE TIME ONLY, we have no way of determining either the period or the frequency. As such, this response is false.]

b.) The amplitude of the wave is 6 meters, its wavelength is 30 meters, and its frequency is impossible to tell with the information given. [The amplitude is not 6, it's 3. This response is false.]

c.) The amplitude of the wave is 3 meters, its wavelength is impossible to tell with the information given, and its frequency is impossible to tell with the information provided. [From above, everything is OK here except the determination of the wavelength, which should be 60 meters. This statement is false.]

d.) None of the above. [This has to be the one.]



14.) A string attached to the ceiling at one end has itself wrapped around an axle whose radius is R at the other end (see sketch). Attached to the axle are two wheels of mass m each, one at each end. The moment of inertia of the entire assembly along the axle's length is $1.4mR^2$. All numerical values are measured in the MKS system. The system is released to freefall. During the fall:

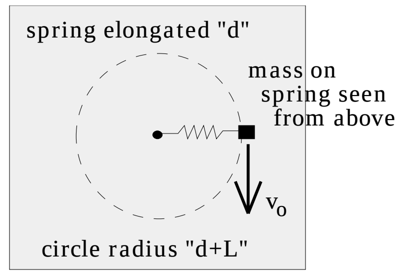
a.) Energy is conserved but angular momentum is not conserved. [In this case, tension does work that is not taken into account via a potential energy function, so energy is not conserved (though the modified conservation of energy equation could be used to determine the velocity of the body after it has dropped some known distance). This statement is false.]

b.) The body's angular acceleration α is numerically greater than the acceleration a of the body's center of mass. [The temptation may be to actually solve this problem to see if α>a. That would be a waste of time. Translational acceleration and rotational acceleration are related by the expression $a = R\alpha$. If R is greater than 1, then the numerical value of α will be less than a. If R is less than 1, the opposite will be the case. In any case, this statement will not always be true, so it must be false.]

c.) The body's angular acceleration α will be numerically less than the acceleration a of the body's center of mass only if the radius of the axle is greater than 1 meter. [According to what was said in Response b, this statement is true.]

d.) Both a and b. [Nope.]

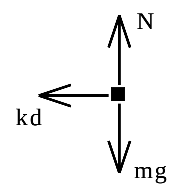
e.) Both a and c. [Nope.]



15.) A spring produces a force equal to kx, where x is the elongation of the spring and k is a constant. Our spring's unstretched spring length is L. One end of the spring is attached to a mass m and the other end of the spring is hooked frictionlessly over a pin at the center of a table. The spring is elongated a distance d beyond L, and the mass is given a constant velocity magnitude v_o so that the whole system moves in a circular path as shown to the right (the view looks down from above).

a.) With the velocity magnitude a constant, the acceleration for this motion must be zero. [For an object to move along a curved path, there must exist within the system a center-seeking force. That, in turn, means that there must be a center-seeking (centripetal) acceleration. This statement is false.]

b.) The acceleration is non-zero, and k = mg. [The acceleration is non-zero, but the rest is nonsense. To get k, we must use N.S.L. The f.b.d. for the mass looking head-on (i.e., as though the mass were coming straight at us) is shown to the right. Summing the forces in the center-seeking direction (kd in this case) and putting that acceleration $a_c = v_o^2/R = v_o^2/(d+L)$, we get:



f.b.d. of mass looking head on--radius of curve "d+L" is directed to the left

$$kd = m[v_o^2/(d+L)]$$
$$\Rightarrow \quad k = mv_o^2/[d(d+L)].$$

This statement is false.]

c.) The acceleration is non-zero, and $k = (mv_o^2)/[x(d + L)]$. [According to the expression derived in Response b, this statement is true.]

d.) The acceleration is non-zero, and $k = (mv_o^2)/(x)$. [This is false.]

16.) Two 2 kilogram masses collide. The first, Body A, has 25 joules of kinetic energy before the collision. The second mass is initially stationary. The two masses experience a perfectly inelastic collision. After the collision, mass B's kinetic energy is:

a.) 12.5 joules. [This is a typically sneaky, A.P. type question. Energy information is given, an energy related question is asked, but energy isn't conserved in the situation, so using conservation of energy will take you nowhere. Cleverly, though, the answer you would have gotten if you had treated energy as conserved is provided. That is, the after-collision kinetic energy of mass B would simply be half the before-collision kinetic energy in the system (remember, the masses are the same), and that value is 12.5 joules. Unfortunately, this response is false.]

b.) 2.5 joules. [To actually do the problem, we know that before the collision, mass A's kinetic energy is $KE_o = .5mv_o^2 = .5(2\ kg)v_o^2 = 25$ joules, or mass A's before-collision velocity is $v_o = 5$ m/s. Using the conservation of momentum through the collision, we can write: $mv_o = 2mv_1$, or the after-collision velocity of the two stuck-together bodies is $v_1 = 2.5$m/s. Plugging that into mass B's after-collision kinetic energy equation, we get: $KE_1 = .5mv_1^2 = .5(2\ kg)(2.5\ m/s)^2 = 6.25$ joules. This response is false.]

c.) 6.25 joules. [From above, this is the one.]

d.) None of the above. [Nope.]

