Chapter 16

MAGNETISM

A.) A Small Matter of Special Relativity:

1.) When you get right down to it, the world of magnetism is a lot odder than you might think. Although you will be primarily studying and using what is called "the classical theory of magnetism," I thought it might be interesting to first take a little more educated look at the subject. You will better understand what I mean by this shortly.

2.) Assume we have a particle of charge $q$ moving with an initial velocity $v_q$ parallel to a current-carrying wire as shown in Figure 16.1.

   a.) Consider the situation from the perspective of the laboratory frame of reference (i.e., the frame in which you and I sit and in which the wire is motionless):

      i.) The positive charges (the protons) are fixed in the wire while the negative charges (the electrons) have some non-zero average velocity $v_e$.

      ii.) There are as many electrons as protons in the wire before the current begins (i.e., the wire is electrically neutral).

      iii.) As many electrons leave the wire as come onto the wire while current flows. As such, the wire is perceived to be electrically neutral even when current is flowing.

   b.) Consider now the situation from $q$'s frame of reference:
**Note:** From this frame of reference, the charge $q$ will be stationary while everything else is moving around it.

i.) In $q$'s *frame of reference* (see Figure 16.2), the wire and all *positive charges* (protons) will move to the left with velocity $v_q$. Meanwhile, *negative charges* (electrons) will move to the left with velocity $v_q - v_e$ (we are assuming $v_q > v_e$). The action is summarized in Figure 16.2.

ii.) Notice that the protons move faster than electrons from this perspective.

3.) Einstein's *Theory of Relativity* suggests that when one object passes a second object, the second object will appear to the first to have *contracted in length*. Called "length contraction," the phenomenon is immediately evident only at very high speeds but does occur microscopically at low speeds.

4.) As all of the charge in the wire moves relative to $q$'s *frame of reference*, the distances between the charges should appear to be closer (relativistic *length contraction*) than would otherwise have been the case if viewed from the *lab frame* (see Figure 16.3). What's more, the protons will appear to be *more tightly packed* because they

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**FIGURE 16.2**

- *wire and protons moving with velocity $v_q$ toward left*
- *electrons moving with velocity $v_q - v_e$ toward left*
- *q stationary*

**FIGURE 16.3**

- *protons well packed due to higher velocity $v_q$*
- *electrons not as well packed due to lower velocity $v_q - v_e$*
- *q stationary*

From $q$'s frame, an *electric field* exists due to the predominance of positive charges in the wire. As $q$ is positive, this *electric field* will push $q$ away from the wire.
are moving faster than the electrons.

a.) In other words, the wire will appear to have more protons than electrons on it. That means charge $q$ will perceive an electric field due to the predominance of positive charge, and that electric field will motivate $q$ to accelerate away from the wire.

b.) If we set up an experiment in which a positive charge is made to move parallel to a current-carrying wire and opposite to the current's direction, we will observe a force on $q$ pushing it away from the wire. The force is due to the relativistic effect we have been discussing, but observers in the previous century did not know that (Einstein's Theory of Relativity wasn't published until 1905). Working strictly from empirical observation, they assumed there must exist a new kind of force—a magnetic force—acting on the moving charge. The theory developed on behalf of that belief is today called "the classical theory of magnetism." It is the subject we are about to consider.

B.) Compasses, Bar Magnets, and Magnetic Fields:

We are about to examine the classical theory of magnetism. To do this, we will start with a series of observations about magnetic phenomena.

1.) When suspended, certain metallic ores are found to have the peculiar ability to orient themselves north/south. They evidently align themselves with some sort of field, a field that in the early days of "modern science" was eventually called a magnetic field.

a.) In experimenting with a piece of such ore, it has been observed that this north/south orientation is always the same. That is, the same face always aligns itself to the north while the opposite face always aligns to the south. To distinguish between the two, one is called "the North Seeking Magnetic Pole $N$," and the other is called "the South Seeking Magnetic Pole $S$."

These observations were, in early times, the basis for what is today called a compass.
2.) As shown in Figure 16.4, a compass put in the vicinity of a theoretically ideal bar magnet (i.e., one that emanates a magnetic field only at its ends) will be found to point in different directions at different places.

a.) A line connecting all of the arrows would create what is called a magnetic field line. Doing this for several paths would produce a sketch of the magnetic field lines for the entire magnetic field (see Figure 16.5).

b.) Magnetic field lines are SIMILAR to electric field lines in the sense that field lines that are close together denote a strong magnetic field whereas field lines that are far apart denote a weak magnetic field.

c.) Magnetic field lines are DIFFERENT from electric field lines in one very important way. Remember, the direction of an electric field line is defined as the direction a positive test charge will accelerate if released in the electric field. In other words, electric fields are nothing more than slightly modified force fields \( (E = F/q) \).

