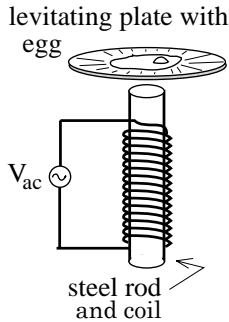


Chapter 17

MAGNETIC INDUCTION



An aluminum dish is gently positioned over a coil through which runs a large AC current. As a consequence, the dish *levitates* six inches above the coil. A raw egg is cracked with its contents deposited onto the plate. The egg begins to fry. What's going on? . . . It's all about the wild world of magnetic induction.

A.) Magnetic Flux:

1.) Before we can start talking about Faraday's Law and induction, there is a mathematical contrivance we need to discuss. It is called *magnetic flux*.

2.) If a magnetic field \mathbf{B} passes through the face of a coil of area A (see Figure 17.1), it will generate what is called a magnetic flux ϕ_m through the coil. The amount of flux is dependent upon several parameters:

a.) The area A of the coil's face (as used here, "face" is defined as *the cross sectional area of the coil*--the larger the face, the more field lines *can* pass through),

b.) The *magnitude* of the magnetic field (it should be obvious that the larger the field, the more *field lines* will pass through a given *face area*), and

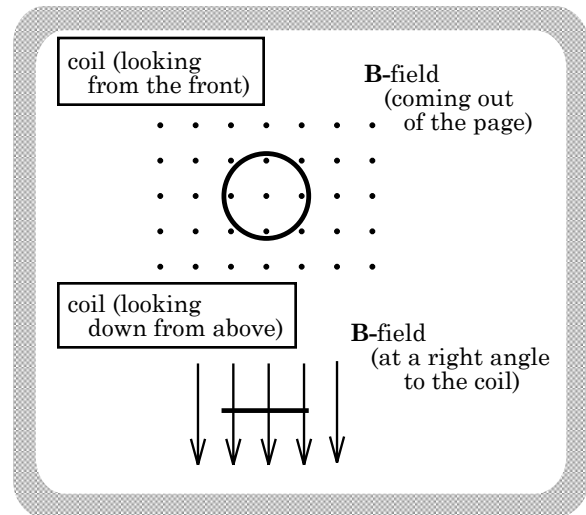


FIGURE 17.1

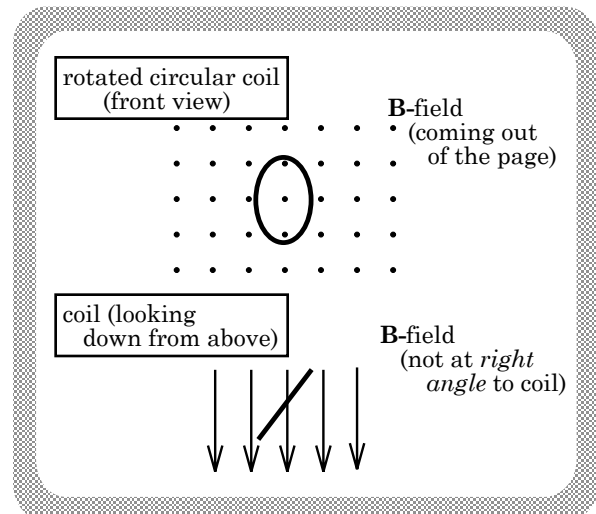


FIGURE 17.2

c.) Figures 17.1, 17.2, and 17.3 show three different angular relationships between the coil's orientation and the direction of the magnetic field.

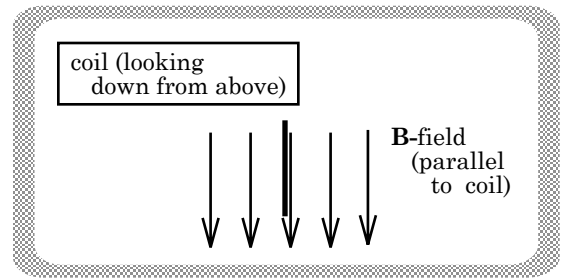


FIGURE 17.3

d.) Bottom line: When the coil is positioned as in Figure 17.1, there is more magnetic flux passing through its face than is the case in Figure 17.2. Notice also that there is *no* magnetic flux through the coil in Figure 17.3.

e.) This angular variation is taken into account in the following way:

i.) Define a vector \mathbf{A} whose *magnitude* is the area of the coil's face and whose *direction* is perpendicularly out from the face (see Figure 17.4). With \mathbf{A} so defined (note: this variable is sometimes denoted as \mathbf{S} , standing for *surface area*--we will use both notations), the magnetic flux through the coil's face is:

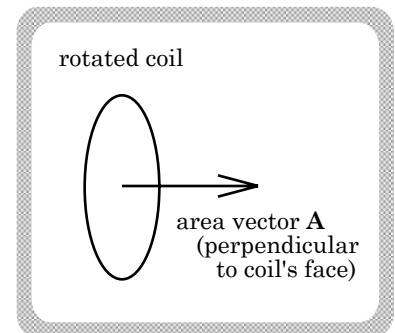


FIGURE 17.4

$$\phi_m = \mathbf{B} \cdot \mathbf{A}.$$

f.) Example of a simple flux calculation: Determine the magnetic flux through the coil depicted in Figure 17.5 (the sketch shows a circular coil FROM ABOVE). Assume that the magnetic field intensity is .02 teslas, the coil's radius is .3 meters, and the angle between the hoop *itself* and the magnetic field is 30° (see sketch).

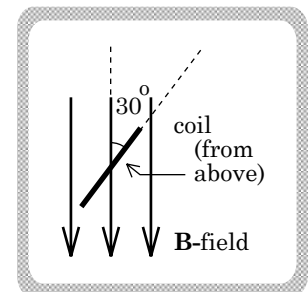


FIGURE 17.5

i.) To begin with, notice that the magnitude of the area of the coil's face is:

$$A = \pi r^2,$$

while the angle θ between \mathbf{A} and \mathbf{B} is 60° . . . that's right, 60° ! *The angle in this dot product is between the line of B and the line of A, where \mathbf{A} is a vector PERPENDICULAR to the face of the coil (see Figure 17.6).*

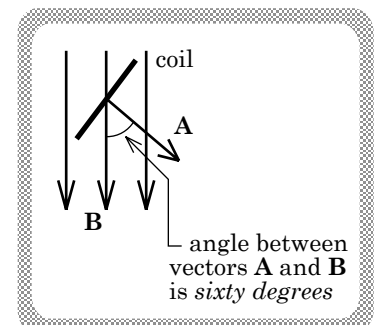


FIGURE 17.6

ii.) Using our definition for flux:

$$\begin{aligned}
 F_m &= \mathbf{B} \cdot \mathbf{A} \\
 &= |\mathbf{B}| |\mathbf{A}| \cos \theta \\
 &= (.02 \text{ T}) [(\pi) (.3 \text{ m})^2] \cos (60^\circ) \\
 &= 2.83 \times 10^{-3} \text{ tesla} \cdot \text{meters}.
 \end{aligned}$$

Note: The unit *tesla·meters* is given a special name--the *weber*. That means the units for a magnetic field could be written as *webers per meter*.

B.) Faraday's Law:

1.) In 1831, Michael Faraday noticed an interesting phenomenon. The following scenario and commentary highlight his observations.

a.) Attach a coil to a galvanometer. There will be no current flowing in the coil because there is no power being provided to the coil.

b.) Put the coil in a constant magnetic field (Figure 17.7). There will be a *magnetic flux* through the coil, but there will still be no current in the coil--there will still be *no power* being supplied to the wire loop.

c.) Change the magnetic flux by either:

i.) Changing the magnetic field strength,

ii.) Changing the area of the coil's face (i.e., somehow changing the coil's radius),

iii.) Changing the orientation of the coil relative to the magnetic field (i.e., changing the angle θ in the dot product $\mathbf{B} \cdot \mathbf{A}$), or

iv.) Some combination thereof.

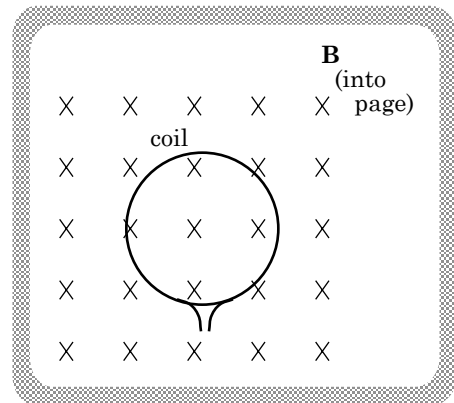


FIGURE 17.7

d.) Observation: Even though there is no standard *power supply* in the circuit, as the *magnetic flux changes*--and ONLY while the flux changes--a *current in the coil* will be registered by the galvanometer.

2.) There are many ways we can explain this. From the point of view of the *free charge* in the wire, decreasing the magnetic field (hence, decreasing the magnetic flux through the coil) is like moving from a region where the \mathbf{B} -field is *bigger* into a region where it is *smaller*. Charges moving in a magnetic field feel a *magnetic force* defined by $q\mathbf{v}\times\mathbf{B}$, so free charges in the wire will feel a force that makes them circulate through the coil.

What is important to note is that Faraday *did not see the situation in these terms*. His evaluation was as follows:

a.) Observation 1: As is stated above, while the *magnetic flux changes*--and *only* while the flux changes--a current in the coil is registered by the galvanometer. Therefore, a *changing magnetic flux* must induce an EMF (this is like a *voltage*) in the coil which, in turn, generates current that lasts as long as the flux continues to change.

Note: *Electromotive force* (EMF) is the part of a power source that motivates charge to move--it is that part that *actually puts energy into the system*. Its units are *volts*. You might wonder why this has a special definition. In the past, we have oversimplified the workings of power supplies. A battery, for instance, has an internal resistance r . That means the battery heats up taking energy out of the system when current is drawn from it (this *internal voltage drop* is equal to ir). To technically delineate between the battery's *terminal voltage* V_o (i.e., the voltage measured across its terminals) and the actual *charge-motivating character* of the battery, the concept of the electromotive force (EMF) was defined. In a battery, the $EMF = V_o + ir$.