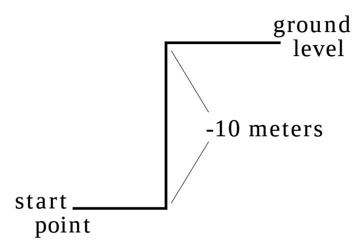
17.) An orbiting 1200 kg satellite has a period of 43 minutes (2580 seconds) in an orbit that is 15,000 meters above a planet of radius 10^6 meters. The satellite's speed is:

a.) 2472 m/s. [This is a little tricky as you don't really need all of the information given. The heart of the problem is in the fact that a satellite's period T and its velocity v are related by the expression: $v = 2\pi r/T$, where r is the radius of the orbit. For this situation, we get: $v = 2\pi(1{,}015{,}000\ meters)/(2580\ sec.) = 2472$ m/s. This response is true.]

b.) 2435 m/s. [If you had taken the radius of motion to be 10^6 meters instead of $(10^6 + 1.5\text{x}10^4)$ meters, you would have gotten this incorrect answer.]

c.) 32.5 m/s. [If you had taken the radius of motion to be $1.5\text{x}10^4$ meters, you would have gotten this incorrect answer.]

d.) None of the above. [Nope.]



18.) A 10 kg projectile moving vertically has 1500 joules of gravitational potential energy at the same time that it has 1500 joules of kinetic energy. (Assume g's magnitude is 10 m/s^2). The potential energy is defined as zero at a point 10 meters below ground level where the body begins its motion (see sketch):

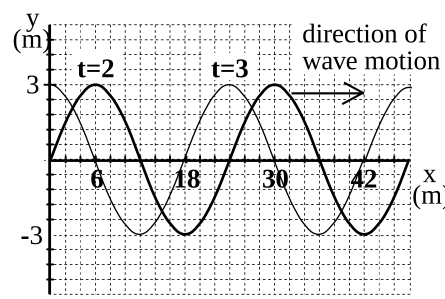
a.) The work gravity does to the top of the body's flight is -3000 joules, and the body's maximum height is 30 meters above ground level. [Knowing where ground level is relative to the body's initial position is inconsequential here. To use the gravitational potential energy function mgy, all you need is knowledge of the level at which the potential energy is zero. As that level is the so-called start point of the motion, the total energy in the system at that point (3000 joules) is all kinetic. Gravity does negative work in bringing the body to rest at the top of the flight, which means the total work gravity does over that portion of flight is, indeed, -3000 joules. As for the maximum height, $mgy_{max} = (10\ kg)(10\ m/s^2)y_{max} = 3000$ joules, or $y_{max} = 30$ meters ABOVE THE ZERO POTENTIAL ENERGY LEVEL. As this is 10 meters below ground level, the body will fly 20 meters above ground level and this statement is false.]

b.) The work gravity does to the top of the body's flight is -3000 joules, and the body's maximum height is 20 meters above ground level. [From the reasoning above, this statement is true.]

c.) The work gravity does to the top of the body's flight is -4000 joules, and the body's maximum height is 30 meters above ground level. [If ground had been the zero potential energy level, then at ground level all of the energy in the system (3000 joules) would be kinetic and the velocity of the body at that point would have been $1/2mv_{gr}^2 = 3000$, or $v_{gr} = 24.5$ m/s. In that case, gravity would have to do -3000 joules of work to bring the body to rest, NOT INCLUDING the work it did while that body went from y = -10 meters to ground level (that amount of work was (10 kg)(10 m/s^2)(-10 meters) = -1000 joules). The total work done in this scenario would, indeed, have been -4000 joules. The height in that case would have been such that $(1/2)mv_{gr}^2 + 0 = mgy_{top}$, or $.5(10 \text{ kg})(24.5 \text{ m/s})^2 = (10 \text{ kg})(10 \text{ m/s}^2)y_{top}$, or $y_{top} = 30$ meters. This statement would have been true under those circumstances, but isn't under the circumstances presented in the original problem.]

d.) The work gravity does to the top of the body's flight is -4000 joules, and the body's maximum height is 20 meters above ground level. [Nope.]