The direction of a magnetic field line is defined as the direction a compass will point if a magnetic field is present. As will be shown shortly, magnetic fields are NOT modified force fields (though they are distantly related to force).
d.) For the sake of completion, it should be noted that the magnetic field generated by an everyday, non-ideal bar magnet will see leakage of field along the edges of the bar (see Figure 16.6). The standard way to demonstrate this is to pour iron filings onto a sheet of paper that is suspended above (or lying directly on) the bar.

i.) Figure 16.6 gives a general sense of how iron filings will align when placed near a non-ideal bar magnet.

e.) It has been found experimentally that north poles attract south poles (and vice-versa), that north poles repulse north poles, and that south poles repulse south poles. In other words, like poles repulse while opposite poles attract. Figure 16.7 on this page and Figure 16.8 on the next page animate these points.

3.) It should also be noted that a constant magnetic field is denoted by field lines that are...
equidistant and parallel as shown in Figure 16.9 below.

4.) The strength of a magnetic field and the direction of the magnetic field is combined together to define the magnetic field vector \( B \). More will be said shortly about \( B \), its relationship to the force on a charge moving in a magnetic field, and its units.

5.) While experimenting with electrical circuits in 1820, a man named Oersted observed that when a compass was placed near a current-carrying wire, the compass would respond. Although we will say more about this later, one of the conclusions he drew from this observation was that magnetic fields are somehow caused by charge in motion.

6.) If a magnetic field is created by charges in motion, what kind of motion creates the magnetic field in an apparently motionless bar magnet? Possibilities: Electrons confined to the atom are constantly in motion. An electron both orbits about its nucleus and spins about its axis. Let's consider both:

   a.) Orbital Motion: While the orbital motion of electrons around the nucleus surely produces a magnetic field, the direction of an electron's motion will be "this way" as much as "that way" (electrons travel around the atom at speeds upward of 150,000 miles per second). Consequently, the net magnetic field produced by electron orbital motion is, on average, zero.

   b.) Spinning On Axis: Electron spin also produces a magnetic field. Due to quantum mechanical effects, electrons spin in only one of two directions. These directions are usually referred to as "spin up" and "spin
down." In most elements, there are as many electrons spinning up as spinning down, which means the net magnetic field generated by all the spinning electrons is zero.

i.) There are some elements whose number of electrons spinning in one direction is noticeably different from the number spinning in the opposite direction. Iron, for instance, has six more electrons spinning one way than the other. As a consequence, the net magnetic field due to electron spin in an iron atom is not zero. Put another way, every iron atom is a mini-magnet unto itself.

ii.) Elements that exhibit this magnetic characteristic are called ferromagnetic materials. The most common are iron, nickel, and cobalt.

7.) Ferromagnetic materials do not always exhibit magnetic effects. Iron nails, for instance, do not usually attract or repulse one another as would be expected if they were magnetized. The question is, "Why?"

a.) Take a structure made of iron (a steel bolt, for example). Within it, there exist microscopic sections called domains. A domain is a volume in which each atom has aligned its magnetic field in the same direction as all the other atoms in the section.

b.) Figure 16.10 shows a side-view blow-up of the domains that reside on the face of a piece of iron. Notice that each domain has its magnetic field oriented in some arbitrary direction. Because none of the domain-fields are aligned, the net (read this average) magnetic field on the face is essentially zero.

This is an example of a ferromagnetic material that is not magnetized.

c.) If the bolt is placed in a relatively strong magnetic field, as shown in Figure 16.11, the domains
will align themselves with the external field and, in doing so, will align themselves with one another. In that case, each face of the bolt will either be a North Pole or a South Pole. That is, we end up with a magnetized piece of iron.

C.) The Earth's Magnetic Field:

1.) As was observed above, North Seeking Magnetic Poles always attract South Seeking Magnetic Poles, and like poles (i.e., N-N or S-S poles) repulse. One of the consequences of this is the peculiar situation we have with respect to the earth's magnetic field.

2.) By definition, the North Seeking Magnetic Pole of a compass points toward the northern geographic region of the earth. But if North Seeking Magnetic Poles are attracted to South Seeking Magnetic Poles, there must exist a South Magnetic Pole in the northern geographic hemisphere. In fact, that is exactly the case. The earth's magnetic field lines leave Antarctica and enter the Arctic (they actually enter in the Hudson Bay region--see Figure 16.12 for the theoretical distribution of magnetic field lines around the earth).

   a.) Note that the field we are seeing in Figure 16.12 is using an ideal bar magnet as its model. In fact, the earth's field acts more like a non-ideal bar magnet, which means the field lines are not so uniformly oriented close to the earth's surface.
b.) In fact, there is what is called dip, which identifies how much the field line dips below the horizon line at a given point.

