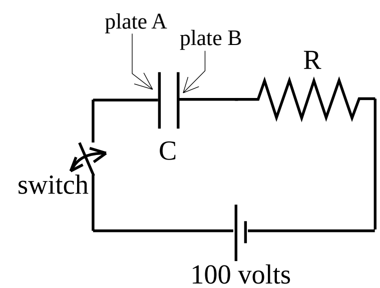
CHAPTER 14 -- CAPACITORS

QUESTION & PROBLEM SOLUTIONS



14.1) You have a power supply whose low voltage "ground" terminal is attached to a resistor whose resistance is $R = 10^4$ *ohms*. The resistor is attached to a plate (we'll call it *plate B*) which is next to, but not connected to, a second plate (we'll call it *plate A*). Reiterating, THERE IS NO CONNECTION between *plate A* and *plate B*. There is, additionally, no initial charge on either of the plates. Attached to *Plate A* is a switch. On the other side of the switch is the high voltage "hot" terminal of the power supply. A sketch of the situation is shown. At $t = 0$, the switch is closed.

a.) Current initially flows between the high voltage terminal and *Plate A*. Why? That is, what's going on?

Solution: There is a voltage difference between the high voltage terminal of the power supply and the chargeless plate. As such, an electric field will set itself up between the two points and current will flow.

b.) Current initially flows from *Plate B* through the resistor, and back to the ground of the power supply. Why? That is, what's going on?

Solution: The electric field between the plates forces charge carriers off *Plate B*. These move through the resistor and back to the ground side of the power supply. The net effect is that current appears to pass through the parallel plate device as though the plates were connected.

c.) What is the two-plate device called?

Solution: This device is called a *capacitor*.

d.) After a while, there is a voltage $V = 10$ *volts* across the plates. At that point in time, there is 10^{-10} coulombs of charge on *plate A*. The ratio of the *charge to voltage* is 10^{-9}.

i.) How much charge is on the *plate B*?

Solution: The charge on *plate B* will be equal and opposite the charge on *plate A*, or -10^{-10} *coulombs*.

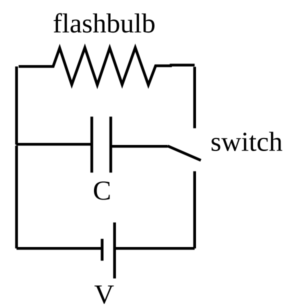
ii.) What is this ratio called?

Solution: This *charge per volt* ratio is called *the capacitance* of the capacitor. It is the constant that identifies how large a capacitor is. (Note: You should be noticing a pattern here. The voltage V_R across a resistor is proportional to the current i through the resistor with the proportionality constant being the resistance R of the resistor. That is, $V_R = iR$. By the same token, the charge q on one plate of a capacitor is proportional to the voltage V_c across the capacitor with the proportionality constant being the capacitance C of the capacitor. That is, $q = CV_c$.)

iii.) At some later point in time, the voltage across the plates is doubled. What is the ratio of *charge to voltage* in that case? Explain.

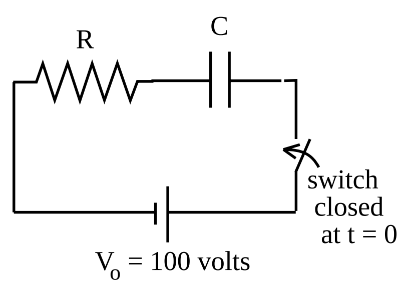
Solution: If the voltage doubles, the charge on one plate will double and the ratio will stay the same.

14.2) What do capacitors (often referred to as *caps*) generally do in DC circuits? Give an example.



Solution: Capacitors store energy in the form of an electric field between the plates. The best example I can think of is a flashbulb circuit (a simple version of a flash circuit is shown to the right). When the flash in a camera is turned on, a switch puts the capacitor in series with a battery allowing the capacitor to charge up. When the photo is taken, the switch flips up putting the charged capacitor in series with the flash (this is shown as a resistor in the circuit). The capacitor discharges through the bulb motivating it to flash. Once discharged, the switch flips back to charging mode and the capacitor recharges (this is why it usually takes a few seconds before you can take the next flash picture).

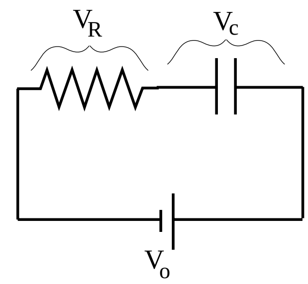
14.3) A 10^{-6} *farad* capacitor is in series with a 10^4 *ohm* resistor, a battery whose voltage is $V_o = 100$ *volts*, and a switch. Assume the capacitor is initially uncharged and the switch is thrown at $t = 0$.



a.) The capacitance value tells you something that is always true no matter what the voltage across the capacitor happens to be. What does it tell you?

Solution: Capacitance tells you how many *coulombs per volt* the plates can accommodate. Unless you change the physical characteristics of the system, this will be a constant for a given capacitor.

b.) What is the initial current in the circuit?



Solution: Notice that at any point in time, the voltage across the battery must equal the voltage V_c across the capacitor added to the voltage V_R across the resistor. Initially, though, the voltage across the capacitor is zero (there is no charge yet on its plates). That means the voltage across the power supply will initially equal the voltage across the resistor. According to Ohm's Law, this can be written as $V_o = i_o R$, or $i_o = V_o/R = (100\ volts)/(10^4\ \Omega) = .01\ amps$.

c.) What is the circuit's current after a long period of time?