19.) A graph of a traveling wave as seen at t = 2 seconds and t = 3 seconds is shown to the right. At the rate it is traveling, how far will it travel between t = 2 seconds and t = 5 seconds?



a.) 12 meters. [According to the graph, the wave is moving 18 meters every second (the distance the wave traveled between the t = 2 seconds and t = 3 seconds is 18 meters, so its velocity is (18 meters)/(3 sec - 2 sec) = 18m/s). Over a 3 second period (i.e., the time between t = 2 seconds and t = 5 seconds) moving at 18 m/s, the wave will travel 54 meters, and this response is false.]

b.) 18 meters. [Nope, although this is the answer you would have gotten if you mistakenly took the distance traveled in one second to be 6 meters.]

c.) 54 meters. [This is the one.]

d.) None of the above. {Nope.]

20.) Newton's Second Law is applied to a system. After a free body diagram is drawn and the forces summed, the equation $-32x^2 = 2a$ emerges, where x is the position and a is the acceleration of the body at an arbitrary point in time.

a.) This equation does not characterize an oscillatory system. [If this equation had been $-32x = 2a$, we would have had the characteristic equation for an ideal spring (i.e., $-kx = ma$). If that had been the case, a positive displacement x would yield a negative force and a negative displacement x would have yielded a positive force. That is, the force would always have been oriented back toward the equilibrium position. With the expression $-32x^2 = 2a$, the $-x^2$ term guarantees that, no matter what the sign of x,, the net force will always be negative. In short, this force function is not a restoring force and, as a consequence, will not produce oscillatory motion of any type. As such, this response is true.]

b.) This equation does characterize an oscillatory system and the motion is simple harmonic in nature. [Nope.]

c.) This equation does characterize an oscillatory system and the motion's frequency is 4 radians per second. [Nope.]

d.) Both b and c. [Nope.]

e.) None of the above. [Nope.]

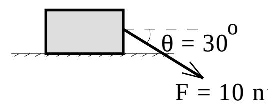
21.) A spring oscillates with frequency 1 cycle/second. What approximate length must a simple pendulum have to oscillate with that same frequency?

a.) 25 cm. [A frequency of 1 cycles/second corresponds to an angular frequency of 2π radians per second. The angular frequency function for a pendulum is $(g/L)^{1/2}$, where L is the pendulum length. Putting it all together, we can write $2\pi = (g/L)^{1/2}$, or $L = g/4\pi^2$, or approximately .25 meters. This response is true.]

b.) 50 cm. [Nope.]

c.) 67 cm. [Nope.]

d.) 90 cm. [Nope.]



22.) A body moves 25 meters. For the entire motion, a 10 newton force acts on the body as shown to the right.

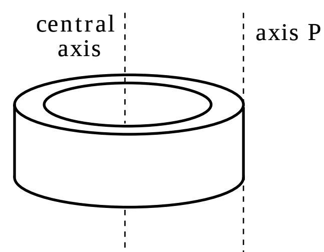
a.) The work done by the force is 250 joules. [This is a tricky question on a number of counts. First, the direction of the motion was not defined. As such, you have no idea whether the work quantity is going to be positive or negative. In short, you can't calculate a number response; Solution e is the answer. If the direction of motion was to the right, $Fd\cos\theta = (10)(25)\cos 30^o = 217$ joules. 250 joules (determined by multiplying the force times displacement without the cosine factor) wouldn't have been the correct response even if the motion had been to the right.]

b.) The work done by the force is -250 joules. [If the motion had been to the left, the calculated work value would have been -217 joules. This answer is obviously wrong.]

c.) The work done by the force is 217 joules. [From above, this would have been correct if the motion had been to the right. As we don't know if that was the case, this response is false.]

d.) The work done by the force is -217 joules. [Same problem. This response if false.]

e.) None of the above. [Given the trickiness of the problem, this is it.]



23.) The moment of inertia about the central axis of a ring is $(1/2)m(R_1^{\,2}+R_2^{\,2})$, where R_1 is the inside radius and R_2 is the outside radius of the ring (see sketch). The moment of inertia about the axis labeled P in the sketch is:

a.) $(1/2)mR_1^{\,2} + (1/2)mR_2^{\,2} + (1/2)m[(R_1+R_2)/2]^2$. [The Parallel Axis Theorem states: $I_p = I_{cm} + mh^2$, where h is the distance between the two axes. Using that expression yields: $I_p = [(1/2)m(R_1^{\,2} + R_2^{\,2}) + mR_2^{\,2}]$. Regrouping yields: $I_p = (1/2)mR_1^{\,2} + (3/2)mR_2^{\,2}$. This response is false.]

b.) $(3/2)mR_1^{\,2} + (1/2)mR_2^{\,2}$. [According to the analysis done above, this response is false.]

c.) $(1/2)mR_1^{\,2} + (3/2)mR_2^{\,2}$. [According to the analysis done above, this response is true.]

d.) None of the above. [Nope.]