The point is that Faraday alluded to a *charge-motivating property* that seems to exist when a coil of wire is placed in a *changing magnetic field*. As such, he related *charge flow* to an induced EMF that, he reasoned, must exist if current was to flow in the circuit.

b.) Observation 2: The *induced EMF* is related to the *rate at which the flux changes* and the *number of winds* N in the coil. That is, if you change the flux slowly, you get a small induced EMF and a small induced current. If you change the flux quickly, you get a large induced EMF and a large induced current. Assuming the change is constant, the induced EMF becomes:

$$\text{induced EMF} = -N \frac{\Delta\Phi_m}{\Delta t}.$$

Note 1: The negative sign in front of the expression will be explained later.

Note 2: The symbol most often used for an EMF is ε .

c.) If the flux-change is *not* constant, the induced EMF becomes:

$$\text{induced EMF} = -N \frac{d\Phi_m}{dt}.$$

THIS IS IMPORTANT: The easiest way to do most Faraday's Law problems is to write out ϕ_m as it exists at any arbitrary point in time, and then take its *time derivative*.

d.) Plugging in our defining expression for magnetic flux (i.e., $\phi_m = \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}||\mathbf{A}| \cos\theta$), we can rewrite Faraday's Law as:

$$\begin{aligned} \text{induced EMF} &= -N \frac{d\Phi_m}{dt} \\ \Rightarrow \varepsilon &= -N \frac{d(\mathbf{B} \cdot \mathbf{A})}{dt}. \end{aligned}$$

In this expression, B is the magnetic field intensity, A is the area of the coil, and θ is the angle between the magnetic field vector \mathbf{B} and the area vector \mathbf{A} (remember, the area vector is directed *normal* to the face of the coil), all evaluated at an arbitrary point in time.

C.) Direction of Induced Current--Lenz's Law:

1.) So far, we have used Faraday's Law to determine the *magnitude* of the *induced EMF* and *current* in a coil through which there is a changing magnetic flux. What about current direction?

Lenz's Law provides a way to determine the direction of an induced current. The approach is somewhat complex but certainly understandable if approached in an orderly manner. The best way to proceed is by looking at an example in pieces, then by putting the pieces together. Consider:

a.) A coil is placed in a constant external magnetic field (see Figure 17.8).

b.) As the external magnetic field passes through the coil's face, there is a magnetic flux through the coil.

c.) At this point, there is *no current in the coil* as there is no *changing flux* through the coil.

d.) Assume now that the external flux somehow *increases* through the coil.

Note: As was said previously, this increase could come as a consequence of an increase in the *magnitude* of the external **B**-field, an increase in the *area* of the coil's face, a change in the *angle* between the area vector **A** and the magnetic field vector **B**, or any combination thereof.

e.) As there is now a changing flux through the coil, there will be an induced EMF which, in turn, will cause an induced *current* in the coil. As we are not yet sure which way the current will flow, assume its direction is as shown in Figure 17.9. If that is the case, the following will be true:

i.) The *induced current* will produce a magnetic field of its own (this will be referred to as *the induced B-field*).

ii.) This field will set up a magnetic flux into the page through the coil's face (this will be referred to as *the induced magnetic flux*--see Figure 17.10). Given the assumed current direction, that *induced flux* will ADD to the increasing external flux through the coil's face (i.e., the net flux through the coil's face will be $\phi_{m,external} + \phi_{m,induced}$).

iii.) The logical consequences of this are as follows: As the *induced flux* adds to the increasing *external flux*, the *net flux change* becomes bigger than it would have been. As such, the *induced current* in the coil becomes bigger than it would

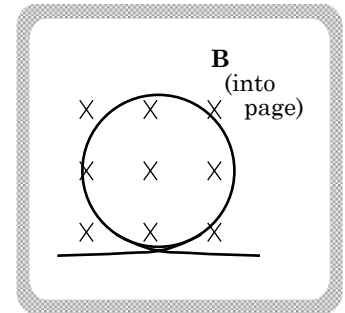


FIGURE 17.8

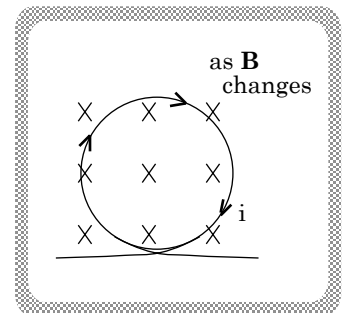


FIGURE 17.9

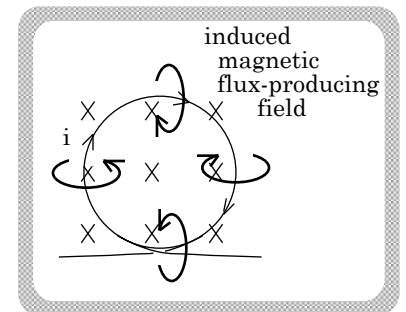


FIGURE 17.10

have been which, in turn, creates an even bigger *induced* **B**-field through the coil. With the *induced* **B**-field larger than expected, the *induced flux* becomes even GREATER, which means the *net induced flux* becomes GREATER, which creates an EVEN BIGGER current in the coil, etc., etc., etc.

iv.) Bottom line #1: If, as the external flux increases, the induced flux ADDS to it (i.e., helps the external flux out as it increases), the consequence will be a runaway situation in which the *conservation of energy* is wholly violated.

v.) Bottom line #2: The induced current must flow such that its magnetic flux does NOT aid the *external flux change*. In fact, the *induced magnetic flux must oppose that change*.

vi.) Note that the negative sign in $\mathcal{E} = -N(d\phi_m/dt)$. It denotes that the EMF is *opposing* the flux change.

2.) Lenz's Law (as actually stated--*not* as is best applied): When an EXTERNAL magnetic flux CHANGES through the face of a coil, an EMF is induced in the coil. That (induced) EMF generates an (induced) current which produces an (induced) magnetic field through the coil's face, which produces an (induced) magnetic flux through the coil's face, which is directed so as to OPPOSE THE CHANGE of the external magnetic flux that started the whole process in the first place.

The *right-thumb rule* (thumb in direction of current--fingers curl in direction of **B** through the coil) allows you to determine the direction of the required induced current. (SEE NEXT SECTION NOW!)

3.) **IMPORTANT**--a *SHORTCUT* to using Lenz's Law (or, NO MATTER HOW LITTLE THE ABOVE MADE SENSE, THE FOLLOWING WILL ALWAYS WORK!!!):

a.) If the external flux INCREASES, no matter how this is accomplished, the direction of the *INDUCED* **B**-field through the coil's face will *always* be OPPOSITE the direction of the *external* **B**-field. If the external flux DECREASES, the direction of the *INDUCED* **B** will be THE SAME AS the direction of the *external* **B**-field.

b.) ANOTHER CLEVER WAY TO GET THE INDUCED CURRENT'S DIRECTION--or, the "*other*" *right-thumb rule*: Determine the direction of the *induced* **B**-field using the logic outlined in *Part 3a*. Orient your right thumb

along that line, pointing it through the coil. Your fingers will curl in the direction of the induced current in the loop.

4.) Using Lenz's Law, determine the direction of the induced current in the situations outlined below:

a.) The external \mathbf{B} -field in Figure 17.11 is increasing out of the page, which means the external flux is increasing. The only way to *oppose* an increasing *magnetic flux* is to produce a second magnetic flux that subtracts from the original flux. In other words, we need a current that will produce a \mathbf{B} -field in the direction opposite the existing \mathbf{B} -field, or *into the page*.

Using the "other" right-thumb rule: Direct your right thumb *into* the page and through the coil. The fingers of your right hand curl clockwise. That's the direction of the induced current.

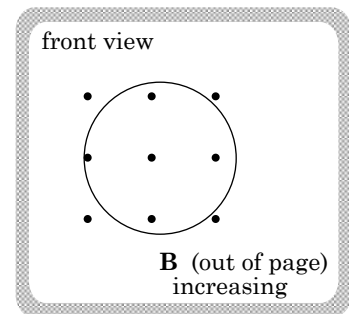


FIGURE 17.11

b.) The external \mathbf{B} -field in Figure 17.12 is decreasing. The only way to *oppose* its *decreasing magnetic flux* is to produce a second magnetic flux that adds to the original flux. In other words, we need a current that will produce a \mathbf{B} -field *in the same direction* as the external \mathbf{B} -field, or to the right.

Direct your right thumb *to the right* and through the coil (you'll obviously have to visualize doing this--the coil's face isn't actually shown). Your right-hand fingers will curl so that the induced current will flow toward the page's bottom in the wire section shown.

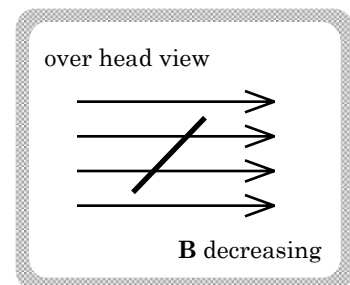


FIGURE 17.12

c.) Rotation as shown in Figure 17.13 depicts a situation in which the external magnetic flux through the coil's face is increasing. The only way to *oppose* an increase in magnetic flux is to generate a magnetic flux that subtracts from the existing flux. In other words, we need a current that will produce a \mathbf{B} -field in the direction opposite the external \mathbf{B} -field, or to the left.

Direct your right thumb *to the left* and through the coil (again, you'll have to visualize doing this as the coil's face isn't

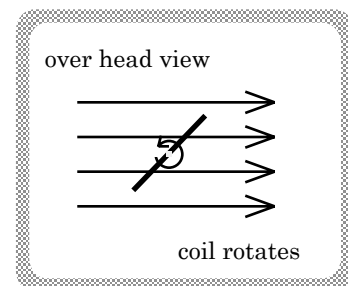


FIGURE 17.13

actually shown). Your right-hand fingers will curl so that the induced current will flow toward the top of the page in the wire section shown.

d.) Rotation as shown in Figure 17.14 depicts a situation in which the external magnetic flux through the coil's face is decreasing (the coil's magnetic flux is at a maximum going down). The only way to *oppose* a decrease in magnetic flux is to generate a magnetic flux that adds to the existing flux. In other words, we need a current that will produce a **B**-field in the same direction as the external **B**-field, or into the page.

Direct your right thumb *into the page* and through the coil. Your right-hand fingers will curl clockwise--that's the induced current's direction.

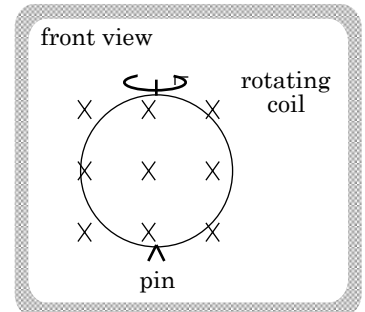


FIGURE 17.14

e.) As the area of the coil in Figure 17.15 becomes smaller, the magnetic flux through the coil's face decreases. The only way to *oppose* a decrease in magnetic flux is to generate a magnetic field that adds to the existing field. In other words, we need a current that will produce a **B**-field in the same direction as the external **B**-field.