Solution: After a long period of time, the accumulated charge on the capacitor's plates will produce a voltage across the capacitor that is equal to the voltage across the power supply. At that point, there will no longer be current in the circuit. (Interesting observation: No current through the resistor means no voltage drop across the resistor as $V_R = iR$ . . . it fits!)

d.) How much charge will the capacitor hold when fully charged?

Solution: The relationship between the charge q on the capacitor at any time and the voltage V_c across the capacitor at that time is $q = CV$. When the capacitor is fully charged, the voltage across the capacitor will equal the voltage across the power supply, and we can write $q = (10^{-6}\ f)(100\ volts) = 10^{-4}\ coulombs$.

e.) How much energy is wrapped up in the capacitor when fully charged?

Solution: The energy wrapped up in a capacitor is equal to $.5CV^2 = .5(10^{-6}\ f)(100\ volts)^2 = .005\ joules$.

f.) Where is the energy stored in the capacitor?

Solution: Energy in a capacitor is stored in the electric field found between the capacitor's charged plates.

g.) You are told that the time constant for the system is 10^{-2} seconds.

i.) What does that tell you about the system?

Solution: The *time constant* gives you a feel for how fast the cap in the capacitor/resistor combination will charge or discharge. Specifically, one *time constant* is the amount of time required for the capacitor to charge up to .63 of its maximum charge (that's 63%) or dump 63% of its charge through the resistor. Two time constants give you 87% charge or discharge, and three time constants gives you 95%.

ii.) How much charge will be associated with the capacitor after a time equal to one time constant?

Solution: The maximum charge on the capacitor will be $10^{-4}\ coulombs$. Sixty-three percent of that is $.63x10^{-4}\ coulombs$.

iii.) Where will the charge alluded to in *Part g-ii* be found?

Solution: The "charge on a capacitor" is, in fact, the amount of charge on *one plate* of the capacitor. The sum of the charge on BOTH plates is *zero*.

h.) After a very long time, the switch is opened. What happens to the capacitor? Will it hold its charge forever?

Solution: Opening the switch disconnects the capacitor from the battery. There will be a trickle of charge flow through the capacitor (the resistance of the insulator is not infinite--there will be some ir action internal to the capacitor with a very large r and a very small i). With time, in other words, the capacitor will lose its charge.

i.) At $t = 1$ *second*, the current is i_1. At $t = 2$ *seconds*, the current is i_2. At $t = 4$ *seconds*, the current is i_4, and at $t = 8$ *seconds*, the current is i_8. Is i_2/i_1 going to give you the same ratio as i_8/i_4?

Solution: Current in a charging circuit follows an exponential function. Its form is $i_1 = i_o e^{-t/RC}$. If time doubles, the current becomes $i_2 = i_o e^{-2t/RC}$. The ratio of $i_2/i_1 = i_o e^{-2t/RC}/i_o e^{-t/RC} = e^{-t/RC} = 1/e^{t/RC}$. From this, it becomes evident that the ratio depends upon the *time* that is being doubled. As t gets bigger, the ratio gets smaller.

14.4) Can you have capacitance if you have only one plate?

Solution: This is obscure, but the answer is *yes.* By putting charge on a plate, you give it a voltage. If you take infinity to be the zero voltage point (i.e., the point where the electric field due to the plate's charge goes to zero), the voltage difference between the charged plate and the fictitious plate at infinity will simply be the voltage of the charged plate. Capacitance is defined as the ratio between the charge on one capacitor plate and the voltage difference between the plates. That is non-zero here, so the single plate does have capacitance. (Note: You will never have to use this--it is something university courses sometimes like to throw in, just for the sake of confusion.)

14.5) You have a series combination of capacitors.

a.) What happens to the equivalent capacitance when you add another capacitor?

Solution: Capacitor combinations are the reverse of resistor combinations. That is, *parallel* resistor combinations (i.e., $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$) have the same equivalence form as *series* capacitor combinations (i.e., $1/C_{equ} = 1/C_1 + 1/C_2 + \ldots$). As such, adding a capacitor to a series circuit will *decrease* the equivalent capacitance (just as adding a resistor to a parallel circuit decreases the equivalent resistance of that type of circuit).

b.) What is common to all the capacitors in the series combination?

Solution: Not only will the current through each capacitor be the same at a given point in time, the charge on each capacitor will also be the same at that time. This makes sense if you think about how charge passes from plate to plate. As charge accumulates on the first plate, it electrostatically repulses an equal amount of like charge off its other plate. Where does that removed charge go? It accumulates on the next plate down the line, repeating the process for each successive capacitor.

14.6) You have a parallel combination of capacitors.

a.) What happens to the equivalent capacitance when you add another capacitor?

Solution: Again, capacitor combinations are the reverse of resistor combinations. Just as a *series* resistor combination (i.e., $R_{eq} = R_1 + R_2 + \ldots$) increases its equivalent resistance when a resistor is added, a *parallel* capacitance combination (i.e., $C_{equ} = C_1 + C_2 + \ldots$) increases its equivalent capacitance when a capacitor is added.

b.) What is common to all the capacitors in the parallel combination?

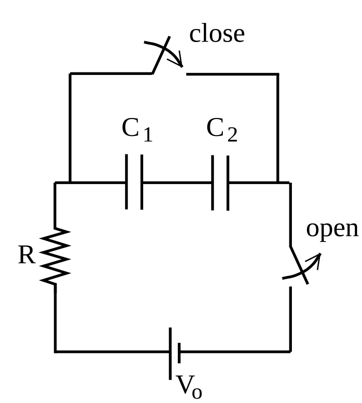
Solution: What is common to *all* parallel-type circuits is voltage. That is, each capacitor in a parallel combination will have the same voltage across its plates (this assumes there is only one capacitor per parallel branch--if there are multiple capacitors in a branch, the common voltage will be across the entire branch).