24.) A .3 kg body moves down a 30^o incline. At a point .7 meters from the bottom the body is found to be moving with a velocity magnitude of .2 m/s. At that point a frictional force of .5 newtons begins to act. Assume the magnitude of the gravitational acceleration is 10 m/s^2.

a.) The body will not make it to the bottom of the ramp. [To make it to the bottom of the ramp, the body needs more total energy at the .7 meter point than friction will remove from the system. On the body's way to the bottom, friction will extract energy in the amount -fd = -(.5 nt)(.7 m) = .35 nt·m. At the .7 meter point, the body has $.5mv^2 + mgh = .5(.3\ kg)(.2\ m/s)^2 + (.3\ kg)(10\ m/s^2)(.7 \sin 30^o$ meters) = 1.06newtons. The body will have enough energy to make it to the bottom of the ramp, and this statement is false.]

b.) The body will just barely make it to the bottom of the ramp. [As shown in Response a, the body will make it to the bottom of the ramp with energy and, hence, velocity to spare. This statement is false.]

c.) The body will make it to the bottom of the ramp, and its velocity at that point will be 4.73 m/s. [We know it will make it to the bottom. We don't know its velocity there. To get it: The potential energy was taken to be zero at ground level in Response a. Continuing in that vein, the energy at the bottom will be the total energy associated with the .7 meter point minus the energy extracted by friction as the body traveled to the bottom. Mathematically, then, E_{bot} = (1.06 joules) - (.35 joules) = .71 joules. As all of that energy will be kinetic, we can write $.5mv^2 = .5(.3\ kg)v^2 = .71$, or $v^2 = 4.73\ m^2/s^2$, or v = 2.18 m/s. The individual doing the calculation here did not take the square root in determining the velocity. This statement is false.]

d.) None of the above. [This is true by default.]

25.) The position function for an oscillating body is $x = 20 \sin (.6t - \pi/2)$. The approximate frequency of the motion is:

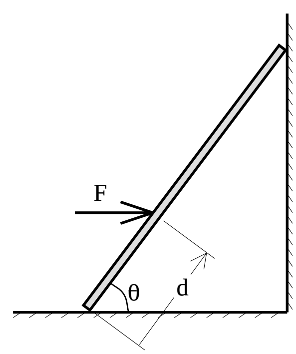
a.) 20 Hz. [As the angular frequency is .6 radians per second, the frequency is (.6 rad/sec)/(2π), or approximately .1 cycles/second. This response is not true.]

b.) 1.57 Hz. [Nope.]

c.) .6 Hz. [Nope.]

d.) .1 Hz. [This is the one.]

26.) A ladder of mass m and length L sits on a frictionless floor perched against a frictionless wall. A force F acting at a distance d units up the ladder (see sketch) keeps the ladder from angularly accelerating.



a.) The torque applied by mg about the point of contact with the floor is negative because mg is directed in the -j direction. [A torque's direction is negative if the force providing the torque motivates the body to angularly accelerate clockwise about the point about which the torque has been taken (this generality was first concluded after using the right hand rule as it applies to rxF). According to that criterion, mg does do this, so the torque mg applies about the contact point at the floor will, indeed, be negative . . . but not for the reason given. This response is false.]

b.) The torque applied by the normal force at the wall about the contact point with the floor is negative because N is directed in the -i direction. [In fact, N produces a positive torque, as defined in the discussion given in Response a. This statement is false.]

c.) About the floor contact, the torque due to F is negative as it tends to motivate the ladder to angularly accelerate clockwise. [This statement is true.]

d.) Both a and b. [Nope.]

27.) A mass m blows into two pieces whose mass ratio is 5 to 9 (call these pieces m_a to m_b):

a.) The ratio of the kinetic energies (E_a to E_b) will be 5 to 9 respectively, and the ratio of the momenta (p_a to p_b) will be 5 to 9. [If the mass is split into a 5 to 9 ratio, that is the same as saying the 5/14 of the mass went one way and 9/14 of the mass went the other. Letting m = 1 kg for simplicity, that observation and the conservation of momentum allows us to write: $0 = (5/14)v_1 - (9/14)v_2$. From this, we get the ratio $v_1/v_2 = 9/5$ (note that the ratio of $v_1^2/v_2^2 = 81/25$--we'll need this information when we look at the kinetic energy of both). Energy isn't conserved, but we can write out an expression for the kinetic energy of both bodies, then take a ratio. Doing so yields: $KE_1 = .5(5/14)v_1^2$ and $KE_2 = .5(9/14)v_2^2$. Taking a ratio of the two yields $KE_1/KE_2 = [.5(5/14)v_1^2]/[.5(9/14)v_2^2] = (5/9)(v_1^2/v_2^2)$. Substituting in for the velocity squared terms, we get $KE_1/KE_2 = (5/9)(81/25) = 9/5$. In short, this has the right numbers, but they are in the wrong order. This statement is false.]