3.) Solar winds are streams of high energy subatomic particles (usually hydrogen ions) that are constantly being emitted by the sun.
   a.) Due to these solar winds, the earth's magnetic field lines are actually compressed in toward the earth on the earth's sun-side while being extruded out away from the earth on the earth's dark side. See Figure 16.13.

4.) The earth's magnetic field is believed to be caused by motion of molten iron at the earth's core. By looking at core samples of the earth's geological history over long periods of time, it has been found that the earth's magnetic field changes direction periodically (the cycle is somewhere around 200,000 to 400,000 years). Although scientists are not completely sure why, one possibility is that a long-period oscillatory variation in the motion of the earth's iron-rich molten interior creates this effect.

D.) Current-Carrying Wires:

1.) Oersted's work did not stop with the conclusion that charge in motion produces magnetic fields. What we now know about current-carrying wires and magnetic field follows.

2.) Magnetic field lines CIRCLE around a current-carrying wire (see Figure 16.14).

   a.) Noting that we are talking about conventional current, the direction of a current-produced magnetic field can be determined by using the following "weird"
right-hand rule (from here on, this rule will be termed the right-thumb rule).

b.) Position the thumb of the right hand so that it points in the direction of conventional current flow. The direction the fingers curl is the direction of the magnetic field's circulation around the wire (Figure 16.15).

c.) Note that if we had used electron current, a similar rule using the left hand would have produced the same magnetic field circulation.

3.) There are several ways to determine magnetic field functions theoretically . . . Ampere's Law and the Law of Biot Savart, to name two. As we are more interested in the conceptual side, I will present the mathematical expressions those approaches yield when used on current configurations of interest without boring you with the mathematical derivations.

4.) The magnitude of the magnetic field produced by a long, current-carrying wire (we are assuming conventional current here) is numerically equal to

\[ B = \frac{\mu_0 i_o}{2\pi r}, \]

where \( \mu_0 \) is the permeability of a vacuum, \( i_o \) is the current in the wire, and \( r \) is the distance from the wire (see Figure 16.16).

a.) Remember, magnetic fields are vectors. To get the direction of a magnetic field produced at a given point in space by a wire:

i.) Draw a circle around the wire and through the point.

ii.) The magnetic field direction will be tangent to that circle in a clockwise or counterclockwise direction. To determine whether it is clockwise or counterclockwise, use the right-thumb rule alluded to above.
5.) A wire wound into a coil is called a solenoid (it's also called an inductor . . . or a coil--clever, eh?) A current through a solenoid will generate a magnetic field down its axis (see Figure 16.17).

a.) The magnitude of the field down the axis of a coil is

\[ B = \mu_0 n i_0, \]

where \( \mu_0 \) is the permeability of a vacuum, \( n \) is the number of winds per unit length, and \( i_0 \) is the current through the coil.

b.) Notice that with the magnetic field lines leaving one end and entering the other end, the magnetic field of a current-carrying coil looks very much like the magnetic field of an ideal bar magnet.

**Note:** Remember, magnetic field lines *leave* NORTH POLES and *enter* SOUTH POLES.

c.) To get the direction of the magnetic field down the axis of the current-carrying wire, we find another use for the right hand. If you grasp the coil with your right hand so that your fingers are oriented in the direction of the conventional current, the direction of your thumb will identify the direction of the magnetic field down the coil's axis. See Figure 16.18.
Note: If you are working with electron current, the same rule works with the exception that you have to use your left hand (try it).

E.) The Force on Charge Moving in a Magnetic Field—Magnetic Interactions:

1.) We need to understand how current-carrying wires and coils interact with external magnetic fields. To do this, we will start by looking at a single charge moving through a magnetic field.

2.) It has been EXPERIMENTALLY OBSERVED that:

   a.) A charge sitting stationary in a magnetic field will feel no magnetic force whatsoever.

   b.) A charge moving along magnetic field lines will feel no force. (Evidently, magnetic fields do not make charges pick up or lose speed.

   c.) A charge that moves across magnetic field lines will feel a force. What's more, the direction of that force will be perpendicular to the direction of motion (i.e., perpendicular to the velocity vector) AND perpendicular to the magnetic field lines.

      i.) Put a little more succinctly, the force will be perpendicular to the plane defined by the velocity and magnetic field vectors.

      ii.) As this force is perpendicular to the line of motion, magnetic forces are, evidently, centripetal in nature.

   d.) It has been experimentally determined that the force vector $F_B$ experienced by a charge $q$ as it moves with velocity $v$ through a magnetic field of strength $B$ is:

      $F_B = q \mathbf{v} \times \mathbf{B}$.

      i.) This cross product yields both the magnitude and direction of the force on a POSITIVE CHARGE moving in the magnetic field. If the charge had been NEGATIVE, its force direction would have been OPPOSITE that determined using the right-hand rule.
Note 1: Remember, a cross product produces a new vector that is perpendicular to the plane defined by the vectors being crossed. That is exactly what we need in this situation.

