

Chapter 6
Energy Chapter Review

EQUATIONS:

- $W_F = \mathbf{F} \cdot \mathbf{d}$ [This is the amount of work a force \mathbf{F} does on a body as the body moves over a distance \mathbf{d} . This definition is applicable as long as both the force and the angle between the force and displacement are constant.]
- $W_F = \int \mathbf{F} \cdot d\mathbf{r}$ [This is the sum of the differential bits of work force \mathbf{F} does on a body moving between two points over successive, differential path lengths each of which are characterized by $d\mathbf{r}$. This definition is always applicable, but is usually used only when the force is variable, the displacement is along an oddball path, and/or the angle between the force and displacement varies.]
- $d\mathbf{r}$ [This is a differential displacement in Cartesian coordinates, or $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$, where \mathbf{i} , \mathbf{j} , and \mathbf{k} are unit vectors. In polar notation, the radial coordinate is usually the only one of interest, so the relationship is written as $d\mathbf{r} = dr\mathbf{r}$, where \mathbf{r} is a unit vector in the radial direction. There is an additional note at the end of this summary about radial coordinates.]
- $KE = (1/2)mv^2$ [This is the amount of kinetic energy an object of mass m has while moving with velocity magnitude v .]
- $W_{net} = \Delta KE$ [This is the Work/Energy theorem. It states that the *total work* done on an object is always equal to the change of the object's kinetic energy.]
- $W_F = -\Delta U$ [The amount of work a *conservative force field* does on an object moving from *point 1* to *point 2* in the field is equal to the change of the potential energy function associated with the force field, as evaluated at *points 1* and *2*.]
- $U(x) - U(\text{where force is zero}) = -\int_{\text{where force is zero}}^x \mathbf{F} \cdot d\mathbf{r}$ [This is the relationship between a force and its potential energy function. The evaluation of this integral is the way potential energy functions are derived.]
- $\mathbf{F} = -\nabla U$ [This is the relationship between a potential energy function and the force that generates it. The evaluation of the *del operator* is the way a force function is derived from its potential energy function.]
- $U_{grav. near earth} = mgy$ [This is the gravitational potential energy function near the earth's surface, where y is the distance above an arbitrarily set *zero potential energy level*. Note that this is arbitrary because there is no place on or near the earth's surface where the gravitational force is zero, hence no preferred spot for the potential energy to be zero.]

- $U_{grav. anywhere} = -G \frac{m_1 m_2}{r}$ [This is Newton's most general gravitational potential energy function. It is especially useful when far from the earth's surface (i.e., where the gravitational force is no longer equal to mg , but rather the more precise $G \frac{m_1 m_2}{r^2} (-\mathbf{r})$].
Note that U is zero at *infinity*, which means that the potential energy function is *derived* assuming the zero potential energy position is at infinity. The kinds of gravitational situations in which this force and potential energy function would be useful will be dealt with later in the course. They have been introduced here primarily because they are force and potential energy functions that aren't equal to zero at the coordinate origin.]
- $\mathbf{F}_{spring} = -kx\mathbf{i}$ [This is the force function for a spring, where k is a spring constant and \mathbf{F}_{spring} equals zero at $x = 0$ where the spring is at equilibrium.]
- $U_{spring} = (1/2)kx^2$ [This is the potential energy function for a spring, where the zero potential energy position is at equilibrium at $x = 0$.]
- $\Sigma KE_1 + \Sigma U_1 + \Sigma W_{extraneous} = \Sigma KE_2 + \Sigma U_2$ [This is the modified conservation of energy theorem, where the sum of KE_1 and U_1 is called the total mechanical energy at *time 1*, the sum of KE_2 and U_2 is the total mechanical energy existing within the system at *time 2*, and $W_{extraneous}$ is the total work done BETWEEN *times 1 and 2* by any force whose work hasn't been taken into account by the potential energy functions (remember, potential energy functions are useful only in that they allow you to determine how much work a conservative force field does as a body moves from one point to another in the field).]
- $P = W/t$ [This is the definition of *power*, being the amount of work *per unit time* that a force provides to a system. Its units in the MKS system are joules/second, or watts.]
- $W_{normal} = 0$ [The amount of work a *normal force* does is always zero because the normal direction is always perpendicular to the line of motion.]
- $W_{friction} = -fD$ [As frictional forces are always directed opposite the direction of motion, the work friction does is always negative and always equals the product of the frictional force and the TOTAL DISTANCE D over which the force is applied.]