b.) The ratio of the kinetic energies (E_a to E_b) will be 9 to 5 respectively, and the ratio of the momenta (p_a to p_b) will be 9 to 5. [From above, the energy part of this is true. What about the momenta? It might be tempting to write out the conservation of momentum equation for this situation, or $0 = (5/14)v_1 - (9/14)v_2$. From this, we get the ratio $v_1/v_2 = 9/5$. Unfortunately, this is the ratio of velocities, not the ratio of momenta. In fact, if we had written out the conservation of momentum equation in a more general fashion, we would have found that: $0 = p_a - p_b$, or $p_a = p_b$. In other words, the momentum ratio is 1:1 and this response is false.]

c.) The ratio of the kinetic energies (E_a to E_b) will be 9 to 5 and the ratio of the momenta (p_a to p_b) will be 1 to 1. [From above, this response is true.]

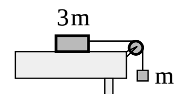
d.) None of the above is true. [Nope.]

28.) Two stars circle in elliptical orbits about their collective center of mass. If one star is three times the size of the second star, which of the following is true?

	momentum of center of mass	angular momentum	energy	for the smaller star, T^2 is proportional to r^3
a.)	conserved	conserved	not conserved	yes
b.)	not conserved	not conserved	not conserved	no
c.)	conserved	conserved	conserved	yes
d.)	conserved	conserved	conserved	no

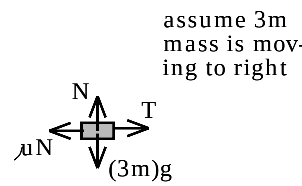
e.) None of the above.

[Commentary: What is conserved and what is not conserved in planetary motion problems? Well, there are no external torques acting on the system, so angular momentum must be conserved (this eliminates Response b). Assuming there is no friction, which is always a good approximation in these kinds of problems, there are no non-conservative forces acting, so energy is conserved (this eliminates Responses a and b). And as there are no external forces acting, the momentum of the system's center of mass will be conserved, so Responses c and d are still alive. The question comes down to whether the square of the smaller star's period is proportional to the cube of its semi-major axis (i.e., whether Kepler's Third Law applies here). What you will hopefully remember is that Kepler called his third Law a law only because Brahe's data made the T^2 proportional to r^3 phenomenon look like a universal truth. What Kepler didn't evidently take into consideration was the fact that Brahe's data had been taken by observing very small objects (i.e., planets) orbiting an immense object (i.e., the sun). In such cases, the system's center of mass and the center of the star are essentially the same, and T^2 is, to a very good approximation, proportional to r^3 (theory predicts this only after approximations are made--see your book if this isn't clear). As the two stars in this problem are almost the same size, the approximation that allows Kepler's Third Law to hold cannot be made, and T^2 is NOT proportional to r^3. In short, Response d is the one.]



29.) The frictional force acting on the 3m mass shown to the right is .6mg. If the hanging mass is m, the system's acceleration is:

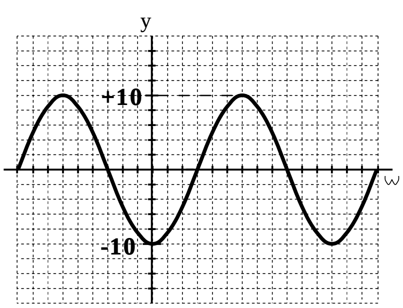
a.) .1g. [This is a tricky question. If you ASSUME the mass is moving to the right, the f.b.d. on the 3m mass will look like the f.b.d. shown to the right. The N.S.L. equation you get from that set-up coupled with the N.S.L. evaluation of the hanging mass will yield an acceleration equal to .1g. Unfortunately, although the direction of acceleration is obvious, we haven't been told the direction of motion of the body (it could initially be moving in either direction). As such, this statement is not necessarily true.]



b.) .4g. [If you assumed the direction of the acceleration was to the left, the direction of the frictional force in Part a's f.b.d. would have been reversed and the acceleration would have calculated to .4g. Again, unfortunately, we haven't been given the motion's direction. This statement is also false.]

c.) .133g. [If you assumed the motion's direction was to the right AND made the mistake of believing that the tension was equal to the weight of the hanging mass (i.e., mg), the acceleration would have calculated to .133g. As these assumptions are not necessarily true, this is a false statement.]

d.) Indeterminable as there isn't enough information. [By default, this is the one.]