Using the *right-thumb rule*, the current direction that generates such a magnetic field through the coil's face will be upward (toward the top of the page) in the section of wire visible to us in the sketch.

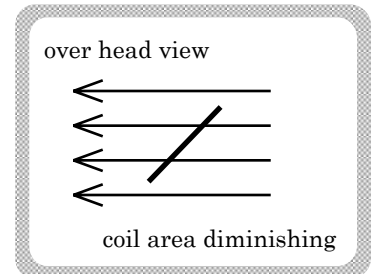


FIGURE 17.15

D.) The Production of AC--Alternating Current:

1.) When a coil is placed in a magnetic field and is rotated, a changing flux $\Delta \phi_m$ will exist through the coil that will produce an induced EMF across the coil's leads. A surprising result is found when Lenz's Law is used to determine the direction of the induced current and, hence, the high and low voltage side of the coil as the coil rotates. Follow along:

a.) Assume the external magnetic field is directed into the page and the coil is initially facing the **B**-field (see Figure 17.16--note that

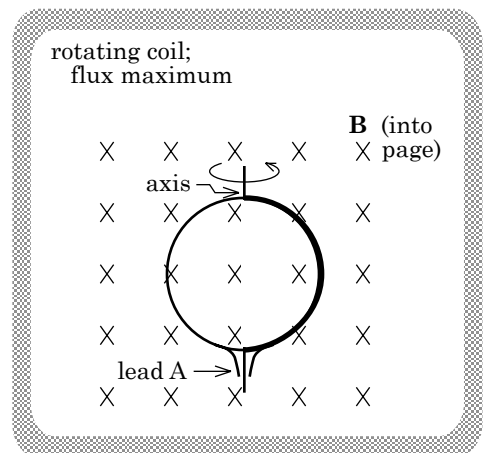


FIGURE 17.16

the right side of the coil--the part that will initially rotate into the page--is darker; this has been done to make it easier to follow the coil through the complete 360° rotation). As the coil begins to rotate, the external flux through the coil's face diminishes (Figure 17.17). The coil generates an induced current which opposes the decrease of flux, which means an induced \mathbf{B} -field is created into the page. The direction of current flow required to do this is clockwise, which means *lead A* must be the positive side of the coil.

b.) The coil rotates through the *no-external-flux* position and into the second quarter of its cycle (see Figure 17.18). In this part of its motion, the external flux is increasing. The induced \mathbf{B} -field required to oppose this increase must be out of the page, which requires an induced current that is counterclockwise (relative to where we are sitting). As such, *lead A* must again be the positive side of the coil.

c.) The coil rotates through the *maximum-flux* position and into the third quarter of its cycle (Figure 17.19). In this part of its motion, the external flux is *decreasing*. The induced \mathbf{B} -field required to oppose this decrease must be into the page, which requires a current that is clockwise relative to ourselves. As such, *lead A* must be *negative*.

d.) *Lead A* will be negative through the last quarter of the rotation (just as in *Part b* above, but with the lead positions switched).

e.) Bottom line: The high-voltage side of a rotating coil alternates from one side to the other as the coil turns. This is how alternating current (AC) is generated.

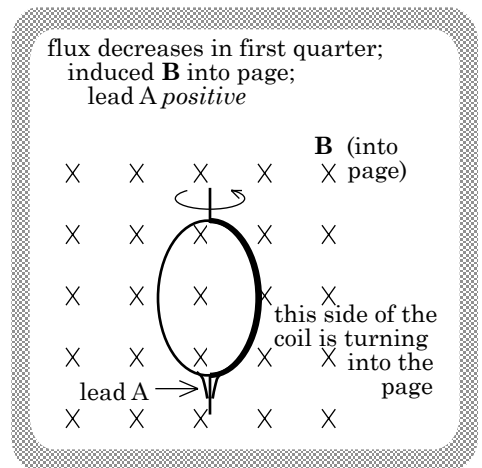


FIGURE 17.17

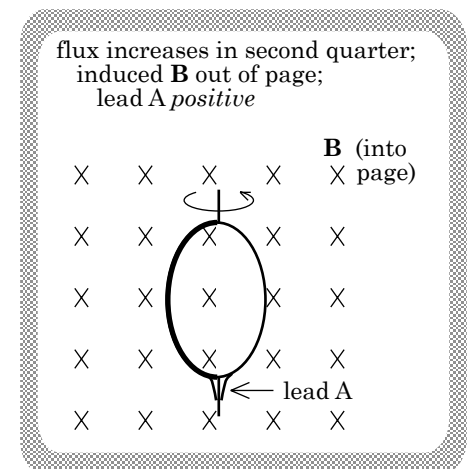


FIGURE 17.18

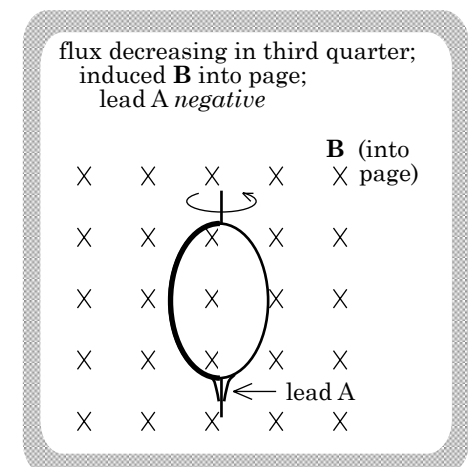


FIGURE 17.19

E.) Induced EMFs in Coils:

1.) The symbol for a *coil* in an electrical circuit is shown in Figure 17.21 (yes, I know, there isn't a Figure 17.20--again, life's tough). Coils are called a number of different things: chokes, solenoids, inductors. The last is the most common usage; the first is slang.

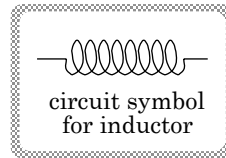


FIGURE 17.21

2.) Consider the coil, resistor, switch, and DC power supply in Figure 17.22. At $t = 0$, the switch is closed. What will the *current versus time* graph look like for this situation?

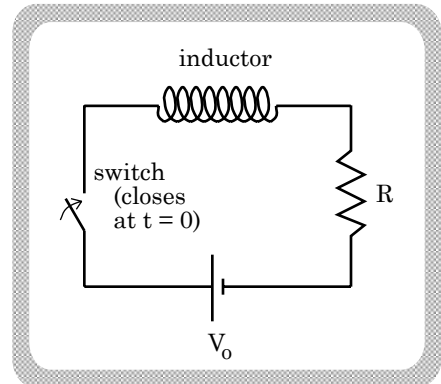


FIGURE 17.22

a.) First, if the circuit were comprised of nothing more than a *resistor* across the power supply, the voltage difference generated by the power source would create an electric field that would motivate all the free charge-carriers in the circuit to move at once. Current in the circuit would immediately be observed (the *electric field* sets itself up at just under the speed of light), its magnitude would be V_0/R , and its *current versus time* graph would be as shown in Figure 17.23.

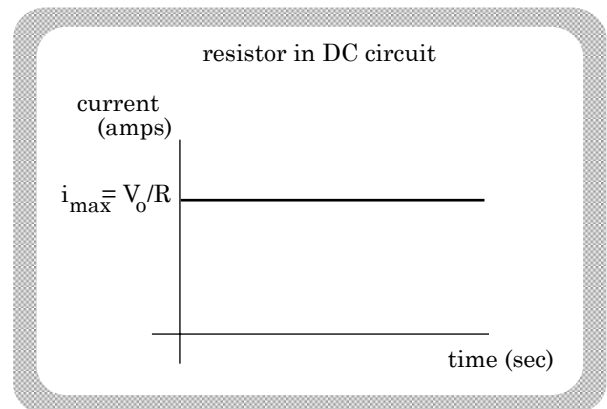


FIGURE 17.23

b.) With a *coil* in the circuit, everything changes.

i.) To begin with, we know that when current moves through a coil it creates a *magnetic field down the coil's axis* and a *magnetic flux through the coil's face*.

ii.) In the case cited above, there is no initial current in the coil, hence no initial magnetic field or magnetic flux. At $t = 0$, the switch is closed. As current begins to flow, the magnetic flux through the coil begins to INCREASE.

iii.) A changing magnetic flux induces an EMF (Faraday's Law) that attempts to set up a **B**-field whose flux through the coil will *oppose* the external flux-change that started the whole process.

iv.) Put another way, the sudden increase in current elicits a back-EMF that attempts to diminish the current increase through the coil. This opposition is not so large that it completely stops current build-up, but it does slow it down. As such, the current in the circuit increases more slowly than would have been expected, and the *current versus time* graph ends up looking like Figure 17.24.

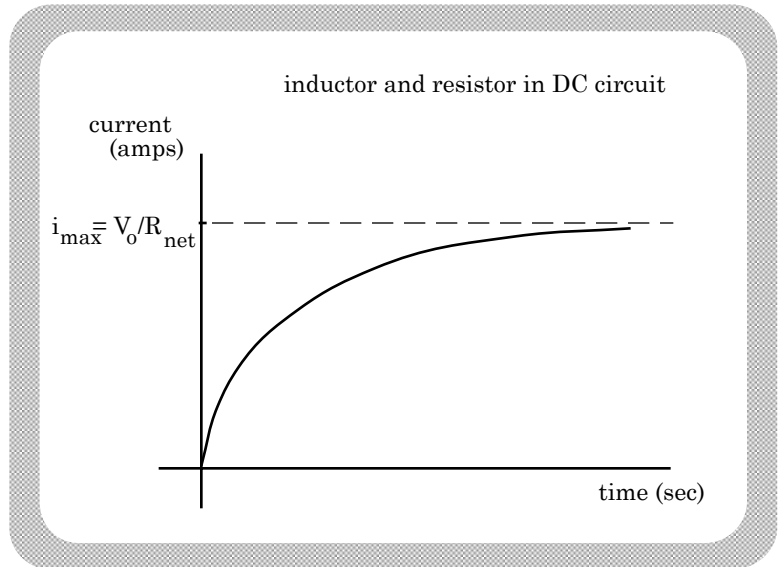


FIGURE 17.24

Note 1: The current will sooner or later reach a maximum steady-state value. At that point, there will be no voltage drop across the coil except that generated by the resistance inherent within the coil's wire. As such, the current is governed by Ohm's Law and equals V_o/R_{net} .