Note 2: From the MKS units for force, charge, and velocity, the units for the magnetic field vector $\mathbf{B}$ must be $nT/[C\cdot(m/s)]$, or $kg/(C\cdot s)$. This set of MKS units is given the special name teslas.

e.) As a very quick example, determine the magnitude and direction of the force on a 4 coulomb charge moving with velocity 12 meters/second in a magnetic field whose strength is 5 teslas if the velocity and magnetic field vectors are as shown in Figure 16.19a.

Note: Vectors pointing perpendicularly into the page are depicted either by a group of circles with crosses in them or simply by crosses. Vectors pointing perpendicularly out of the page are depicted by a group of circles with points at their centers or simply by points.

i.) The magnitude of the force is

$$|\mathbf{F}| = q|\mathbf{v}||\mathbf{B}| \sin \theta,$$

where $\theta$ is the angle between the line of $\mathbf{v}$ and the line of $\mathbf{B}$ (note that the sketch is a bit tricky-- $\theta$ should be the angle between the line of $\mathbf{v}$ and the line of $\mathbf{B}$--this is not the angle given in the figure). Putting in the numbers, we get:

$$|\mathbf{F}| = (4 \text{ C})(12 \text{ m/s})(5 \text{ T})(\sin 150^\circ)$$

$$= 120 \text{ newtons}.$$

ii.) The direction is found using the right-hand rule for a cross product. The right hand moves in the direction of the line of the first vector ($\mathbf{v}$); the fingers of the right hand curl in the direction of the line of the second vector ($\mathbf{B}$). Doing so yields a force direction for this situation into the page.

iii.) For the sake of completeness, this force can be written as a vector in unit vector notation as:

$$\mathbf{F}_B = (120 \text{ newtons})(-\mathbf{k}).$$
3.) A current-carrying wire is nothing more than a thin strand of metal with individual charges moving through it. Putting a current-carrying wire in a magnetic field should, therefore, generate a force on the wire if the wire is oriented correctly. In fact, that is exactly what happens—a current-carrying wire in an external magnetic field will experience a force.

**Note:** That's right, folks. There are two possible things happening when dealing with a current-carrying wire. It will produce its own magnetic field and it will feel a force if placed in an external magnetic field.

**a.)** If the wire is oriented along magnetic field lines, there will be no force on it as current passes through it.

**b.)** If the wire crosses magnetic field lines, there will be a force on it when current passes through it.

**i.)** It has been observed experimentally that the force on the wire is perpendicular to the plane defined by the magnetic field vector and the line of current flow.

**c.)** Mathematically, it is possible to manipulate $F_B = q v \times B$ to derive an expression for the force $F_B$ on a current-carrying wire of length $L$ in a magnetic field $B$. That force expression is

$$F_B = i L \times B,$$

where $i$ is the conventional current in the wire and $L$ is a vector whose direction is the direction of conventional current flow and whose magnitude is the length $L$ of the wire.

4.) A quick example: Figure 16.19b shows a current-carrying wire of length .5 meters in a magnetic field. If the current is $2 \times 10^{-3}$ amps, the magnetic field is 4 teslas, and the orientation between $i$ and $B$ is as shown in the sketch, what is the magnitude and direction of the force on the wire?

**a.)** What is the angle for the cross product? The line of $L$ is up and to the right while the line of $B$ is down. The angle between the two lines is $120^\circ$ (not $30^\circ$).
b.) The magnitude is:

$$|F| = (2 \times 10^{-3} \text{amps})(0.5 \text{ m})(4 \text{ T})(\sin 120^\circ)$$

$$= 3.46 \times 10^{-3} \text{ newtons.}$$

c.) There are two ways to get the direction.

i.) The first is to notice that you are dealing with a cross product. Falling back on the right-hand rule that is normally used to determine the direction of a cross product (i.e., putting your right hand along the line of the first vector, or \( L \), then waving toward the second vector, or \( B \)), we get a direction into the page.

ii.) The second approach is to remember that cross products always generate vectors that are \textit{perpendicular} to the plane defined by the two vectors being crossed into one another. As both \( L \) and \( B \) are in the plane of the page, that means the direction is either \textit{into} or \textit{out of} the page. Which is it? Go back to option A above and you will deduce that it is \textit{into} the page.

5.) To determine the direction of force on a current-carrying wire in a magnetic field, we can use the expression quoted above and the right-hand rule associated with the cross product. There is another way to look at the process, though.

a.) Instead of thinking about how an external magnetic field will interact with a current-carrying wire, think about how an external field will interact with the magnetic field set up by the current-carrying wire.

b.) Not clear? Think back.