30.) A graph of the position function for a body oscillating with an angular frequency of π radians per second in simple harmonic motion is shown to the right.

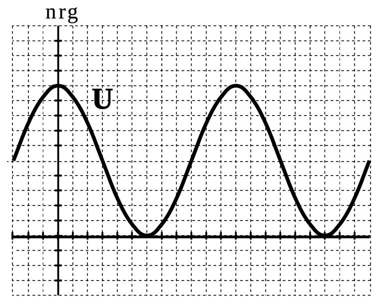
a.) The amplitude is 10, the period is 2 seconds per cycle, and the phase shift is $\pi/2$ radians. [The amplitude for this wave is 10 meters, which eliminates Response b. The period is the inverse of the frequency, where the frequency in this case is the angular frequency (i.e., π radians) divided by 2π, or .5 cycles/second. This yields a period of 2 seconds per cycle. So far, so good for this selection. As for the phase shift, if you were to shift a sine wave $\pi/2$ radians to the right (i.e., a quarter of a cycle), you would end up with a cosine wave. That is not what the graphs shows (in fact, the phase shift is $-\pi/2$). This response is false.]

b.) The amplitude is 20, the period is 5 seconds per cycle, and the phase shift is $-\pi/2$ radians. [The amplitude is off and this is false.]

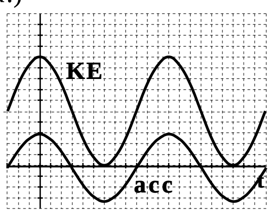
c.) The amplitude is 10, the period is 2 seconds per cycle, and the phase shift is $-\pi/2$ radians. [From what has been said above, this is the one.]

d.) None of the above. [Nope.]

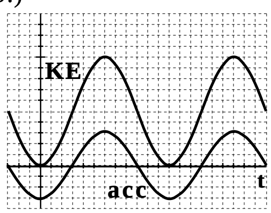
31.) The potential energy vs. time graph for an oscillating system is shown to the right. The kinetic energy vs. time and acceleration vs. time graphs look like:



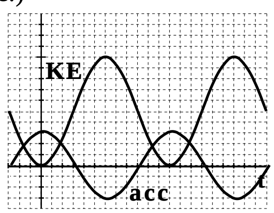
a.)



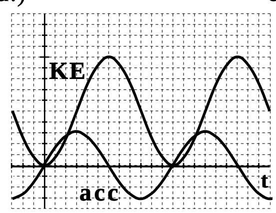
b.)



c.)



d.)



e.) None of these.

[Commentary: Looking at the potential energy graph, the maximum potential energy does not appear to be diminishing with time. That means that we can assume that energy is, to a good approximation, conserved. Due to this, the kinetic and potential energy functions should be mirror images of each other (this eliminates Graph a).

The first time I did this problem, my commentary continued on to say that because velocity and acceleration functions are out of phase with one another by a quarter of a cycle (see next sentence for justification of this should it not be obvious), the kinetic energy function (a velocity dependent quantity) should be out of phase with the acceleration by a quarter of a cycle, and graphs b and c could not be "it."