Note 2: Figure 17.25 shows the *current versus time* graph of an inductor/resistor circuit that is continuously turning *on* (i.e., the switch is closed) and *off* (i.e., the switch is opened). Just as the coil sets up an EMF that opposes an *increase* in current when the switch *closes*, so also will it set up an EMF that opposes a

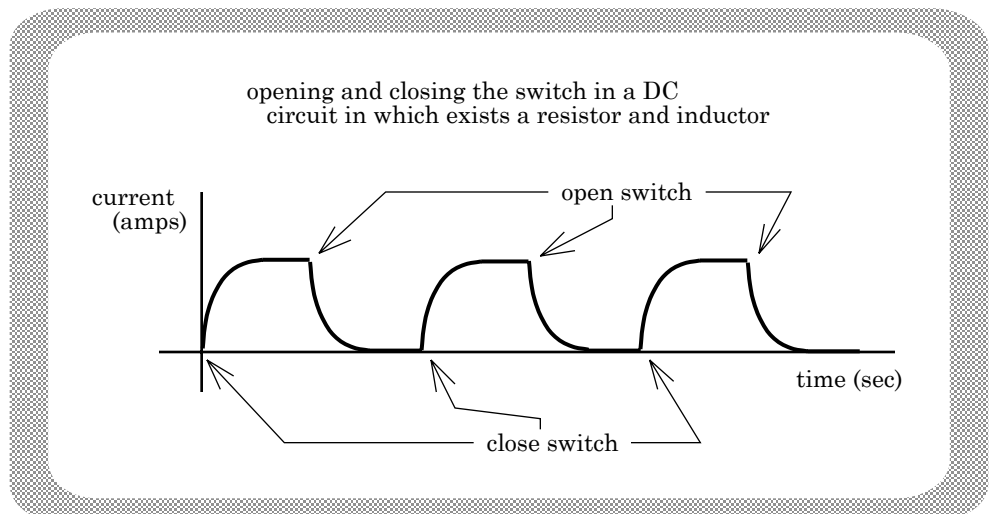


FIGURE 17.25

decrease in current when the switch *opens*. (Look at the graph!)

F.) Inductance:

1.) According to Faraday's Law, when the current through a coil changes, an EMF is produced that works to oppose the change of the external flux. Until now, we have calculated EMFs using the relationship:

$$\text{EMF} = -N (\Delta\phi_m)/(\Delta t).$$

The problem with this expression, at least when dealing with a coil in a real-life situation, is convenience. *Flux* is not a quantity we can easily measure with normal laboratory equipment.

2.) As it is the *change of current* in the coil that caused the change of flux, we could link our induced EMF to the *change of current* instead of the flux change. That is, we could define a proportionality constant L that allows us to relate the *induced EMF* across the coil with the *change of current* in the coil.

Mathematically, this would look like:

$$\text{EMF} = -L \frac{di}{dt}.$$

3.) The proportionality constant L is called the coil's *inductance* (you can now see why a coil in an electrical circuit is usually called an *inductor*). Its units are *henrys* (it is not uncommon to find people laughing at inductors because of the name of their units). It's also not uncommon to find inductors in the milli-henry range (a milli-henry is $10^{-3} H$ and is symbolized as mH).

Qualitatively, inductance tells us how large an induced EMF (in *volts*) we can expect across the coils of an inductor *per change of current per unit time*.

4.) Consider an electrical circuit in which there is an inductor whose inductance is L , a resistor whose resistance is R , an ideal DC power supply, and a switch (for now, we will assume that any resistance wrapped up in the wires making up the coil is negligible). Figure 17.24 shows the graph of *current versus time* for this situation, assuming that the switch was closed at $t = 0$.

a.) What function defines how the current will act with time in this situation? That is, what is $i(t)$?

b.) To determine this, we need to start with Kirchoff's Second Law and write a *loop equation* for our circuit. Doing so, then manipulating, yields:

$$\begin{aligned}
 -L \frac{di}{dt} - iR + V_o &= 0 \\
 \Rightarrow \frac{di}{dt} + i \left(\frac{R}{L} \right) &= \frac{V_o}{L}.
 \end{aligned}$$

c.) What the differential equation is asking us to do is to find a current i such that when you take its derivative di/dt and add it to a constant times itself $(R/L)i$, you always get the same number (i.e., V_o/L). I'm not going to subject you to the pain of this derivation, but the bottom line--the current function that does the job--turns out to be:

$$i = i_o(1 - e^{-(R/L)t}).$$

d.) What is interesting about this is that the time dependent expression for current in an RL circuit is similar to the charging function in an RC circuit. As was the case with capacitors, we can define a *time constant* τ_L for our circuit. As before, one time constant will equal the amount of time needed for $e^{-Rt/L}$'s exponent to numerically equal -1 . In our case, this occurs when $t = L/R$.

i.) The consequences of this can be seen by doing the math:

$$\begin{aligned}
 i(t) &= i_o(1 - e^{-Rt/L}) \\
 \Rightarrow i\left(t = \frac{L}{R}\right) &= i_o\left(1 - e^{-R\left(\frac{L}{R}\right)/L}\right) \\
 &= i_o(1 - .37) \\
 &= .63i_o.
 \end{aligned}$$

ii.) Knowing the *time constant* allows us to determine how fast the current will rise or fall when a DC-driven RL circuit is turned on or off. According to the math, after one *time constant* the current will be 63% of its maximum (after two *time constants*, the current will be 87% of its maximum).

iii.) A graph of the current function and the *time constant* is shown in Figure 17.26. Notice that it is exactly the graph we deduced using hand-waving arguments in previous sections.

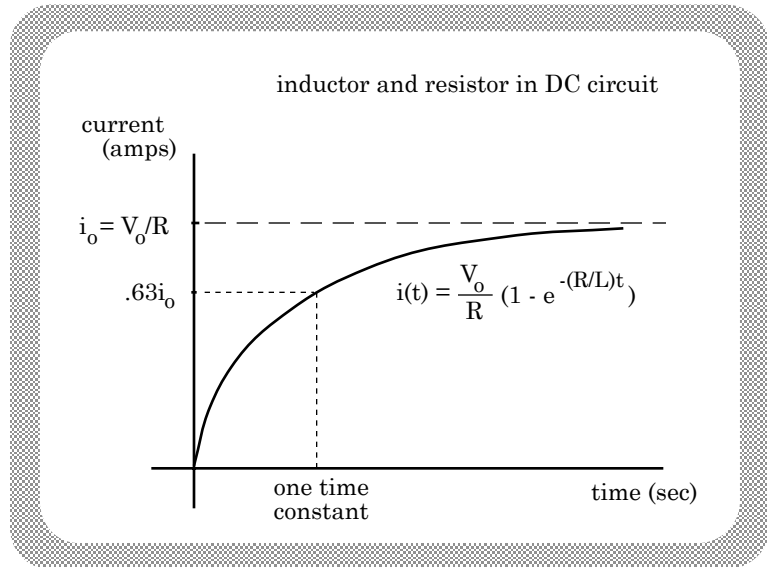


FIGURE 17.26

G.) Energy Stored in an Inductor:

1.) To increase the current in a coil, extra work must be done to overpower the coil's tendency to resist changes in its magnetic flux. Where does that energy go? Some of it is stored in the inductor's magnetic field. This section deals with *how much* energy a current-carrying coil can store.

2.) To determine the amount of energy wrapped up in an inductor's magnetic field:

a.) Reconsider Kirchoff's loop equation. That is:

$$L \frac{di}{dt} + iR = V_0.$$

Multiplying by i , we get:

$$iL \frac{di}{dt} + i(iR) = iV_0.$$

b.) Noting that the iV_0 term equals the amount of power provided to the circuit by the power supply, and the i^2R term equals the amount of power dissipated by the resistor, it's a good bet that the $Li(di/dt)$ term equals the amount of power dissipated by the inductor in the circuit.

The resistor pulls energy out of the circuit as heat. The inductor pulls energy out of the system by storing it in the coil's magnetic field.

As *work per unit time* (i.e., the power) equals dW/dt , we can write:

$$\frac{dW}{dt} = Li \frac{di}{dt},$$

or

$$dW = Li(di).$$

Assuming we want to sum all the work done on the inductor between the time the current is zero to the time it is at some maximum current i_0 , we can integrate both sides. This will give us the total energy stored by the inductor. Doing so yields:

$$\begin{aligned} \text{energy} &= \int dW \\ &= \int_{i=0}^{i_0} Li(di) \\ &= \frac{1}{2} Li_0^2. \end{aligned}$$

Note: The energy in an inductor is stored in a magnetic field governed by current flow in the coil, and the energy expression is $(1/2)Li^2$. The energy in a capacitor is stored in an electric field governed by a voltage across the capacitor's plates, and the energy expression is $(1/2)CV^2$. Nice symmetry, eh?

H.) Inductors in AC Circuits:

1.) So far, all we have dealt with have been inductors as they act in DC circuits. Whenever the current attempts to change, they fight the change.

In AC circuits, inductors are constantly fighting a change. This makes for some very fun times.

2.) Consider the *RL circuit* shown in Figure 17.27.

3.) The *inductor* in the circuit will have a certain amount of resistance r_L inherent within the wires that make up its coils. That resistance will act like any other resistor-like element in an AC circuit.