What happened when we brought two ideal bar magnets close to one another (see Figure 16.20)?

i.) If the poles were alike (say, both were north poles), the magnetic field lines from the two magnets were found to be \textit{parallel} to one another and repulsion was observed. Extended conclusion? \textit{Whenever} field lines from a source are parallel to external field lines,
the source will experience a force that pushes it *away from* the parallel lines.

**ii.** If the poles were opposites, the magnetic field lines from the two magnets were found to be anti-parallel to one another and attraction was observed. Extended conclusion? *Whenever* field lines from a source are anti-parallel to external field lines, the source will experience a force that pulls it *toward* the anti-parallel lines.

c.) With the conclusions drawn above, think about the magnetic field lines generated by the current-carrying wire shown in Figure 16.21. There will be places where those field lines are *parallel* to the external field lines in the problem, and places where those field lines are *anti-parallel* to the external field lines. Knowing what we do about parallel and anti-parallel field lines, it should be obvious that the current-carrying wire will feel a force on it that is *upward and to the left*. That is, a force directed *toward* anti-parallel field lines and *away from* parallel field lines.

**i.** In fact, if you use the *cross product* approach to determine the direction of the force on the wire due to the external field, that is exactly the direction the right-hand rule predicts for the force.
6.) Consider the force on a current-carrying wire that runs between the jaws of a horseshoe magnet (see Figure 16.22a). Assume the current moves into the page.

a.) The magnetic field lines for the two magnetic sources (i.e., the wire and the horseshoe magnet) are anti-parallel above the wire and parallel below the wire. That means the wire will feel a force upward.

b.) If we use $LxB$ (i.e., from the expression $F = iLxB$), noting that $L$ is into the page while the external field $B$ is to the left, the right-hand rule yields a force direction on the wire that is upward.

c.) The conclusions for both approaches match. As always, isn't this fun?

7.) Two last devices. Look at the sketches shown in Figures 16.22b and 16.22c and see if you can tell what each device does. The answers are found in the homework solutions.

F.) A Current-Carrying Coil in an External B-Field . . . and Analog Meters:

1.) Consider a pinned, single-looped rectangular coil whose side-lengths are equal to $a$ and whose top and bottom lengths are equal to $b$. If a current $i$ passes
through the wire while in a magnetic field (see Figure 16.23), the moving charges will feel a force according to $F_B = i L \times B$.

**Note 1:** As seen from above, the magnetic field lines generated by bar magnets are not constant (see Figure 16.24). Nevertheless, we will assume a constant B-field for simplicity.

**Note 2:** The direction of the force will depend upon the current's direction relative to the magnetic field vector. Looking at the wire and charge flow from above (see Figure 16.25), we can make the following observations:

a.) The section of wire with current moving out of the page (side A in Figure 16.25) will feel a force whose direction is toward the top of the page (think $iLxB$).

b.) The section of wire with current moving into the page (side C in Figure 16.25) will feel a force whose direction is toward the bottom of the page.

c.) Each of these forces will produce a torque on the coil about the pin which will motivate the coil to rotate about the pin.
2.) We have established that a current-carrying coil in a magnetic field can have torques applied to it. With that observation, consider the magnetically engulfed, current-carrying coil shown in Figure 16.27 (yes, there's no Figure 16.26--that's just life!). A spring attached to the bottom of the coil produces a counter-torque when the coil rotates. When rotation occurs, a needle attached to the coil also rotates. If that needle is placed over a visible, calibrated scale, we end up with the prototypical current-sensing meter.

3.) The most basic version of a current-sensing meter is called a galvanometer (the sketch shown in Figure 16.27 is actually that of a galvanometer).

   a.) A coil in a known magnetic field has attached to it a spring that is just taut enough to allow the needle to rotate full-deflection (i.e., to the end of the scale) when 5x10^-4 amps flow through it. In that way, if an unknown current flows through the galvanometer and the needle fixes at half deflection, the user knows that the current is half of 5x10^-4 amps, or 2.5x10^-4 amps.

   b.) ALL GALVANOMETERS ARE MADE TO SWING FULL DEFLECTION WHEN 5x10^-4 AMPS FLOW THROUGH THEM. This uniformity is the reason galvanometer scales are labeled 1 through 5 without any other hint as to the meaning of the numbers. It is assumed that if you know enough to be using a galvanometer, you know that its units are "x10^-4 amps" (quite a conceit if you think about it).

   c.) As all galvanometers are made to the same specifications throughout the industry, they are the cornerstone in the production of all other meters, voltmeters and large-current ammeters alike.