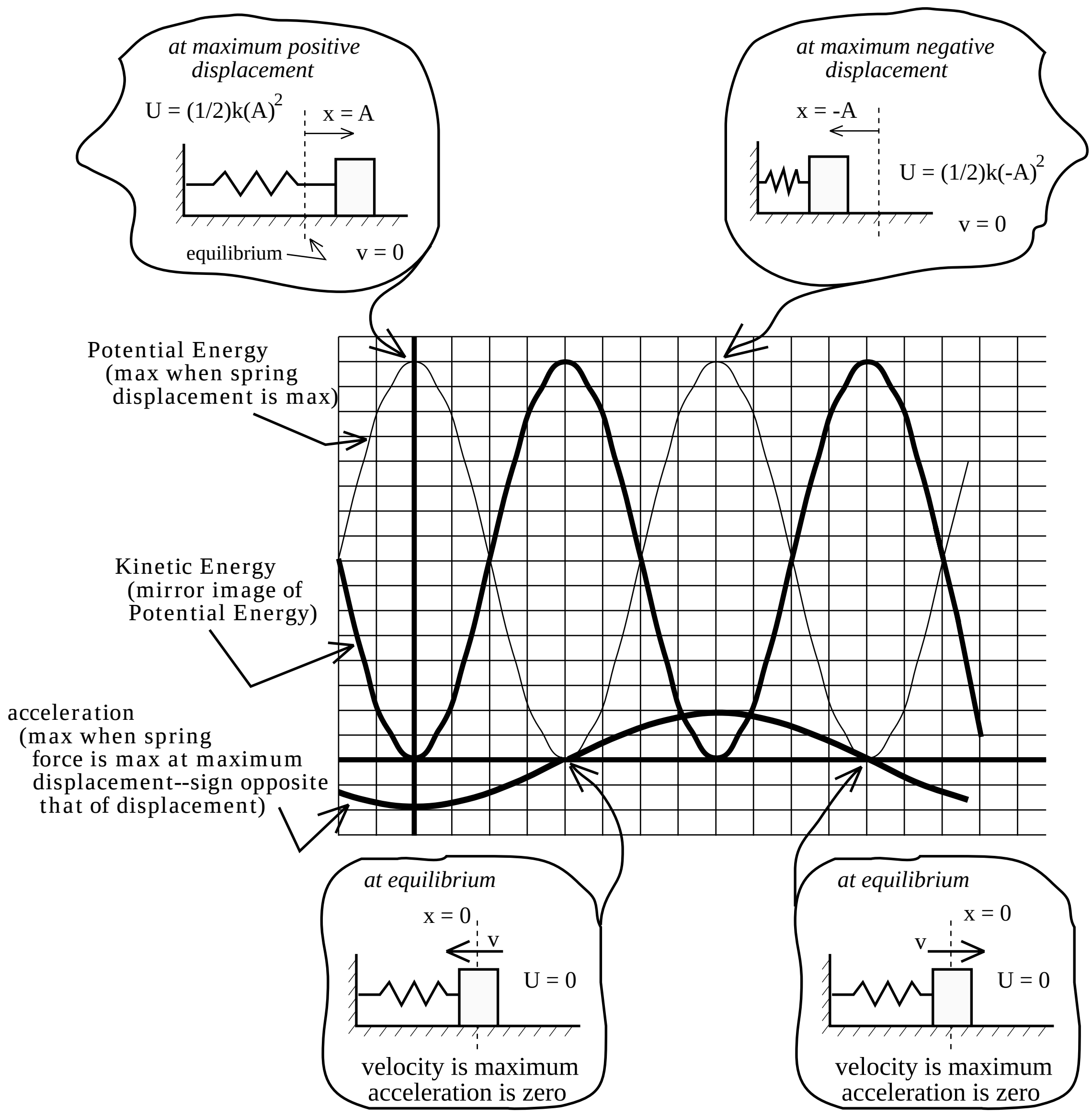
(Note that the justification for the velocity/acceleration phase relationship goes as follows: if we assume the velocity function is a sine wave--the acceleration, or derivative of the velocity function, will be a cosine wave. As sine curves are out of phase with cosine curves by a quarter of a cycle, the velocity and acceleration functions should be out of phase with one another by a quarter of a cycle.)

So what about graph d? When I generated that graph, I used the fact that the maximum kinetic energy occurs when a body is at equilibrium (visualize this by thinking about a mass attached to a spring), and that at equilibrium the force and acceleration are zero. Look at graph d's first KE peak (that's where I started when I constructed the graphic). The acceleration at that point is graphed as zero. To finish off the graphic, I constructed a sine wave around that point and came up with what you see. In fact, I presented this as the right answer, thereby committing the sin I yell and scream at you folks NOT to commit. Specifically, don't get so involved in the math that you completely lose track of the reality of the situation.

What am I talking about? A student noticed that although the graph may match at the first kinetic energy peak, it doesn't match at the origin of the axes. At that point, the kinetic energy is zero and the acceleration is shown to be zero--a clear no-no. When brought to my attention, I did the more complete evaluation shown on the next page.

The key to the problem? Kinetic energy is NOT a function of velocity, it's a function of velocity SQUARED. Squaring the velocity essentially halves the amount of time it takes for the kinetic energy to go from being maximum to being maximum again. That is, in the time it takes to go from a positive maximum acceleration to a negative maximum acceleration (this is half the acceleration cycle), the kinetic energy goes from positive maximum to positive maximum. Squaring the velocity halves the period of the kinetic energy waveform.

The answer is e. When I mess up, I do it good.



32.) At a particular point in its motion, a 10 kg projectile moving vertically has 1500 joules of gravitational potential energy at the same time that it has 1500 joules of kinetic energy. The body starts its motion at ground level, and it takes t_1 seconds to reach the point alluded to above.