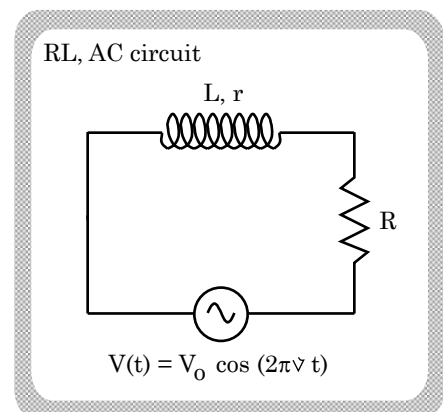


FIGURE 17.27

4.) In addition to r_L , the inductor also has a *resistive nature* that is frequency-dependent. Not obvious? Follow along.

a.) When an alternating current passes through an inductor, Faraday's Law demands that an induced EMF be generated across the leads of the coil that will ultimately produce an induced magnetic flux that opposes the changing magnetic flux through the coil's face. From the previous chapter, the magnitude of this induced EMF equals:

$$\mathcal{E}_L = L (di/dt).$$

b.) Writing a Loop Equation for an RL circuit (see Figure 17.28 for the voltages associated with each element), we get:

$$(-\mathcal{E}_L - ir) - iR + V_0 \cos(2\pi vt) = 0.$$

In this expression, \mathcal{E}_L is the induced, frequency-dependent voltage drop across the inductor (i.e., $L di/dt$), $2\pi v$ is the angular frequency of the power supply (a *cosine* function has been used to characterize the varying voltage across the power supply--I've done this because it will make life easier when we do the evaluation that is to follow--we could as well have used a sine function, but the resulting expression would have been a bit messy). Substituting in and rearranging this expression, we get:

$$L (di/dt) + ir + iR = V_0 \cos(2\pi v t).$$

c.) The resistor-like resistance inherent in an inductor is sometimes negligibly small and sometimes not. For the sake of simplicity we will lump it with R to get R_{net} . Doing so, the above expression becomes:

$$L (di/dt) + iR_{net} = V_0 \cos(2\pi vt).$$

d.) Though you will never have to derive this on a test, we need an expression for the resistive nature of the inductor *excluding* the resistor-like resistance inherent within its wires. To do this:

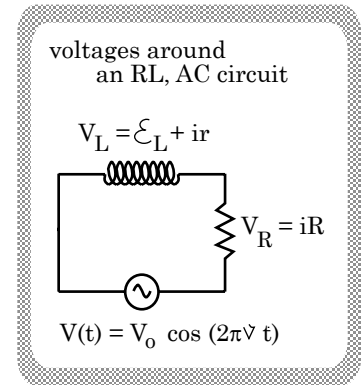


FIGURE 17.28

i.) Assume the resistance of (and, hence, voltage across) all of the resistor-like elements in the circuit is negligible (i.e., that $R_{net} = 0$). In that case, Kirchoff's Loop Equation becomes:

$$L (di/dt) = V_o \cos (2\pi vt).$$

ii.) We know the voltage across the power supply and inductor. We'd like an expression for the current through the circuit. To determine this, we need to manipulate and integrate. Doing so yields:

$$\begin{aligned} L \frac{di}{dt} &= V_o \cos(2\pi vt) \\ \Rightarrow L di &= V_o \cos(2\pi vt) dt \\ \Rightarrow L \int_{i=0}^i di &= V_o \int_{t=0}^t \cos(2\pi vt) dt \\ \Rightarrow Li &= V_o \left(\frac{1}{2\pi v} \right) [\sin(2\pi vt)] \\ \Rightarrow i &= \frac{V_o [\sin(2\pi vt)]}{(2\pi v L)} \quad \text{(Equation A).} \end{aligned}$$

5.) Important points:

a.) Ohm's Law maintains that the *current* through an element must equal the *voltage* across the element divided by a quantity that reflects the *resistive nature* of the element. In the above expression, the voltage across the element is evidently $V_o [\sin(2\pi vt)]$. That means the *resistive nature* of the inductor must be $2\pi v L$.

b.) Even though inductors don't adhere to Ohm's Law (their current isn't proportional to their voltage), this is, nevertheless, the *frequency-dependent* resistive nature of an inductor. It is called the *inductive reactance*, its symbol is X_L , and its units are *ohms*. Summarizing, we can write:

$$X_L = 2\pi v L \text{ (ohms),}$$

where the inductance L must be written in terms of *henrys* (versus *milli-henrys* or whatever).

c.) Although we assumed the resistor-like resistance of the circuit was negligible to do the derivation, in fact this inductive reactance expression is true whether the resistor-like resistance is big or small.

6.) Side-Note: Does the frequency-dependent expression for the *resistive nature* of an inductor (i.e., its inductive reactance) make sense?

a.) Consider a general RL circuit (i.e., one in which R_{net} is not small) hooked across an AC power supply that runs at low frequency.

Note: A *low frequency voltage* means that although the amplitude of the voltage of the power supply may be large or small, the *rate at which the voltage changes* is very slow.

i.) A *low frequency* voltage will produce a low frequency current.

ii.) A low frequency current means that di/dt will be small (the current is changing slowly if it is low frequency).

iii.) A small di/dt means the induced voltage drop across the inductor (Ldi/dt) is small.

iv.) A small induced voltage drop across the inductor implies a *relatively large voltage drop across the resistor* (at any instant, the two have to add up to the voltage across the power supply--a quantity that can be large).

v.) As the voltage drop across a resistor is directly proportional to the current through the resistor, a large voltage drop across the resistor implies a relatively large current through the resistor and, hence, through the circuit.

b.) Bottom line #1: The current in an RL circuit will be relatively large when a *low frequency signal* passes through the circuit. That means we would expect the inductive reactance (the resistive nature of the inductor) to be small at low frequencies. This is exactly what our derived expression predicts (i.e., when ν is small, $X_L = 2\pi \nu L$ is small).

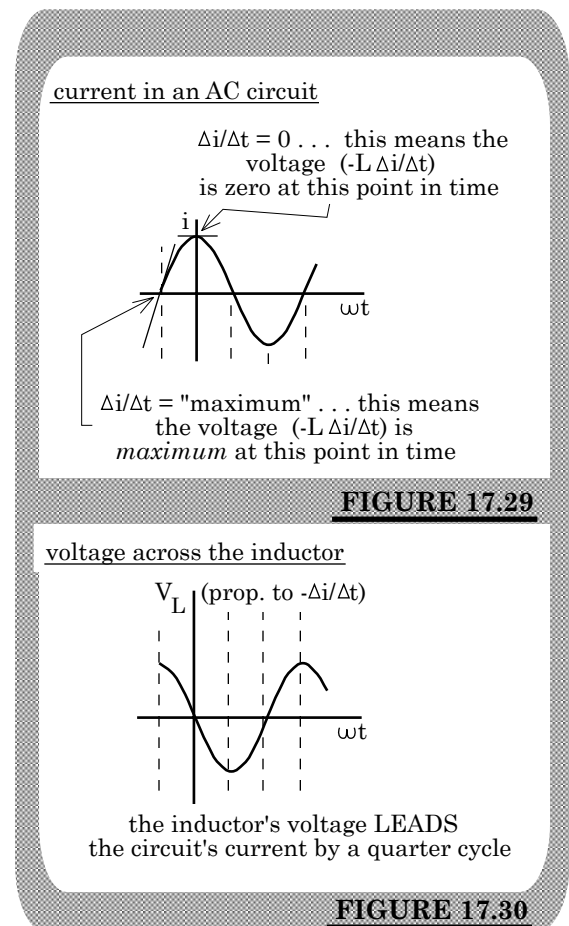
c.) Using similar reasoning, a power supply running at high frequency creates a high frequency current that will produce a very large di/dt value. In such a case, the voltage drop across the inductor ($L di/dt$) is relatively large and the voltage drop across the resistor is relatively small. A small voltage drop across the resistor suggests a *small current* flowing in the circuit.

d.) Bottom line #2: The current in an RL circuit will be relatively small (i.e., approaching zero) when a *high frequency signal* passes through the circuit. That means we would expect the inductive reactance to be big at high frequencies. This is exactly what our expression predicts (i.e., when ν is large, X_L is large).

e.) Summary: An *inductor* in an AC circuit passes *low frequency signals* while damping out *high frequency signals*. As such, inductors are sometimes referred to as low pass filters.

7.) The second point to note about *Equation A* again has to do with its form. By assuming a power supply voltage that is proportional to $\cos(2\pi \nu t)$, and assuming that the net resistance in the circuit is zero (i.e., $R_{net} = 0$ so that the voltage across the inductor is the same as that across the power supply), we find that the circuit's current is proportional to $\sin(2\pi \nu t)$. Examining the graph of these two functions (the current is shown in Figure 17.29 and the voltage shown in Figure 17.30) allows us to conclude that in this situation the *voltage* across the inductor *leads* the *current* through the inductor (i.e., the circuit's current) by $\pi/2$ radians.

Note: This $\pi/2$ phase shift exists **ONLY** if there is no resistor-like resistance in the circuit. As there will never be a case in which there is absolutely no resistor-like resistance in a circuit, the phase shift in a real AC circuit will never be $\pi/2$. Calculating the real shift is something you will run into later.



I.) Transformers:

1.) Consider the iron yoke (the doughnut-shaped structure) in Figure 17.31.

a.) The *primary circuit* is comprised of a switch and power supply connected to a coil whose winds are wrapped around the yoke. The *secondary circuit* is comprised of a galvanometer connected to a secondary coil the winds of which are also wrapped around the yoke.

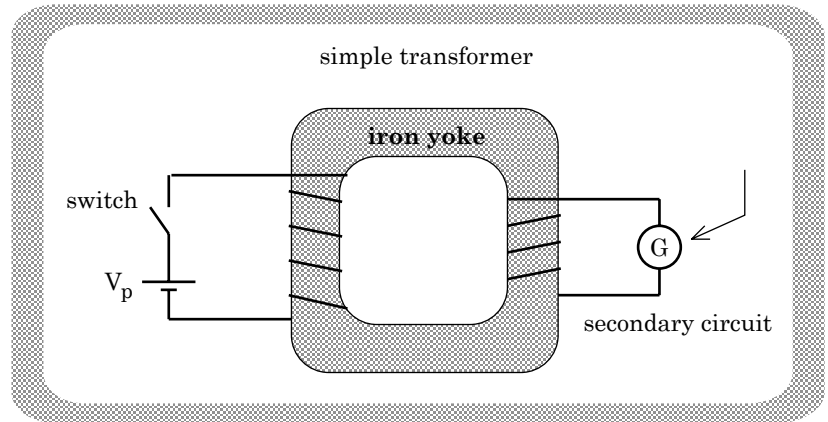


FIGURE 17.31

b.) At $t = 0$, the switch is closed. Current establishes itself in the primary circuit, building slowly due to the presence of the coil.