**Note:** Although it is not evident in Figure 16.27, a galvanometer's needle always points toward the center of the scale when no current is passing through the meter. In that way, the needle can deflect either to the right or the left, depending upon which meter-terminal the high voltage is connected to. Galvanometers are the
only meters that have this "center-zero" setup. All other meters have their zero to the left, swinging to the right when current passes through them. That means they depend upon you, the user, to hook the high voltage leads to the correct terminal.

4.) The Ammeter: The sketch in Figure 16.28 shows the circuit for a 12 amp ammeter (the sketch is general to all ammeters; I have arbitrarily chosen 12 amps for the sake of a number example). Notice the design requires a galvanometer (designated by the resistance $R_g$) and a second resistor $R_s$. Assume the resistance of the galvanometer is 5 ohms. The rationale behind the design is as follows:

a.) We want the galvanometer's pointer to swing full-deflection when twelve amps of current passes through the ammeter. In other words, when twelve amps flow into the meter, we want $5\times 10^{-4}$ amps to flow through the galvanometer.

b.) The parallel design allows current passing through the meter to split up. If we pick just the right size resistor $R_s$ (this is called a shunt resistor because it shunts off current from passing through the galvanometer), all but $5\times 10^{-4}$ amps will flow through that resistor whenever twelve amps flow into the device. The trick is in finding the proper value for the shunt resistor. To do so:

i.) Noticing that the voltage across $R_g$ is the same as the voltage across $R_s$ (the two resistors are in parallel), we can write:

$$i_{g,\text{max}} R_g = i_{s,\text{max}} R_s.$$
ii.) We know that $i_{g,\text{max}}$ will be $5 \times 10^{-4}$ amps when 12 amps flow into the circuit, so the amount of current passing through $R_s$ must be whatever is left over, or:

$$i_{s,\text{max}} = (12 \text{ amps}) \cdot (0.0005 \text{ amps}) = 11.9995 \text{ amps}.$$ 

iii) Putting it all together, we get:

$$i_{g,\text{max}} R_g = i_{s,\text{max}} R_s = (5 \times 10^{-4} \text{ amps}) (5 \Omega) = (11.9995 \text{ amps}) R_s \Rightarrow R_s = 2.08 \times 10^{-4} \Omega.$$ 

c.) A short piece of wire will have resistance in this range. In other words, a typical ammeter is nothing more than a galvanometer with a measured piece of wire hooked in parallel across its terminals.

5.) The Voltmeter: The sketch in Figure 16.29 shows the circuit for a 12 volt voltmeter (the sketch is general to all voltmeters; I have arbitrarily chosen 12 volts for the sake of a number example). Notice the design requires a galvanometer (designated by the resistance $R_g$) and a second resistor $R_1$. Assuming the galvanometer's resistance is 5 ohms, the rationale behind the design is as follows:

a.) We want the galvanometer's pointer to swing full-deflection when twelve volts are placed across the voltmeter (i.e., when the voltmeter is hooked across an electrical potential difference of twelve volts). In other words, when 12 volts are placed across the meter, we want $5 \times 10^{-4}$ amps of current to flow through the galvanometer.

b.) The series design requires that voltage across the voltmeter be split up between the two series resistors (i.e., the voltage drop across the galvanometer plus the voltage drop across the second resistor must sum to 12 volts). If we
pick just the right size resistor $R_1$, a current of $5 \times 10^{-4}$ amps will flow through both resistors whenever *twelve volts* are placed across the meter. The trick is in finding the proper value for the second resistor. To do so:

1.) When the total voltage across the meter is 12 volts, the galvanometer's voltage must be $i_{g,\text{max}} R_g$ while the second resistor's voltage must be $i_{g,\text{max}} R_1$ (the two resistors are in series, hence the current is common to both). As such we can write:

$$
V_o = (i_{g,\text{max}} R_g) + (i_{g,\text{max}} R_1)
$$

12 volts $= (5 \times 10^{-4} \text{ amps}) \times (5 \Omega) + (5 \times 10^{-4} \text{ amps}) \times (R_1)

\Rightarrow R_1 = 2.3995 \times 10^4 \Omega.$$

c.) In short, a typical voltmeter is nothing more than a galvanometer hooked in series to a large resistor. As would be expected, they draw very little current when hooked across an element in a circuit.

6.) **Bottom line:** All analog meters (i.e., meters that are not digital) are based on the galvanometer, and all galvanometers are based on the proposition that current moving through an appropriately pinned coil in a magnetic field will feel a torque-producing force which is proportional to the amount of current passing through the coil.

G.) **A Current-Carrying Coil in an External Magnetic Field . . . and MOTORS:**

1.) We have already established that a current-carrying coil will feel a rotation-producing torque when placed in a fixed, external magnetic field. We have also seen how such a set-up can be used to measure voltages and currents. What we are about to examine is how this design can be used to turn electrical energy into mechanical energy (translation--we are about to look at *motors*).