14.7) You charge up two single capacitors that are in parallel. You disconnect the battery. What happens to the current in the system when you do this?

Solution: If the capacitors are in parallel, the voltage across each will be the same. If that is the case, there will be no voltage difference between the high voltage sides of the caps, and no new current will flow.

14.8) You charge up two unequal capacitors that are in series. You disconnect the battery by opening S_1, then reconnect the two capacitors by closing S_2.

when switches are thrown



a.) What happens to the current in the system when you do this?

Solution: What is common in series combinations of capacitors is the *charge* on each capacitor. When the charged capacitors are reconnected, assuming the capacitances are different, there will be a voltage difference between the caps. Current will flow until that voltage difference disappears.

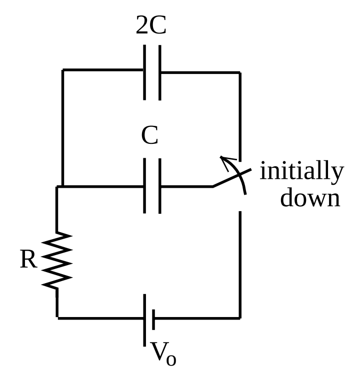
b.) Out of curiosity, why was the resistor included in the circuit?

Solution: There is always some resistance in a circuit. When you are dealing with a capacitor circuit, the resistance works with the capacitance to govern the rate at which the capacitor charges up. In other words, in this problem, the resistance information won't be used. To be kosher, though, a resistor needs to be included in all capacitor circuits.

c.) What kind of circuit do you have after both switches are thrown? That is, what kind of relationship will exist between the capacitors after the throw?

Solution: Though it isn't immediately evident, charge will flow until the voltage across each cap is the same. If you think about it, this is characteristic of a parallel combination. This would have been more evident if C_2 had been located right next to the upper switch. Nevertheless, that's the case.

14.9) You use a battery whose voltage is V_o to charge up a capacitor C. When fully charged, there is q*'s* worth of charge on the cap. You then disconnect the capacitor from the battery and reconnect it to a second uncharged capacitor whose capacitance is *2C* (in the sketch, this disconnection, then reconnection is done with the switch). After the switch is thrown:



a.) What is the voltage across the second capacitor?

Solution: Before charge redistributes itself, the voltage across the 2C capacior will be zero (remember, $V_{cap} = q/C$--no q, no V_c). Once charge has redistributed and the battery has also provided some current, both caps will have voltage V_o across them.

b.) How will the charge redistribute itself? That is, how much charge ends up on the second capacitor?

Solution: If capacitance C tells you how much *charge per volt* the cap can hold, a capacitor that is twice as large (*2C*) will hold twice the charge. If the first cap gets Q*'s* worth of charge, the second cap will get $2Q$*'s* worth of charge. With the total being $3Q$*'s* worth of charge, the first cap will get *1* part of the $3Q$ and the second cap will get *2* parts of the $3Q$. In short, C_2 gets two-thirds of the original charge on C_1.

14.10) You charge up a parallel plate capacitor that has air between its plates. Once charged, you *disconnect it* from the battery, then insert a piece of plastic (an insulator) between the plates. The amount of charge on the capacitor does not change (being disconnected from the circuit, it has no place to go), but the voltage across the capacitor does change.

a.) What is the insulator usually called in these situations?
Solution: In such cases, the insulator is called *a dielectric*.

b.) How and why does the voltage change (up, down, what?)?
Solution: In the presence of the capacitor's charged plates, each of the insulator's plate-facing surfaces will take on the appearance of being charged. Electrons in the insulator will stay in their orbitals (remember, valence electrons in insulators can't wander about the way valence electrons in metallically bonded structures can), but the plate charge will motivate them to spend most of their time close to the positive plate. The consequence of this polarization (it is a Van der Waal phenomenon) is that the polarized charge will set up a weak, reverse electric field through the insulator and between the plates. That field will subtract from the electric field set up by the capacitor's plates. With an effectively diminished electric field across its plates, the plate voltage *decreases*.

c.) What happens to the capacitance of the capacitor?
Solution: Capacitance is the ratio of *charge on one plate* to *the voltage across the plates*. The charge on one plate hasn't changed, but the voltage has gone down, so the new capacitance gets larger ($C = q/V$ . . . when V gets smaller, C gets larger).

d.) What happens to the energy content of the capacitor? If it goes up, from whence did the new energy come? If it goes down, where did the energy go?

Solution: Energy in a capacitor is calculated using $.5CV^2$. In this case, the voltage has decreased while the capacitance has gone up. As voltage is squared in the relationship, though, it looks like the energy content has diminished (don't believe me? $C = q/V$, so $.5CV^2 = .5(q/V)V^2 = .5qV$ . . . q is constant here while V decreases--the energy goes down). Where does the energy go? When the dielectric is inserted, it is actually pulled into the region between the plates. The work done to do that pulling is where the energy goes.

14.11) You have a parallel plate capacitor with air between its plates hooked up to a power supply whose voltage is V_o. *Without disconnecting the battery*, you carefully insert a piece of plastic between the plates.

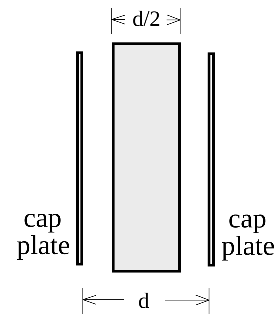
a.) What happens to the voltage across the capacitor?
Solution: This is an interesting question. When you slip the insulator between the capacitor's plates, you will get an induced charge polarization on the insulator's surfaces. That will drop the effective voltage across the plates which, in turn, will create a voltage difference between the high voltage plate of the capacitor and the high voltage plate of the power supply. Remembering that current ceases in capacitor circuits only when the voltage across the capacitor is the same as the voltage across the power supply, this voltage difference creates an electric field in the wire that motivates current to flow. The short-lived current puts more charge on the plates thereby bringing the plate voltage back up to V_o. In short, the voltage across the capacitor won't change at all.

b.) What happens to the capacitor's capacitance?