Note: Why does the current increase slowly? You should know from the last section! If you do not, read on:

Initially there is no magnetic flux through the *primary coil* as there is no current (the switch is initially open). When the switch is closed, the current in the primary coil begins to increase. In doing so, a magnetic field down the primary coil's axis appears which produces a magnetic flux through the primary coil's face. According to Faraday's Law, this generates an induced EMF in the primary coil which opposes the increasing flux through the primary coil. As the flux-producing magnetic field is generated by the current in the primary circuit, this back-EMF effectively fights that current. That is why the primary circuit's current grows slowly.

c.) The yoke is made of iron. As the current increases in the primary circuit (see Figure 7.32), the primary coil's

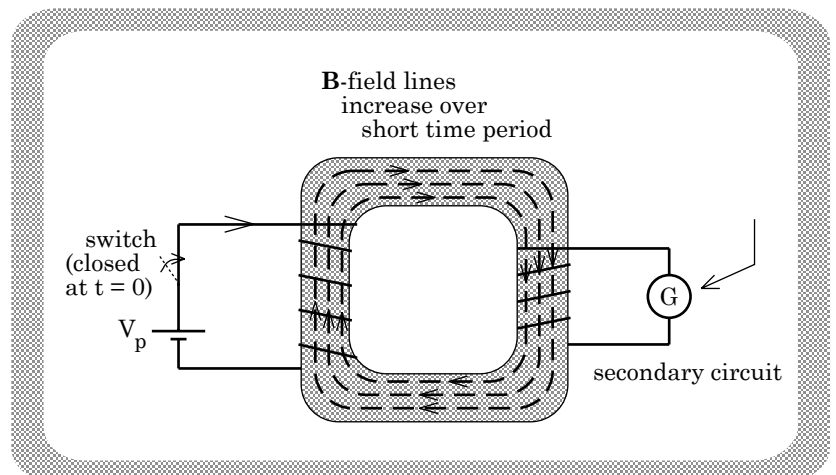


FIGURE 17.32

magnetic field magnetizes the yoke. Being one piece, this magnetic field sets itself up throughout the yoke and passes through the face of the *secondary coil*.

d.) Because the magnetic field through the secondary coil is changing, an induced EMF will be set up in the secondary coil to oppose the changing flux. As such, current will flow in the secondary circuit. **THIS CURRENT FLOW WILL CONTINUE ONLY AS LONG AS THE *MAGNETIC FLUX CHANGES* IN THE SECONDARY COIL.** During that time, the galvanometer will register charge flow.

e.) Once the current in the primary coil has built to steady-state, there will be a magnetic field in the yoke and the secondary coil, but there will *not* be a *CHANGING* magnetic field. As such, there will be no induced EMF in the secondary coil and the galvanometer will read *zero* for that part of the system.

f.) If, after a time, the switch in the primary circuit is *opened*, the exact opposite scenario will occur. The current in the primary circuit will decrease relatively slowly, decreasing the magnetic field through the primary coil. The diminishing magnetic field through the primary coil will diminish the magnetic field setup in the yoke which, in turn, will diminish the magnetic flux through the secondary coil. The secondary coil will respond by producing an induced EMF to oppose the change of flux through its face, generating a current in the secondary circuit for as long as the changing magnetic field exists.

A graph of the *current* in the secondary circuit as a function of *time* is shown in Figure 17.33.

2.) This device is called a *transformer*. It allows us to transfer power from one electrical circuit (the primary) to another electrical circuit (the secondary) without electrically connecting the two. It is not particularly useful in DC circuits where the only change in current occurs when a switch is opened or closed, but it is very useful in AC circuits.

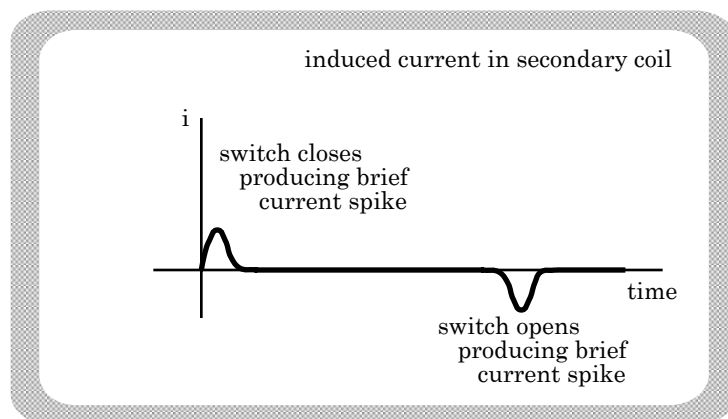


FIGURE 17.33

3.) The symbol for a transformer in a circuit is shown in Figure 17.34. The symbol is supposed to represent two coils connected by a common magnetic flux.

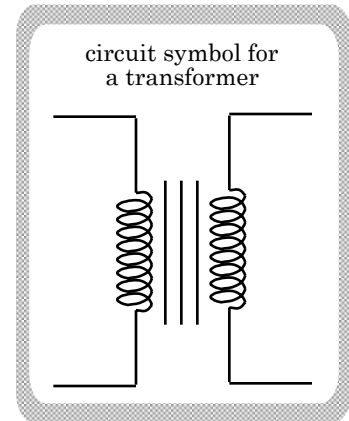


FIGURE 17.34

J.) Transformers and AC Circuits:

1.) In a DC circuit, the power source supplies a terminal voltage which sets up a constant electric field. Free charge carriers in the circuit all respond to that field by moving in one direction only.

In an AC circuit, the power source supplies an alternating *terminal voltage* that sets up an electric field that changes both in magnitude and direction. Free charge carriers in an AC circuit respond to this alternating field by jiggling back and forth.

2.) With this in mind, consider what happens when an AC power supply is put in place of the DC power supply originally used in the transformer's primary circuit (see Figure 17.35).

a.) As stated above, the current in the primary circuit will be constantly changing in *direction* and *magnitude*.

b.) A constantly changing, alternating current will produce a constantly changing, alternating magnetic field in the yoke. This, in turn, will produce a constantly changing, alternating magnetic flux through the secondary coil.

c.) An alternating magnetic flux through the secondary coil will produce an alternating induced EMF across the secondary coil's terminals which, in turn, will produce an induced alternating current in the secondary circuit.

d.) Bottom line: A transformer is useful if one wants to transfer AC power from one circuit to another without electrically hooking the two

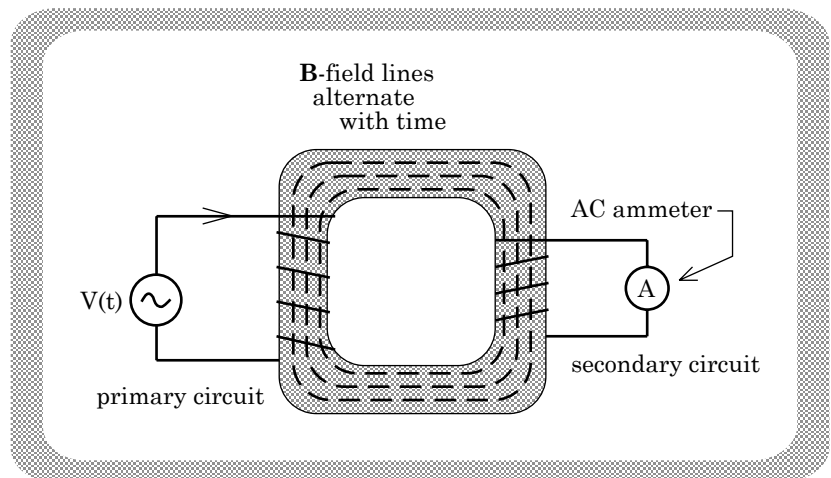


FIGURE 17.35

circuits together. As the current in the secondary coil is constantly varying, power in such circuits is transferred continuously.

Note: Putting an AC power source in place of the DC source in Figure 17.35 will additionally require changing the current-measuring device in the secondary circuit. Galvanometers are DC ammeters; we would need an AC ammeter.

3.) Transformers do more than just transfer power from one part of a circuit to another.

a.) To see this, assume ϵ_p is the induced EMF across an ideal transformer's primary circuit at a given instant, i_p is the induced current through the primary circuit at that same instant and assume 100% efficiency (that is, assume all the power provided by the magnetic component of the primary coil is transferred to the secondary coil). From this, we can model the power transfer as:

$$\begin{aligned} P_p &= P_s \\ i_p \mathcal{E}_p &= i_s \mathcal{E}_s. \end{aligned}$$

b.) We know that $\epsilon = -N(\Delta\phi_m)/(\Delta t)$, and we know that the change in flux $\Delta\phi_m/\Delta t$ through the primary and secondary coils will be the same (both coils have the same face-area and the same magnetic field passing through them). With this information we can write:

$$i_p \mathcal{E}_p = i_s \mathcal{E}_s$$

$$i_p \left[-N_p \left(\frac{\Delta\Phi}{\Delta t} \right) \right] = i_s \left[-N_s \left(\frac{\Delta\Phi}{\Delta t} \right) \right].$$

After canceling out the $\Delta\phi/\Delta t$ variables on both sides of the equal sign and manipulating, we get:

$$N_p / N_s = i_s / i_p.$$

Important Note 1: The turns-ratio N_p/N_s is inversely proportional to the ratio of currents in the primary and secondary coils.

c.) Going back to the power relationship, we know that:

$$\begin{aligned} i_p \mathcal{E}_p &= i_s \mathcal{E}_s \\ \Rightarrow i_s / i_p &= \mathcal{E}_p / \mathcal{E}_s. \end{aligned}$$

Substituting this into our turns relationship

$$i_s / i_p = N_p / N_s$$

yields

$$N_p / N_s = \mathcal{E}_p / \mathcal{E}_s.$$

Important Note 2: The turns-ratio N_p/N_s is directly proportional to the voltage ratio between the primary and secondary coils.

d.) Summary:

i.) When $N_p > N_s$:

$$\mathcal{E}_p > \mathcal{E}_s \text{ and } i_p < i_s,$$

where it is still true that $N_p/N_s = \mathcal{E}_p/\mathcal{E}_s$ and $N_p/N_s = i_s/i_p$.

ii.) When $N_p > N_s$, we have what is called a step-down transformer (it is called *step down* because the voltage *decreases* as we go from the *primary* to the *secondary coil*). Notice that with a step-down transformer, the current in the secondary coil is GREATER than the current in the primary.

iii.) When $N_p < N_s$:

$$\mathcal{E}_p < \mathcal{E}_s \text{ and } i_p > i_s,$$

where it is still true that $N_p/N_s = \mathcal{E}_p/\mathcal{E}_s$ and $N_p/N_s = i_s/i_p$.

iv.) When $N_p < N_s$, we have what is called a step-up transformer (it is called *step up* because the voltage *increases* as we go from the *primary* to the *secondary coil*). Notice that with a step-up transformer, the current in the secondary coil is LESS than the current in the primary.