2.) At this point, you should be able to look at the device shown in Figure 16.30 (next page) and be able to make some sense of it. In the off-chance you can't:

a.) The power supply in the circuit will motivate *conventional current* to flow clockwise as shown in the sketch.
b.) Current will enter the coil via the sliding contact (brushes provide this contact) over the upper half-cylinder conducting sheath (this is the sheath that is closest to "side A").

c.) That current will flow through the coil, exiting at the sliding contact made via the lower brushes (these are the brushes closest to "side C").

d.) Laying the right hand on the coil so that the fingers follow the current, it is evident that the magnetic field set up by the coil down its axis will have a north pole at side C and a south pole at side A.

e.) The coil's north pole will be attracted to the south pole of the bar magnet on the right while the coil's south pole will be attracted to the north pole of the bar magnet on the left. The net effect is that the entire coil will rotate counterclockwise.
f.) After accelerating angularly, side A will sooner or later find itself exactly opposite the north pole of the bar magnet on the left. When that happens, two things will be true.

i.) Because it has angular momentum, the coil will continue to move as it passes the bar magnet's north pole (see Figure 16.31).

![Figure 16.31](image_url)

ii.) And because the brushes will have each switched their sliding contacts to the opposite sheathes, the current in the coil will reverse.

iii.) With the reversed flow, the coil's "side A" will become a north pole. That pole will be repulsed by the left bar magnet's north pole. At the same time, the coil's "side C" will become a south pole. That pole will be repulsed by the right bar magnet's south pole. These repulsions will motivate the coil to continue to accelerate counterclockwise. In other words, the coil's motion will continue.
g.) This device is called a DC motor.

i.) The part of the device that rotates (i.e., the coil and half-sheaths) is called the armature.

ii.) The part of the device that stays stationary (i.e., the brushes) is called the stator.

Note/Observation: All motors, whether they be DC or AC driven, have three non-negotiable requirements. There must be current flowing in a coil (this will produce one magnetic field), there must be a secondary magnetic field, and one of the two magnetic fields must alternate while the other stays stationary. (In the design shown above, the coil's magnetic field alternates while the field of each bar magnet stays fixed.)
QUESTIONS & PROBLEMS

16.1) What is the symbol for a magnetic field? What are its units? Also, what are magnetic fields, really?

16.2) What are magnetic forces? That is, how do magnetic forces act; what do they act on; what, in general, do they do?

16.3) Give two ways you can tell if a magnetic field exists in a region of space.

16.4) The direction of an electric field line is defined as the direction a positive test charge would accelerate if put in the field at the point of interest. How are magnetic field lines defined?

16.5) What does the magnitude of a magnetic field tell you?

16.6) What kind of forces do magnetic fields produce?

16.7) You put a stationary positive charge in a magnetic field whose direction is upwards toward the top of the page. Ignoring gravity:
   a.) What will the charge do when released?
   b.) How would the answer to Part 4a change if the charge had been negative?
   c.) In what direction would the charge have to move to feel a magnetically produced force into the page? If allowed to move freely, would the charge continue to feel that force into the page?
   d.) How would the answer to Part 4c change if the charge had been negative?
   e.) The positive charge is given an initial velocity of 2 m/s directed upward toward the top of the page. How will its velocity change with time?

16.8) Six particles with the same mass move through a magnetic field directed into the page (Figure II).
   a.) Identify the positively charged, negatively charged, and electrically neutral masses. (Hint: How would you expect a positively charged particle to move when traveling through a B-field directed into the page?)
b.) Assuming all the particles have the same charge-magnitude, which one is moving the fastest? (Hint: For a fixed charge, how is charge velocity and radius of motion related? Think!)

c.) Assuming all the particles have the same velocity, which one has the greatest charge? (Same hint as above, but reversed.)

16.9) A positive charge \( q = 4 \times 10^{-9} \) coulombs and mass \( m = 5 \times 10^{-16} \) kilograms accelerates from rest through a potential difference of \( V_0 = 2000 \) volts. Once accelerated, it enters a known magnetic field whose magnitude is \( B = 1.8 \) teslas.

a.) On the sketch in Figure IV, draw in an approximate representation of the charge's path.

b.) We would like to know the velocity of the charge just as it enters the B-field. Use conservation of energy and your knowledge about the electrical potentials to determine the charge's velocity at the end of the acceleration (yes, this is a review-type question).

c.) Determine the particle's radius of motion once in the B-field.

16.10) In what direction is the magnetic field associated with a wire whose current is coming out of the page?

16.11) You have two current-carrying wires, one on the left and one on the right, positioned perpendicularly to the page. The magnitude of the current in each is the same. You are told that the current flow in the wire on the left is into the page. If you are additionally told that there is no place between the wires where the magnetic field is zero, in what direction is the current in the wire on the right?