Solution: From above, we've decided that the plate voltage stays the same but the *charge on each plate* increases. That means the capacitance ratio q/V increases. Note that this is just what you would expect. Whenever a dielectric is placed between the plates of a capacitor, the capacitance always go up.

c.) What happens to the charge on the capacitor's plates?

Solution: As was pointed out in *Part 11a*, the charge on the plates increases.

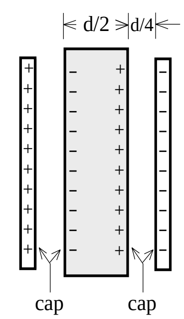


14.12) Between the plates of one air-filled capacitor, you insert a dielectric whose dielectric constant is k and whose thickness is half the plate separation. Between the plates of a second cap, you insert a piece of metal whose thickness is also half the plate separation. (Both situations look like the sketch.) After some nasty Calculus, the capacitance expression for the dielectric situation is found to be

$$C = \left(2\varepsilon_o \frac{A}{d}\right)\left(\frac{2k}{1+k}\right).$$

a.) Which modified capacitor will end up with the greater capacitance? Justify.

Solution: We have already established that putting a dielectric between the plates of a capacitor will increase the capacitance. We don't know by how much in this case because we don't know the dielectric constant. Nevertheless, if the capacitance goes down with the insertion of a conductor between the plates, we are done In short, what we need to determine before doing anything else is what a conductor will do to the capacitance when placed between the plates.



The most conceptually elegant way to do this is to think about how a conductor will act when charged capacitor plates are placed on either side of it. In that case, the conductor's electrons will attract to the capacitor's positive plate (the left plate in this case) *in the amount equal to the charge on the capacitor's positive plate*. In doing so, the other side of the condutor will become electrically positive (see sketch). In other words, you will end up with what looks like two equal sized capacitors in series with one another.

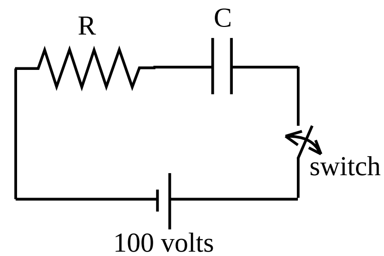
As you know, the equivalent capacitance of a series combination of capacitors is always *smaller* than the smallest capacitor in the combination. In fact, with all else held constant, the equivalent capacitance of two *equal* capacitors in series will be $C/2$. The problem is that we have done more than simply make one cap into two. We have also cut down on the *gap* between the capacitor plates. In fact, each plate is now $d/4$ units apart. According to the capacitance relationship for a parallel plate cap, this should increase the capacitance of each cap by a factor of 4 (remember, $C = \varepsilon_o A/d$). The net net change of these two alterations means the new metal-plate capacitance is $2\varepsilon_o A/d$.

Mathematically comparing the given capacitance of the dielectric-filled capacitor ($2[\varepsilon_o A/d][2k/(1+k)]$) and the calcuated capacitance of the metal filled capacitor ($2[\varepsilon_o A/d]$), we can see that the two capacitances will be equal if $k = 1$ (i.e., if the dielectric is *air*) and the dielectric-filled cap will have a greater capacitance for all other values of k.

b.) What is the ratio of the two capacitances?

Solution: Again, it depends on how big the dielectric constant is., but you know a factor of $2k/(1+k)$ is going to be in there somewhere.

14.13) You have a capacitor in series with a switch, a resistor, and a power supply. At $t = 0$, you throw the switch and current begins to flow.



a.) For the amusement of it, draw the circuit.

Solution: See sketch.

b.) If the capacitor had been half as big, how would the current have flowed? That is, would the cap have charged faster or slower? Justify your response.

Solution: The time constant RC tells you how fast the cap in a cap/resistor circuit will charge. If the capacitance is halved, the time constant will halve and the capacitor will take *less time* to charge to 63%. In other words, the current will be greater with the smaller cap.

14.14) Assuming there is no charge initially on any capacitor, answer all the following questions for the capacitor circuit in *sketch a*. When done, repeat the process for the circuit shown in *sketch b*:

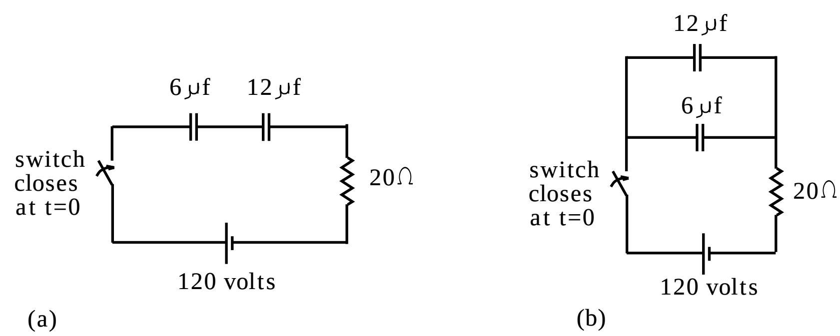


FIGURE I

Note: Initially, an uncharged capacitor will allow current to flow through it as though it has no *resistance to charge flow* at all (i.e., it will act like a short-circuit). As time progresses and the capacitor charges, current through the cap decreases as it becomes more and more difficult to force still more charge onto its plates. After a long enough time, current will cease completely and the totally charged capacitor will act like a break in the circuit (i.e., an open-switch circuit). We will do the entire problem for *Circuit a* first, then do the problem for *Circuit b*.