4.) Now that you have some idea about how transformers work and what they do, we can talk briefly about *power production*.

a.) Consider a hydroelectric plant. A waterfall turns a turbine, the shaft of which is attached to a giant coil. The coil is suspended in a fixed magnetic field. As the coil rotates, AC is produced across its leads. Sliding contacts tap the alternating voltage.

b.) Transport: Sending the electrical power to the city requires the use of wires. The single biggest source of *energy loss* during this transfer is due to the heating-up of those wires. As high currents generate lots of heat, the trick is to keep the current as low as possible.

i.) Using a step-up transformer, the voltage is stepped up to, say, 50,000 volts. This drops the current down quite low for the span between the power generator and the city.

ii.) As there are few toasters that can handle 50,000 volts, a step-down transformer at the city is used to step the voltage down to either 110 or 220 volts AC.

iii.) This shoots the current capacity skyward. In that way, power companies can accommodate hundreds of thousands of homes at once.

c.) The entire process is displayed in Figure 17.20. (Oh, look, there *was* a Figure 17.20.)

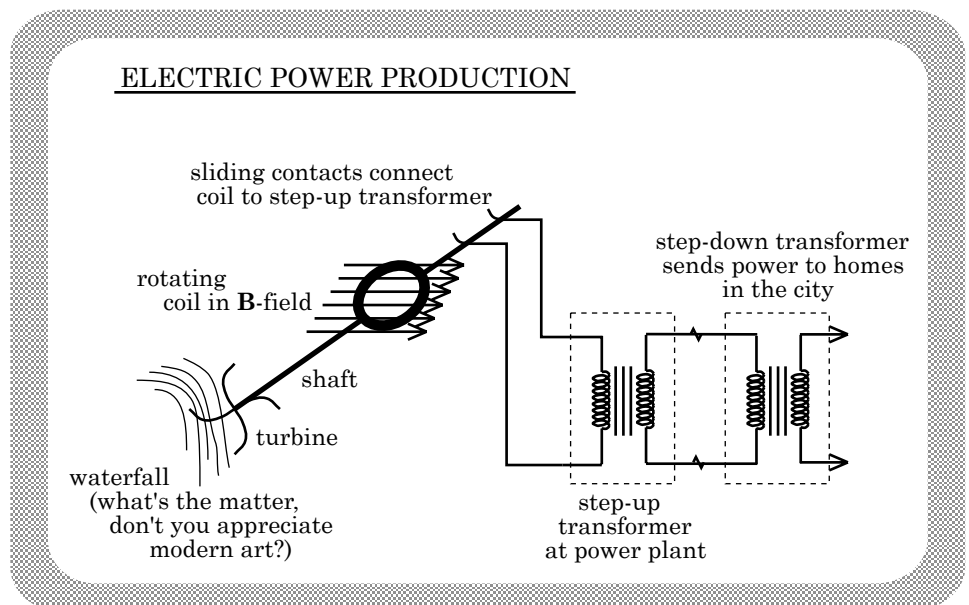


FIGURE 17.20

K.) Impedance Matching and Transformers:

1.) There is one more topic of importance, especially to anyone who is interested in speakers and stereo systems. It is best introduced with an analogy.

When optical light passes through an interface (i.e., a boundary) between two media, it will normally experience partial reflection caused by the fact that the two media have different densities. The only time a light beam will not reflect is when it passes into a second medium whose "density environment" is exactly the same as the first.

2.) Consider any complex electrical circuit--say, a stereo system connected to speakers. If the impedance of the stereo and the impedance of the collective speakers is the same, the signal will pass from the one to the other just as light passes through two *common-density environments* (i.e., there will be no reflection at the interface between the two systems). If, on the other hand, the *resistive nature* of the circuitry from which the signal comes (i.e., the stereo) is different from the resistive nature into which the signal must go (i.e., the speakers), reflection will occur at the interface. Put another way, maximum power will be transferred from the stereo to the speakers when the impedance of both is the same.

3.) The problem frequently confronting circuit designers is the fact that stereo systems have large impedances whereas speaker circuits have only tiny impedances. The question is, "How does one trick the signal into thinking the circuit it is entering has the same impedance as the circuit it is leaving?"

The answer involves the use of a transformer and is wrapped up in what is called *impedance matching*.

4.) A quick review of transformers: A transformer is essentially a pair of coils linked via a common magnetic field and, hence, a common magnetic flux. The turns-ratio (N_s/N_p) dictates how the *secondary* voltage and current are related to the *primary* voltage and current. That is:

a.) $N_s/N_p = \varepsilon_s/\varepsilon_p = i_p/i_s$ (this is true in all cases). Additionally;

b.) If $N_s < N_p$, the secondary voltage is smaller than the primary voltage ($\varepsilon_s < \varepsilon_p$) and the transformer is called a *step-down transformer*. In step-down transformers, the current in the secondary is *larger* than the current in the primary (i.e., $i_s > i_p$).

c.) If $N_s > N_p$, the secondary voltage is larger than the primary voltage ($\varepsilon_s > \varepsilon_p$) and the transformer is called a *step-up transformer*. In step-up transformers, the current in the secondary is *smaller* than the current in the primary (i.e., $i_s < i_p$).

5.) Having so reviewed, consider the following situation. A $1200\ \Omega$ stereo system ($Z_{st} = 1200\ \Omega$) is hooked up to a set of 8 W speakers ($Z_{sp} = 8\ \Omega$). From the signal's standpoint, how can we use a transformer to make 8 Ω speakers look like $1200\ \Omega$ elements?

a.) The first thing to notice is that as the signal comes into the transformer, it sees a net impedance ($Z_{transf.+load}$) made up of the primary coil's impedance, the secondary coil's impedance, and the load's impedance (Z_{load}). This net impedance is what we want to numerically equal the stereo's impedance (Z_{st}). Put another way, the signal sees an entire package which, if the transformer system has been designed optimally, will appear to have an impedance of $1200\ \Omega$.

b.) We know that the primary coil's current i_p will be the current coming from the stereo (i.e., $i_p = i_{stereo}$) while the primary coil's voltage is some value V_p . From Ohm's Law, the impedance of the stereo circuit (Z_{st}) will be:

$$Z_{st} = V_p / i_p \quad (= 1200\ \Omega \text{ for our example}).$$

c.) As the current from the stereo is i_p and the impedance of the stereo is Z_{st} , the energy provided by the stereo to the primary coil will be:

$$P_p = i_p^2 Z_{st}.$$

d.) Assuming an ideal transformer, the power provided by the primary will be completely transferred to the secondary. That is:

$$P_p = P_s.$$

e.) We know that the energy provided to the secondary circuit (i.e., the power available to the secondary circuit from the primary coil) will be dissipated by the load (the speakers). That is:

$$P_s = i_s^2 Z_{load}$$

f.) Equating the power terms yields:

$$\begin{aligned} P_p &= P_s \\ i_p^2 Z_{st} &= i_s^2 Z_{load} \\ \Rightarrow Z_{st} &= (i_s^2 / i_p^2) Z_{load} \end{aligned} \quad (\text{Equation A}).$$

g.) We have already established the relationships that exist between the secondary and primary currents and the turns-ratio of the transformer. Specifically, we know that:

$$N_p / N_s = i_s / i_p.$$

h.) Using this to eliminate the current terms in *Equation A* leaves us with:

$$Z_{st} = (N_p^2 / N_s^2) Z_{load}$$

i.) What does this relationship mean? Z_{st} and Z_{load} are fixed.

Evidently, for the signal to transfer without reflection, the turns-ratio of the transformer must be such that:

$$(N_p / N_s)^2 = Z_{st} / Z_{load}$$

j.) Bottom line: To modify the speaker-load to suit the incoming signal (i.e., to *impedance match*), we must use a transformer whose turns-ratio is such that:

$$(N_p / N_s)^2 = Z_{st} / Z_{load}$$

where Z_{load} is the true load resistance (i.e., that of the speakers) and Z_{st} is the impedance of the signal's source.

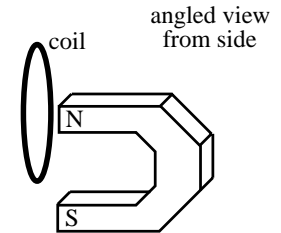
k.) For our situation, $N_p/N_s = (1200 \Omega / 8 \Omega)^{1/2} = 12.24/1$. If the winds are $N_p = 1224$ and $N_s = 100$, the signal will see the load as 1200Ω , and no reflection will occur.

QUESTIONS & PROBLEMS

17.1) What is magnetic flux? How is it defined? What does it do?

17.2) A coil is placed in the vicinity of a horseshoe magnet.

- a.) Once in place, is there a flux through the coil?
- b.) Once in place, is there a current in the coil? If so, why? Also, if so, in what direction will the current flow?



17.3) The coil alluded to in *Problem 17.2* is placed in the vicinity of the same horseshoe magnet, but this time the coil is rapidly pulled away from the magnet.

- a.) Is there an initial flux through the coil?
- b.) What happens to the flux as the coil is pulled away?
- c.) From the standard perspective associated with magnetic fields and charges moving in magnetic fields, would you expect a current to flow in the coil as the coil was pulled away from the magnet? If so, why? Also, in what direction would the current flow?
- d.) From Faraday's perspective, would you expect a current to flow in the coil as the coil was pulled away from the magnet? If so, how would Faraday explain the current? Also, how would he determine the direction of current flow?

17.4) Each of the loops in the figure are identical. Each has a length of .2 meters, a width of .08 meters, and a resistance of 4 ohms. Each is moving with a velocity magnitude of .28 m/s, and *Loops A, C, and F* each have .05 meters of their lengths *not in the magnetic field* at the time shown in the sketch (that is, the length *outside the field* at the time shown is .05 meters for each of those loops). The magnetic field in the shaded region is *into the page* with a magnitude of $B = 3 \times 10^{-2}$ teslas.

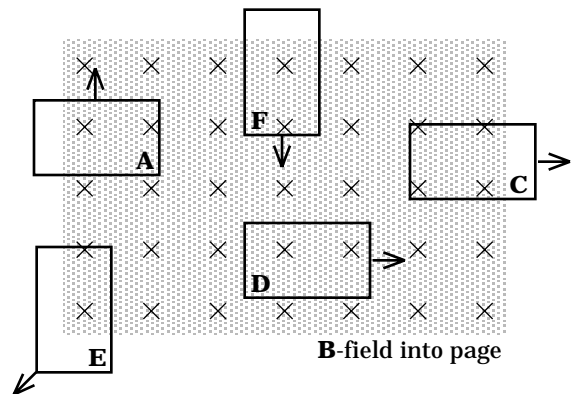


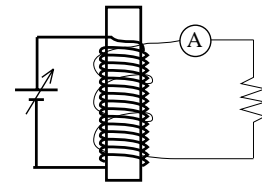
FIGURE II

- a.) What is the direction of the induced current for each loop at the instant shown in the sketch?
- b.) What is the induced EMF generated in *Loops A, C, and F* at the instant shown?

c.) What is the magnitude and direction of the induced magnetic force felt by *Loop F* at the instant shown?