16.12) You have two current-carrying wires in the plane of the page. The magnitude of the current in the upper wire is twice the magnitude of the current in the lower wire. Do a quick sketch of the magnetic field between the wires if:

a.) Both currents are to the right.

b.) The top current is to the right while the bottom current is to the left.
16.13) Three wires with different currents as shown are perpendicular to the page as depicted in the sketch. In what direction is the magnetic field at the center of the triangle?

16.14) A group of current-carrying wires is shown to the right. The current is the same in each wire and the direction of the magnetic field is shown at various places in the configuration. From what you have been told, identify the direction of each wire's current.

16.15) Two parallel wires have equal currents passing through them. The currents are toward the left. The top wire's current produces a magnetic field which, impinging upon the current-carrying bottom wire, produces a force on the bottom wire. The bottom wire produces a similar force on the top wire.
   a.) Draw on the sketch the direction of both forces.
   b.) Are the two forces alluded to in Part 15a N.T.L. force couples? Explain.
   c.) If you doubled the distance between the wires, how would the force change?

16.16) A negative charge passes through a magnetic field. It follows the path shown in the sketch.
   a.) In what direction is the field?
   b.) In what direction would a positive charge take in the field?
   c.) If the size of the magnetic field had been doubled, how would the radius of the motion have been changed?
   d.) If the magnitude of the velocity had been doubled, how would the radius of the motion have been changed?

16.17) Two charges move through a given magnetic field as shown.
   a.) If we assume the velocities and masses are the same, which charge must be larger?
   b.) If we assume the charges and masses are the same,
which charge must have the larger velocity?

c.) If the magnetic field is oriented out of the page, what is the sign of each charge (i.e., positive or negative)?

16.18) An electric field $E$ is oriented toward the bottom of the page. In the same space is a magnetic field $B$. A negative charge passes straight through the region moving in the $+x$ direction. As a consequence of both fields, the negative charge moves through the region without changing its direction of motion. Ignoring gravity:

a.) What is the direction of the magnetic force in this case?

b.) What is the direction of the magnetic field in this case?

16.19) Galvanometers are based on what principle?

16.20) An ammeter can be built using a galvanometer and what kind of circuit? How do you determine the value for any extra resistors used in the circuit (i.e., extra beyond the resistance of the galvanometer)?

16.21) A voltmeter can be built using a galvanometer and what kind of circuit? How do you determine the value for any extra resistor(s) used in the circuit (i.e., extra beyond the resistance of the galvanometer)?

16.22) How can one piece of iron be magnetized while a second piece is not?

16.23) What does the earth's magnetic field really look like, and why?

16.24) Magnet $A$ is a light, weak, bar magnet. Magnet $C$ is a heavy, strong, bar magnet. You place magnet $A$ on a table so that it can move freely.

a.) If you pick up magnet $C$ and approach magnet $A$ so that $C$'s north pole comes close to $A$'s south pole, what will happen and why?

b.) If you pick up magnet $C$ and approach magnet $A$ so that $C$'s south pole comes close to $A$'s south pole, what would you expect to happen and why?
c.) If you said the magnets would repulse one another for Part 24b, you could be wrong. In fact, there is a good chance that if you actually tried this, the two magnets would attract. THIS DOESN'T MEANS LIKE POLES ATTRACT! What does it mean?

16.25) A wire carries 8 amps. The earth's magnetic field is approximately 6x10^{-5} teslas.

a.) How far from the wire will the earth's magnetic field and the wire's magnetic field exactly cancel one another?

b.) How must the wire be oriented (i.e., north/south, or south-east/north-west, or what?) to effect the situation outlined in Part a? (Assume there is no "dip" in the earth's B-field)?

16.26) The Hall Effect was an experiment designed to determine the kind of charge that flows through circuits (electrons were suspected but there was no proof). The device is shown in Figure VI. It consists of a battery attached to a broad, thin plate that is bathed in a constant magnetic field. Using the device, how might you determine the kind of charge carriers that move in electrical circuits?

Step #1: Assume electrons flow in the circuit. What path, on the average, will those negative charges take as they pass through the plate in the magnetic field? Which side of the plate will be the high voltage side?

Step #2: Do the same exercise as suggested in Step #1 assuming positive charge flow.

Culmination: If you didn't know whether the situation depicted in Step 1 or Step 2 was the real situation, how could the use of a voltmeter help?

16.27) Assuming the resistance of a galvanometer is 12 ohms, draw the circuit design for and determine all pertinent data required to build:

a.) A 300 volt voltmeter;

b.) A .25 amp ammeter.
16.28.) What do you suppose will happen when AC is piped through the coil of the device shown in sketch a, and through the circuitry shown in the sketch b?

16.29) What is the difference between a motor and a generator?