Circuit a finds two capacitors in series. Series elements have common currents. For capacitors, that means the magnitude of the *charge accumulated on each capacitor plate* will be the same for all caps in the series combination. It also means that for different size capacitors, the *voltage* across each capacitor will be different (remember, $V_1 = Q/C_1$).

a.) Determine the initial current in the circuit when the switch is first thrown.

Solution: The initial current through the circuit will be that of a resistor in series with a battery (the uncharged caps will act like "shorts"), or:

$$i = V_R/R$$
$$= (120 \text{ volts})/(20\ \Omega)$$
$$= 6 \text{ amps.}$$

b.) A long time after the switch is thrown (i.e., by the time the caps are charged up fully), how much charge is there on each plate?

Solution: To begin with, when the capacitors are totally charged, there will be *no current* through the circuit (the charged capacitors will act as open circuits). That means the entire 120 volt *voltage drop* will be across the capacitor combination (none across the resistor as $i = 0$ . . . remember, the voltage across a resistor is iR).

The charge on each individual capacitor will be the same as the charge on the circuit's *equivalent capacitor*. The equivalent capacitance for a series combination is such that:

$$1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$$
$$C_{eq} = [1/(6\text{x}10^{-6} \text{ f}) + 1/(12\text{x}10^{-6} \text{ f})]^{-1}$$
$$= 4\text{x}10^{-6} \text{ farads.}$$

From $C = Q/V$ we get:

$$Q = CV,$$

where C is the capacitance of the capacitor in question, Q is the charge on one of the capacitor plates, and V is the voltage across the cap. With $C = C_{eq}$ and $V =$ *120 volts*, we get:

$$Q = C_{eq}V_o$$
$$= (4 \text{ x}10^{-6} \text{ f})\ (120 \text{ volts})$$
$$= 4.8 \text{ x}10^{-4} \text{ coulombs.}$$

Each capacitor will hold $4.8\text{x}10^{-4}$ coulombs per plate when fully charged.

c.) What is the voltage across the 6 μf capacitor when fully charged?

Solution: Knowing the charge on the 6 μf cap, we can use $C = Q/V$ to determine the voltage across the cap:

$$V_6 = Q/C_6$$
$$= (4.8 \text{ x}10^{-4} \text{ C})/(6 \text{ x}10^{-6} \text{ f})$$
$$= 80 \text{ volts.}$$

Note: As the total battery charge is 120 volts, that means the other capacitor has 40 volts across it when fully charged.

d.) How much energy does the 6 μf capacitor hold when completely charged?

Solution: The energy wrapped up in a charged capacitor equals:

$$\text{Energy} = (1/2)CV^2,$$

where C is the cap's capacitance and V is the voltage across the cap. Using this yields:

$$\begin{aligned}\text{Energy} &= .5(6 \text{ x}10^{-6} \text{ f})(80 \text{ volts})^2 \\ &= .0192 \text{ joules.}\end{aligned}$$

e.) Determine the RC circuit's *time constant*. What does this information tell you?

Solution: The RC time constant (t) tells you the amount of time required for the capacitor in the circuit to charge up to 63% of its total charge. *Two time constants* is the time it takes to charge up to 87%, and *three time constants* is the time it takes to charge to 95% of its maximum charge. (By the same token, a charged system will dump 63% of its charge in a time interval equal to *one time constant*, 87% in a time interval equal to *two time constants*, and 95% in a time interval equal to *three time constants*).

The relationship between the *time constant*, the *net resistance* in the circuit, and the *net capacitance* of the circuit is:

$$\begin{aligned}\tau &= RC_{eq} \\ &= (20\ \Omega)(4\text{x}10^{-6} \text{ f}) \\ &= 8\text{x}10^{-5} \text{ seconds.}\end{aligned}$$

f.) How much charge is there on the 6 μf capacitor after a time interval equal to one time constant passes?

Solution: As stated above, 63% of the charge will be lost in the first time constant. That means 37% will be left. As such, we can write:

$$.37(4.8 \text{ x}10^{-4} \text{ coul}) = 1.8 \text{ x}10^{-4} \text{ coulombs}$$

will be left.

Circuit b finds two capacitors in parallel. Parallel elements have *voltages* in common. For different size capacitors, that means the amount of charge on each cap will be different (remember, $Q_1 = C_1V$).

a.) Determine the initial current in the circuit when the switch is first thrown.

Solution: As before, the caps will act like shorts when uncharged. Current must go through the resistor to return to the battery, so the initial current again will be governed by the size of the *resistor* in the circuit with the entire *voltage drop* occurring across that element:

$$\begin{aligned} i &= V_R/R \\ &= (120 \text{ volts})/(20\ \Omega) \\ &= 6 \text{ amps.} \end{aligned}$$

b.) A long time after the switch is thrown (i.e., by the time the caps are charged up fully), how much charge is there on each plate?

Solution: When the capacitors are totally charged, there will be *no current* through the circuit (the charged capacitors will act as open circuits). That means the ENTIRE 120 volt *voltage drop* will be across EACH parallel capacitor.

For the 6 μf cap:

$$\begin{aligned} Q_6 &= C_6V_o \\ &= (6 \text{ x}10^{-6} \text{ f})(120 \text{ volts}) \\ &= 7.2 \text{ x}10^{-4} \text{ coulombs.} \end{aligned}$$

For the 12 μf cap:

$$\begin{aligned} Q_{12} &= C_{12}V_o \\ &= (12 \text{ x}10^{-6} \text{ f})(120 \text{ volts}) \\ &= 1.44 \text{ x}10^{-3} \text{ coulombs.} \end{aligned}$$

c.) What is the voltage across the 6 μf capacitor when fully charged?