COMMENTS, HINTS, and THINGS to be aware of:

- **Work**--what is it?
--Newton observed that a net force acting on a body over a given distance would change the body's motion. The two quantities that affect the magnitude of the change are *the magnitude of the force component in the direction of motion* and *the distance over which the force acts*. Multiplying those two quantities yields a number that gives us a relative feel for how much motion change we can expect. Called *work*, it is this defined quantity from which the entire theory of *energy* begins.

- **Energy**--what is it?
 - Objects are said to have energy if they have the ability to do work, either due to the fact that they are moving (quantified by *kinetic energy*--the energy of motion), or due to the fact that their position in a force field will allow them to pick up motion at some time in the future (quantified by *potential energy*--the energy of position--this *potential to pick up kinetic energy* function can be defined only for so-called *conservative force fields*).
- **Conservative forces** are forces that do work that is path independent. That is, the amount of work a conservative force does on a body moving from one point to another in the field is determined solely by where the endpoints are, not by how far or in what direction(s) the body traveled in traversing between the points.
- If you are **given a potential energy function** and asked to **derive its force function**, don't let the derivation scare you. The partial derivative in a **del operator** is nothing more than a derivative taken with respect to *one variable* at a time.
- If you are **given a force function** and asked to **derive its potential energy function**, don't let the derivation scare you. **The integral** simply determines how much *work* the force field does as a body moves from the *zero potential energy point* (this will normally be where the force is zero) to an arbitrary point located somewhere out from the origin.
- The **energy approach** is a book keeping technique. What you are doing is keeping track of the amount of energy of motion (kinetic energy) and energy of position (potential energy) that a body or system of bodies has at various points in time.
- **Conserved quantities** are quantities that *do not change with time*. If the mechanical energy of a system is said to be *conserved*, that means that the sum of the system's kinetic and potential energies at any given instant will always add to the same number.
- Think about what the **conservation of energy theorem** says in its strictest form. It says: Determine the total mechanical energy in a system (i.e., the sum of the kinetic and potential energies of all the bodies in the system) as evaluated *at a particular point in time*. Do the same at a second point in time. The individual kinetic and potential energy quantities may change over the time interval, but those two SUMS will be the same UNLESS there is work being done by forces that you can't take into account using potential energy functions (i.e., forces that are non-conservative--friction, for instance--or forces that are conservative but for which you don't have potential energy functions). If there are no extraneous forces doing work, $\sum KE_1 + \sum U_1 = \sum KE_2 + \sum U_2$ and the *total mechanical energy* within the system is said to be *conserved*.
- If **non-conservative forces** exist within a system, or if there exist forces that are conservative but for which you don't have a potential energy function, you can still use the **modified conservation of energy expression**. It says: The total mechanical energy in a system evaluated at a particular point in time will be different than the total mechanical energy evaluated at a later point in time. The variation will be due to work done by extraneous forces for which you don't have a potential energy function. You must either use $\mathbf{F} \cdot \mathbf{d}$ or $\int \mathbf{F} \cdot \mathbf{dr}$ for each of those forces to determine that extraneous work. This is denoted in the

modified conservation of energy theorem as $W_{extraneous}$. Symbolically, this can be written as $\Sigma KE_1 + \Sigma U_1 + \Sigma W_{extraneous} = \Sigma KE_2 + \Sigma U_2$.

- **Gravitational potential energy near the earth's surface** can be made zero anywhere only because the gravitational force near the earth's surface is a constant--it isn't *zero* anywhere. If there had been a preferred *zero force position*, that would be where U_{grav} would equal zero.
- A MINOR POINT about coordinate systems that will become important later: In a Cartesian system, $-i$ is a unit vector that is always directed *to the left* in the negative x direction. In a **polar spherical coordinate system**, on the other hand, a *radial* unit vector r always proceeds *from the origin out to the point of interest*. That means that a **negative radial vector**, characterized as $-r$, will always be directed *inward* toward the origin. This kind of notation is very useful when dealing with, say, the theoretical expression for the gravitational force produced by the earth. In that case, as gravity is always directed toward the earth's center no matter where you are on the earth, the gravitational force is conveniently characterized theoretically as being oriented in the negative radial (i.e., $-r$) direction.