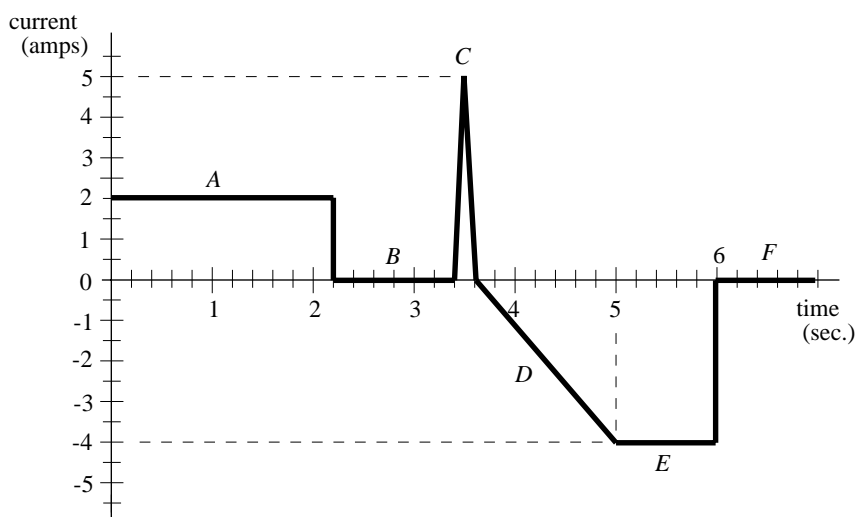
d.) What is the direction of the induced magnetic force on *Loops A, C, and D* at the instant shown?

17.5) Two coils share a common axis but are electrically isolated from one another (that is, they aren't electrically connected). The coil on the left is attached to a variable power supply (we'll call this *the primary circuit*). The coil on the right is attached



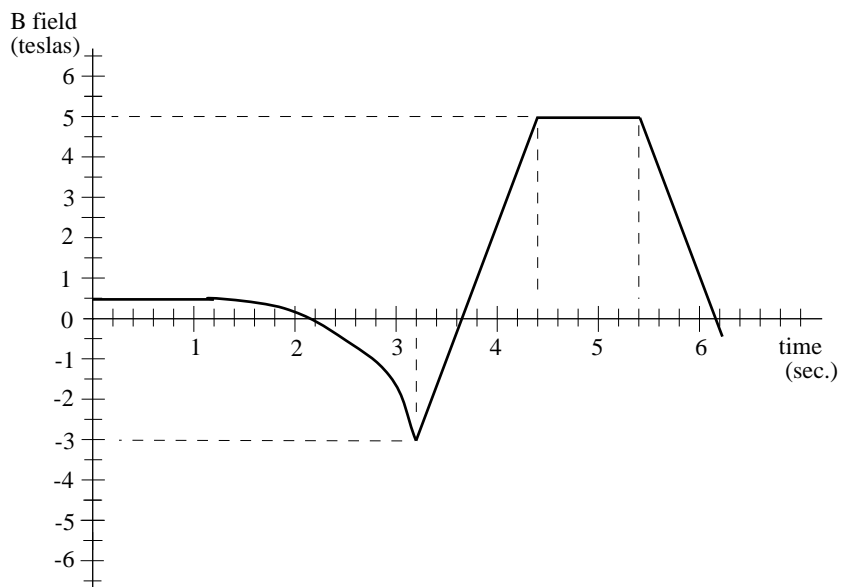
only to a resistor and an ammeter (we'll call this *the secondary circuit*).

One of the more hyperactive students in the crowd begins to play with the voltage across the primary coil power supply while a second student records, then graphs the current in the SECONDARY coil. That graph is shown in the sketch. There are



six time intervals identified by letters on the graph (i.e., *A* corresponds to the current during the period between $t = 0$ and $t = 2.2$ seconds, etc.). Explain what must be happening to the power supply in the *primary circuit* during each of those time periods.

17.6) The magnetic field down the axis of a coil varies with time as graphed to the right. On the graph, sketch the induced EMF set up in the coil.



17.7 If the graph in *Problem 17.6* had been of the EMF set up in the coil as a function of time, what could you say about the magnetic flux through the coil?

17.8) A 6-turn circular coil whose radius is .03 meters and whose net resistance is 12Ω 's is placed squarely (that is, \mathbf{A} and \mathbf{B} are parallel to one another) in a magnetic field whose direction is *out of the page* and whose magnitude is 2.3 teslas.

- a.) What is the coil's initial magnetic flux?
- b.) If the field increases at a rate of .6 teslas per second, what is the magnitude and direction of the induced current in the coil?
- c.) Go back to the original situation. The coil is made to rotate about its vertical axis at an angular frequency of $\omega = 55 \text{ radians per second}$. That means the induced EMF is AC.
 - i.) What is the frequency of the AC current generated?
 - ii.) Determine an expression for the *induced EMF* in the circuit.

17.9) For the *RL circuit* shown in Figure III, the inductance is 1.5 henrys and the inductor's internal resistance is 6 ohms. A current of 2.5 amps has been flowing in the circuit for a long time. At $t = 0$, the power is switched off and the current begins to die.

- a.) What is the voltage across the inductor BEFORE $t = 0$?
- b.) After .05 seconds, the current has dropped to approximately one-third of its original value. Determine the resistance of the resistor R . (Hint: think about the *time constant* of an RL circuit and what it tells you).
- c.) How much POWER does the inductor provide to the circuit over the .05 second time period alluded to in *Part b*? (Hint: Think about the definition of power and what you know about stored energy in a current-carrying inductor).
- d.) The power given up by the inductor: where did it go?

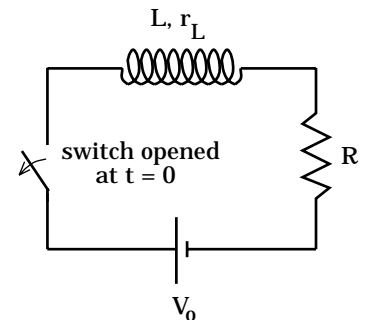


FIGURE III

17.10) A rectangular coil of area A_0 has N turns in it. It is rotated in a time-varying magnetic field (see Figure V) equal to $B_0 e^{-kt}$, where k is a constant and B_0 is the amplitude of the magnetic field. Assuming the frequency of the rotation is n :

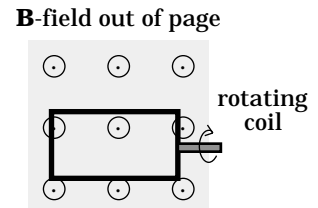


FIGURE V

- a.) Determine the EMF in the coil as a function of time, and;
- b.) At what point in time will the magnitude of the EMF be at maximum?

17.11) A fixed circular coil of radius R is placed in a magnetic field that varies as $12t^3 - 4.5t^2$. If the coil has N winds and \mathbf{A} is defined *out of the page* (i.e., in the $+\mathbf{k}$ -direction):

- a.) Is \mathbf{B} into or out of the page at $t = .2$ seconds?
- b.) Derive a *general expression* for the magnetic flux through the coil.
- c.) What is the *general expression* for the induced EMF in the coil?
- d.) Determine the *two* points in time when the induced EMF is zero.
- e.) What is the direction of the current flow:
 - i.) Just before $t = .25$ seconds?
 - ii.) Just after $t = .25$ seconds?
- f.) Derive the general expression for the induced electric field setup in the coil.
- g.) An electron is placed at $R/2$ in the field. Derive an expression for its acceleration at time $t = 3.3$ seconds. For this, assume $N = 15$ and $R = .2$ meters.

field direction is time-dependent--the direction of \mathbf{A} is out of the page

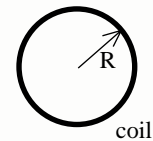


FIGURE VI

17.12) The transformer shown in Figure VIII has 1200 winds in its primary coil and 25 winds in its secondary. The resistance in its primary is 80Ω 's, the resistance in its secondary is 3Ω 's, and the primary's inductance is $L_p = 10 \text{ mH}$. A 110 volt DC power supply is hooked into the primary providing an 8.25 amp current to the system. The switch has been closed for a long time. When the switch is opened, the current drops to zero in .04 seconds.

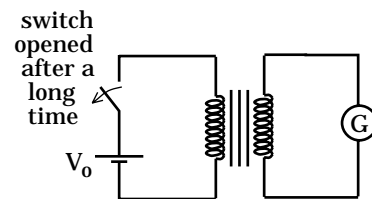


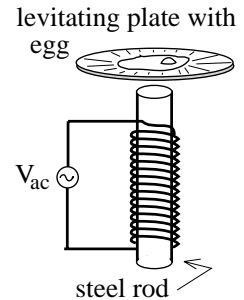
FIGURE VIII

- a.) What is the induced EMF across the primary before the switch is opened?
- b.) What is the induced EMF across the primary during the current change?
- c.) What is the current in the primary during the current change (i.e., after the switch is opened)?
- d.) What is the current in the secondary before the switch is opened?

- e.) What is the current in the secondary after the switch is opened and DURING the current change?
- f.) Is this a step-up or step-down transformer?

17.13) An AC source is attached to a coil that has a vertical, steel bar down its axis. When the power is turned on, an alternating magnetic field is set up along the axis of the bar. An aluminum plate is centered over the bar at its upper end. When power is provided to the coil, the plate levitates.

- a.) Is aluminum a magnetizable material?
- b.) Why does the plate levitate?
- c.) An egg is broken onto the plate. What will happen to the egg . . . and why?



17.14) What is inductance? How is it comparable to resistance and capacitance?

17.15) How do transformers work?

17.16) You have just built from scratch a stereo system. You have 8 ohm speakers you would like to plug into the system, but as it stands the system's unrestricted output impedance is 60 ohms. Assuming you don't want to completely redesign the entire system, what would you have to do so that your set-up could run the 8 ohm speakers without power loss? Be specific and include numbers where applicable.