Solution: When fully charged, the maximum voltage across the 6 μf cap will be $V_{max} = 120$ *volts* (as stated above).

d.) How much energy does the 6 μf capacitor hold when completely charged?

Solution: The energy wrapped up in the 6 μf cap when fully charged equals:

$$\begin{aligned} \text{Energy} &= .5(6 \text{ x}10^{-6} \text{ f})(120 \text{ volts})^2 \\ &= .0432 \text{ joules.} \end{aligned}$$

e.) Determine the RC circuit's *time constant*. What does this information tell you?

Solution: The previous *Part e* explained what the time constant means. Determining it for this circuit requires that we determine the equivalent capacitance for the circuit. For parallel combinations of capacitors, we just add the capacitances. That means:

$$\begin{aligned} C_{eq} &= (6 \text{ x}10^{-6} \text{ f}) + (12 \text{ x}10^{-6} \text{ f}) \\ &= 18 \text{ x}10^{-6} \text{ farads.} \end{aligned}$$

Knowing that:

$$\tau = RC_{eq}$$
$$= (20\ \Omega)(18\ x10^{-6}\ f)$$
$$= 3.6\ x10^{-4}\ \text{seconds.}$$

f.) How much charge is there on the 6 μf capacitor after a time interval equal to one time constant passes?

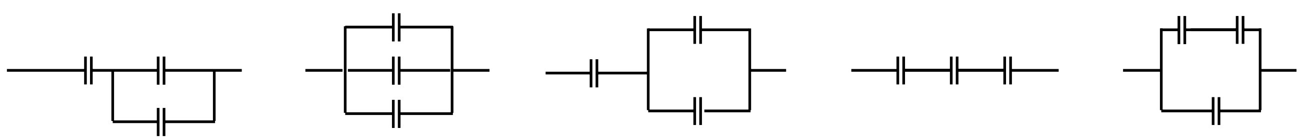
Solution: As stated above, 63% of the charge will be lost in the first time constant. That means 37% will be left. As such, we can write:

$$.37(7.2\ x10^{-4}\ C) = 2.7\ x10^{-4}\ \text{coulombs}$$

will be left.

14.15) Three identical capacitors are connected in several ways as shown in Figure II.

a.) Order the combinations from the smallest *equivalent capacitance* to the



(a) (b) (c) (d) (e)

FIGURE II

largest.

Solution: By inspection, the equivalent capacitance for each combination is: (a.) (2/3)C; (b.) 3C; (c.) (2/3)C (this configuration is exactly the same as *Part a*--only the sketch has been rendered differently); (d.) C/3; (e.) (3/2)C. That means the order should be: *d, a* and *c, e,* and *b*.

b.) Which combination has the potential of storing the most energy?

Solution: The energy content of a capacitor combination is such that:

$$(1/2)C_{eq}V^2,$$

where C_{eq} is the equivalent capacitance of the capacitor combination in question. For a given voltage, that means the most energy-storing capacity will go to the combination with the largest equivalent capacitance. That will be the pure, parallel capacitor combination in the sketch.

14.16) A *parallel plate capacitor* is connected to a 20 volt power supply. Once charged to its maximum possible Q, the capacitor's plates are separated by a factor of four (that is, the distance between the plates is quadrupled) while the capacitor is *kept hooked to the power supply*. As a consequence of this change in geometry:

a.) How will the capacitor's *capacitance change*?

Solution: The capacitance of a single parallel plate capacitor is:

$$C = \varepsilon_o A/d,$$

where A is the area of one plate, d is the distance between the plates, and ε_o is the *permittivity of free space* and is equal to $4\pi x 10^{-7}$ farads per meter.Neither ε_o nor A is changing in this situation, whereas d gets larger by a factor of 4. That means, according to $C = \varepsilon_o A/d$, that the capacitance should diminish by a factor of 4, giving us $(1/4)C_{original}$.

b.) How will the *charge* on the capacitor *change*?

Solution: The charge on the capacitor is related to the voltage across the capacitor and the size of the capacitor by the relationship:

$$C = Q/V \quad \text{or} \quad Q = CV.$$

V hasn't changed but C is now a quarter of its original value. That means Q_{new} must equal $(1/4)Q_{old}$.

c.) How will the *energy stored* in the capacitor *change*?

Solution: The energy equals:

$$\text{Energy} = (1/2)CV^2.$$

This implies that the energy in the cap will decrease by a factor of 4 also.

d.) If a dielectric ($k_d = 1.6$) had been placed between the plates of the *original setup*, what would the *new capacitance* have been?

Solution: Knowing the dielectric constant allows us to determine the new capacitance knowing the old capacitance. That is:

$$C_{new} = k_d C_{old}.$$

That means $C_{new} = 1.6\ C_{old}$.

14.17) Determine:

a.) The equivalent capacitance of the circuit shown in Figure III.