(Assume the gravitational potential energy function is defined to be zero at ground level, and assume g's magnitude is 10 m/s^2). At the point:

a.) The maximum height the body reaches will be 15 meters. [The sum of the potential and kinetic energies is 3000 joules. This is the total mechanical energy in the system. At the top of the flight, all of that is wrapped up in potential energy. That is: mgy_{max} = 3000 joules. Putting in the numbers yields a maximum height y_{max} = 30 meters. This statement is false.]

b.) It will take $2t_1$ seconds for the body to reach the top of its flight. [With half the energy being potential, and with gravitational potential energy equaling mgy near the earth's surface, the assumption that most people would make here is that the body is halfway to the top. It is, indeed, halfway from the top, but it is not at all clear whether it is moving upward toward the top or downward away from the top. Both situations have the same potential and kinetic energy associated with them (remember, energy is not a vector). As such, we don't really know if it took t_1 seconds to get from ground level to the halfway point, or t_1 seconds to get from ground level to the top and then back down to the halfway point. Not knowing this makes it impossible to evaluate the situation, which means this statement must be false. For those who care, if the body was halfway going toward the top, the governing equation relating the body's y position and time would be the kinematic equation $(y_2 - y_1) = v_o t + .5at^2$. With the body at the halfway point, it would have to travel 2d to get to the top. From the equation, it should be clear that that would take $(2)^{1/2}t_1$ seconds to accomplish (try some numbers if you don't believe me). On all counts, this statement is false.]

c.) The body is exactly halfway to the top of its flight. [From Response b, this is true.]

d.) None of the above. [Nope.]

33.) A skater with arms outstretched goes into a spin during which her arms are brought in near her body. During the movement:

a.) Energy is conserved and angular momentum is conserved. [Although it may not be obvious, energy is not conserved here. How so? Chemical energy, which is neither kinetic nor potential in nature--is required to motivate the muscles of the arms to pull them in (they would naturally fly outward if given the freedom). That means energy is being drawn into the system from a source that is not part of the mechanical energy family, and this statement is false.]

b.) Energy is conserved and angular momentum is not conserved. [This statement is false on the grounds that energy is not conserved.]

c.) Energy is not conserved and angular momentum is conserved. [We have already established that energy is not conserved, so that part is true. Because the forces that pull the arms in produce no torque about the axis of rotation (a force aimed at the axis of rotation will produce no torque about the axis of rotation), angular momentum will be conserved. As for momentum, skaters are not usually moving when they perform spins, but even if they were, standard momentum would be conserved as long as there were no external forces acting on the body to change the momentum. In short, momentum is also conserved and this response is true.]

d.) Energy is not conserved and angular momentum is not conserved. [Nope.]

34.) A sound wave moving at 330 m/s has a frequency of 220 Hz. What is the phase difference between two points .3 meters apart?

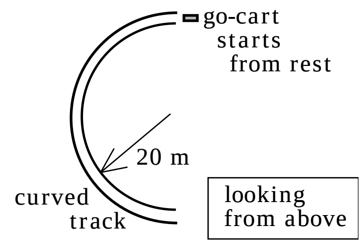
a.) (1/5) of a cycle, or $.4\pi$ radians. [The velocity of a wave will numerically equal the product of the wave's frequency and wavelength. Knowing that the frequency is 220 Hz and the wave velocity is 330 m/s, the wavelength must be 1.5 meters. A distance of .3 meters is one-fifth of 1.5 meters, so the points are one-fifth of a wavelength away from each other. As there are 2π radians in a wavelength, one fifth of that will be $2\pi/5 = .4\pi$, which is to say that this response is correct.]

b.) (2/3) of a cycle, or $4\pi/3$ radians. [Not so.]

c.) (3/2) of a cycle, or 3π radians. [Not so.]

d.) None of the above. [Not so.]

35.) A go-cart sitting on a flat, circular section of track of radius 20 meters accelerates at a rate of 1 m/s^2. If the car's mass is 100 kg and the coefficient of static friction between the road and its tires is .5, approximately how long will it take before friction can no longer hold the cart in circular motion and the cart spins out? Approximate g to be 10m/s^2.



a.) 2.4 seconds. [There is no way to do this problem shorthand--you have to work it out. To start, we need to determine the maximum velocity the body can handle, given what we know about the coefficient of static friction. Noting that in this situation, the normal force N equals the weight of the car mg, we can sum the forces in the center-seeking direction and write:

$$\mu_s N = ma_c$$
$$\Rightarrow \mu_s(mg) = m(v_{max}{}^2/R)$$
$$\Rightarrow v_{max} = (\mu_s gR)^{1/2}$$
$$= [(.5)(10\ m/s^2)(20\ m)]^{1/2}$$
$$= 10\ m/s.$$

An acceleration of 1 m/s^2 means the body changes its velocity by 1 m/s every second. At that rate, it will get to 10 m/s in 10 seconds. This statement is false.]

b.) 8.2 seconds. [From above, this statement is false.]

c.) 10.0 seconds. [From above, this statement is true.]

d.) 13.6 seconds. [From above, this statement is false.]