Chapter 7
Momentum Chapter Review

EQUATIONS:

- $x_{cm} = \dfrac{\sum_{i=1}^{n}(m_i x_i)}{M}$ [This is the definition of the *x coordinate* of the center of mass of a system of *n* discrete bodies (i.e., point masses). When done for all three axes, the center of mass yields a kind of weighted *average position* for the system, all relative to the coordinate grid being used.]

- dV [A differential volume, *dV's* size depends upon the geometry you are considering. For instance, if it is a differentially thin cylindrical shell, *dV* will equal the circumference $2\pi r$ of the shell times the length *L* of the shell times the differential thickness *dr* of the shell, or $dV = (2\pi r L)dr$. Other geometries require different expressions. Note that in all cases, there *has* to be a differential length like *dr* or *dx* in the expression.]

- $\rho = \dfrac{mass}{unit\ volume} = \dfrac{dm}{dV}$ [Called a *volume density function*, ρ (rho) is a RATIO that quantifies the amount of *mass per unit volume* associated with a distribution of mass. Differentially, $\rho = \dfrac{dm}{dV}$ with *dm* being the differential mass associated with the differential volume *dV*. The expression is useful because it allows you to write *dm* in terms of ρ and *dV*, or $dm = \rho dV$. If the structure is homogeneous (i.e., the mass is uniformly distributed throughout), ρ also equals the total mass in the structure divided by the total volume, or *M/V*. If, on the other hand, the structure is inhomogeneous, *M/V* is nonsense and a function for ρ must be provided (i.e., something like $\rho = kr$, where *k* is a constant and *r* is a variable that defines the distance between *dm* and an axis of interest). In any case, the $dm = \rho dV$ expression is ALWAYS true.]

- dA [A differential area, its size depends upon the geometry you are considering. For instance, if it is a flat, differentially thin hoop, *dA* will equal the circumference $2\pi r$ of the hoop times the hoops differential thickness *dr*, or $dA = (2\pi r)dr$. Other geometries require different expressions. Note that in all cases, there *has* to be a differential length like *dr* or *dx* in the expression.]

- $\sigma = \dfrac{mass}{unit\ area} = \dfrac{dm}{dA}$ [Called an *area density function*, σ (sigma) is a RATIO that quantifies the amount of *mass per unit area* there is behind a given area on the surface of a body. Differentially, $\sigma = \dfrac{dm}{dA}$ with *dm* being the differential mass behind the differential surface area *dA*. This function is used whenever there is mass variation in two dimensions, or in uniformly distributed three-dimensional situations that just *look* easier to do in two dimensions--a rectangular solid is a good example. The expression is useful because it allows you to write *dm* in terms of σ and *dA*, or $dm = \sigma dA$. If the structure is homogeneous, σ is equal to the total mass within the structure divided by the total surface the x direction at *time 2*, and $\sum F_{x,external}\Delta t$ is the net impulse provided by all of the EXTERNAL FORCES acting on the system in the x direction between *times 1* and *2*.]

COMMENTS, HINTS, and THINGS to think about:

- **Momentum**--what is it?
  --If you will remember, **inertial mass** was defined as a *relative measure* of a body's resistance to changing its motion. In other words, if you are told that a certain object has, say, 3 kilograms of mass associated with it, you know that the weight has *three times the resistance to changing its motion than does the standard in Paris*. Because you have had experience with objects measured in kilograms, that has meaning for you.
  --**Momentum** is a lot like inertial mass in the sense that it, too, is **a relative measure of a quality associated with massive objects**. It gives you a relative feel for the force needed to stop the object in a given amount of time. Newton realized that the two parameters that governed the difficulty or ease of changing a body's motion are mass and velocity. By taking the two relevant parameters and multiplying them together, he came up with a quantity that allowed him to quantify this characteristic of matter. That quantity, mv, he called *momentum*.
  --In short, a body with a lot of momentum will be hard to stop in a short period of time. A body with little momentum will be easier to stop in a short period of time.

- If you are dealing with a SINGLE MASS, the **Impulse Equation** relates the mass's *change of momentum* and the *total external impulse* ($F_{net,external}\Delta t$) applied to the body over the allotted time interval Δt.

- For a collision, the **area under** a ***force versus time*** **graph** yields $F\Delta t$--the impulse applied during the collision. This is useful as it is related to the body's *change of momentum*.

- **External forces** are forces that impinge upon pieces of a system but are not due to the interaction of those pieces with other pieces of the system. Gravity, for instance, is normally an external force. The force a wall applies to a ball when the ball is thrown against it is an external force UNLESS the wall is free to move after the collision (i.e., it is on rollers) and the initial and final momentum of the wall can be calculated. In that case, the wall can be considered *part of the system*, the force considered internal, and the total momentum of the ball and wall conserved. If the wall is secured to the ground so that it isn't free to move, it will provide an external force to the ball and the ball's momentum will *not* be conserved through the collision.

- Don't get **internal forces** and **conservative forces** confused. For *energy to be truly conserved*, the work done by each force acting on a system must have a potential energy function associated with it. That can happen only if the work done by each force is path independent. As *path independence* is the defining characteristic of the work done by a conservative force, *conservation of energy* in its strictest form can only happen if the forces in the system are conservative. For *momentum to be truly conserved*, each of the forces within a system must be due to the interaction of the pieces of the system. The forces must be *internal* to the system. What all of this means is that although a force like friction will never be conservative (hence, energy will never truly be conserved with friction present), it could exist as a consequence of the interaction of the pieces within the system (hence, could exist within a system in which *momentum* was conserved).