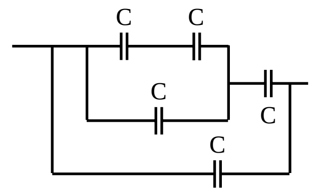
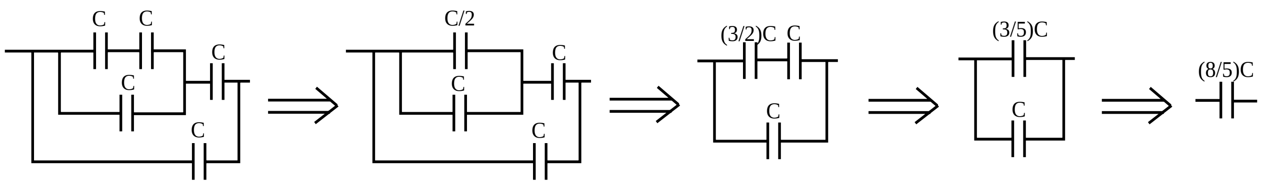


FIGURE III

Solution: The circuit evaluation to determine C_{eq} is shown below (remember that the equivalent capacitance rules are the mirror image of equivalent resistance rules).



b.) Assuming each capacitor's capacitance is 25 mf, how much energy can this system store if it is hooked across a 120 volt battery?

Solution: Using the equivalent capacitance, we can write:

$$\begin{aligned} E &= (1/2)CV^2 \\ &= .5[(8/5)(25\text{x}10^{-3}\text{ f})](120\text{ v})^2 \\ &= 288\text{ joules.} \end{aligned}$$

14.18) The capacitors in the circuit shown in Figure IV are initially uncharged. At $t = 0$, the switch is closed. Knowing the resistor and capacitor values:

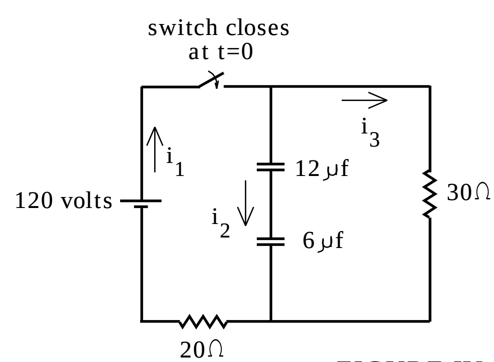


FIGURE IV

a.) Determine all three *initial* currents in the circuit (i.e., the currents just after the switch is closed).

Solution: This question is just down right tricky.

--When the switch is closed, the 30 Ω resistor is in parallel with the two series-connected capacitors. Being in parallel, the net voltage drop across the capacitors and across the resistor must be the same.

--Initially, the capacitors have no charge on them. That means the capacitors initially have no voltage drop across them.

--No initial voltage drop across the capacitors means no initial voltage drop across the 30 Ω resistor.

--With no initial voltage drop across the 30 Ω resistor, there will be no initial current through that resistor. As such, the initial current i_3 equals zero.

--Meanwhile, the capacitors initially act like open circuits (with no charge on them, there is nothing to motivate them to do otherwise), which means the initial current *will* flow freely through them and $i_1 = i_2$.

--That means the entire 120 volts from the battery is initially dropped across the 20Ω resistor, and we can write initial currents as:

$$i_1 = (120 \text{ v})/(20\ \Omega) = 6 \text{ amps;}$$
$$i_2 = i_1 = 6 \text{ amps; and}$$
$$i_3 = 0.$$

b.) Determine all three currents in the circuit after a long period of time (i.e., at the theoretical point $t = \infty$).

Solution: After a long period of time:

--The capacitors will be fully charged so that $i_2 = 0$;

--The currents i_1 and i_3 will be equal to one another.

--The full 120 volt drop will be across the 30 Ω and 20 Ω resistors in series. As such, the steady-state current will be:

$$i_1 = i_3 = (120 \text{ v})/(50\ \Omega) = 2.4 \text{ amps, and } i_2 = 0.$$

c.) Without solving them, write out the equations you would need to solve if you wanted to determine the currents in the circuit at any arbitrary point in time. Be sure you are complete.

Solution: Note first that this is also a bit tricky. Why? Because the fact that there are three unknown currents might lead you to believe that you need only three equations. The problem is that there are also *capacitors* and *unknown charge quantities* with which to deal. There are three loop equations possible, but only two are independent of one another. In short, we need two loop equations, one node equation, and one other equation. Starting with Kirchoff's Laws (presented in general algebraic terms first) and noting that the equivalent capacitance of the series combination of capacitors is 4 μf, we can write:

Node equation: $i_1 = i_2 + i_3.$

Left inner loop: $V_o - q/C_{eq} - i_1R_{20} = 0$

$$\Rightarrow \quad 120 - q/(4\text{x}10^{-6}) - 20i_1 = 0.$$

Right inner loop: $-i_3R_{30} + q/C_{eq} = 0$

$$\Rightarrow \quad -30i_3 + q/(4\text{x}10^{-6}) = 0.$$

Where does the last equation come from? The rate at which charge q is deposited on the capacitor's plates and the current dq/dt in that part of the circuit are the same, so we can write:

$$i_2 = dq/dt.$$

These are the four equations we need to determine i_1, i_2, *and* i_3.

Note: You could have used the outside loop instead of either of the two loops used. Doing so would have yielded the equation:

$$V_o - i_3R_{30} - i_1R_{20} = 0.$$

d.) Determine the *total charge* the 6 mf capacitor will accumulate (i.e., the amount of charge on its plates at $t = \infty$).

Solution: The charge on the 6 mf capacitor will be the same as the charge on the 12 mf capacitor (they are in series). By definition, this will also be the same as the charge on the equivalent capacitor.

When the capacitors are fully charged, the current through the circuit will be $i_1 = i_3 = 2.4$ *amps*, as calculated above. That means the voltage drop across the 20 Ω resistor will be:

$$\begin{aligned} V_{20} &= i_1R_{20} \\ &= (2.4 \text{ amps})(20\ \Omega) \\ &= 48 \text{ volts.} \end{aligned}$$

Adding voltage drops as we go, the voltage drop across the battery will equal the sum of the voltage drops across the two capacitors (or their single equivalent capacitor) and the 20 Ω resistor. At maximum, that is:

$$\begin{aligned} V_o &= V_{c,max} + V_{20} \\ \Rightarrow \quad 120 &= V_{c,max} + 48 \\ \Rightarrow \quad V_{c,max} &= 72 \text{ volts.} \end{aligned}$$

Knowing the voltage across the equivalent capacitor, we can use the definition of capacitance to determine the charge on that capacitor:

$$\begin{aligned} q_{max} &= C_{equ} \quad V_{c,max} \\ &= (4\text{x}10^{-6} \text{ farads})(72 \text{ volts}) \\ &= 2.88\text{x}10^{-4} \text{ coulombs.} \end{aligned}$$

e.) Once totally charged, how much energy do the capacitors hold?

Solution: The total energy on the capacitors when fully charged will be:

$$\begin{aligned}\text{Energy} &= (1/2)C_{equ}V_{c,max}^{2} \\ &= .5(4\text{x}10^{-6}\text{ farads})(72\text{ volts})^2 \\ &= 1.04\text{x}10^{-2}\text{ joules.}\end{aligned}$$

f.) After a very long time (i.e., long after the capacitors have fully charged), the switch is opened. How long will it take for the two capacitors to dump 87% of their charge across the 30 Ω resistor?

Solution: It will take the capacitor two *time constants* to dump 87% of its charge. As the 20 Ω resistor is out of the circuit when the switch is open:

$$\begin{aligned}t &= 2\tau \\ &= 2[RC_{equ}] \\ &= 2(30\ \Omega)(4\text{x}10^{-6}\text{ farads}) \\ &= 2.4\text{x}10^{-4}\text{ seconds.}\end{aligned}$$

.

14.19) An AC voltage source is found to produce a 12 volt peak to peak signal at 2500 hertz.

a.) Characterize this voltage as a *sine function.*

Solution: This is really a review question designed to remind you of the nuts and bolts of an AC circuit. For this situation, $V_o = 6$ *volts* (i.e., half the *peak to peak* voltage). The frequency is $\nu = 2500$ *hertz*, so the sine's argument, $2\pi\nu = 2\pi(2500\text{ hertz}) = 15700$, yields a time dependent voltage function of $V(t) = V_o \sin(2\pi\nu)t = 6 \sin 15700\, t$.

b.) Determine the *RMS voltage* of the source.

Solution: The *RMS* value will be .707 of the amplitude, or *.707(6) = 4.24 volts*. If you sketch this on the non-existent graph from *Part a*, it would look like a straight line positioned at 4.24 volts.

c.) It is found that when a *capacitor* and *resistor* are placed across the source as characterized above, an ammeter in the circuit reads 1.2 amps. What is the *maximum current* drawn from the source?

Solution: The maximum current will be the amplitude of the current function. AC ammeters read *RMS* values, so the maximum current through that branch will be $i_o = (1.2\text{ amps})/(.707) = 1.7$ *amps*.

14.20) An RC circuit is hooked across an AC power supply. Which of the following statements are true (there can be more than one)? Explain each response.

a.) The RMS voltage across the resistor is the same as the average voltage across the resistor.

Solution: The average voltage across a resistor in an AC circuit is *zero*. This statement is false.

b.) The RMS voltage across the resistor is equal to R times the RMS current through the resistor.

Solution: This, by Ohm's Law, is true.

c.) The RMS voltage across the resistor will be very large if the capacitive reactance is very large.

Solution: The capacitive reactance is a measure of the resistive nature of the capacitor. If the capacitive reactance is large, that means there is a lot of resistance to charge flow in the circuit and the RMS current will be small. The RMS voltage across the resistor reflects the RMS current through the resistor, so this situation will yield only a small RMS voltage across the resistor and this statement is false.

d.) The RMS current in the circuit will be very large if the capacitive reactance is very small.

Solution: As was suggested in *Part c*, a small capacitive reactance means that there is relatively little resistance to charge flow in the circuit (assuming the resistor itself isn't large). That suggests that the RMS current will be large, relatively speaking, when the capacitive reactance is small. This statement is true.

e.) A decrease in frequency will increase the voltage across the capacitor.

Solution: This is a little bit tricky. A capacitor is what is called a *high pass filter*. That is, it allows high frequency signals to pass through it while dampening out low frequency signals. In an RC circuit, the sum of the voltages across the resistor and capacitor at any instant have to equal the net voltage across the power supply. If, at low frequency, the current in the circuit is small (this is what we concluded above about lowering the frequency), little voltage drop will exist across the resistor. Why? Because the voltage across a resistor is *directly proportional to the current through the resistor*, and if the current is small, the voltage drop across the resistor has to be small. The only way this can happen, though, is if most of the voltage drop happens across the capacitor. The more the current drops (i.e., the more the frequency is lowered), the higher the voltage across the capacitor and this statement is true.

f.) An increase in the capacitance will increase the current in the circuit for a given frequency.

Solution: The *capacitive reactance* is inversely related to capacitance, so an increase in capacitance will create a decrease of *capacitive reactance*. This, in turn, will mean less net resistance in the circuit and more current. This statement is true.

g.) A decrease in frequency will increase the voltage across the resistor.

Solution: A decrease in frequency means less signal will pass through the circuit (remember, capacitors are *high pass* filters). This corresponds to less current which, in turn, corresponds to a decrease of voltage across the resistor (REMEMBER, again, what current is doing in a circuit is exactly mirrored by the voltage drop across a resistor in the circuit.).

14.21) Why won't a capacitor allow low frequency AC current to flow through it?

Solution: The voltage across a capacitor at a particular instant is related to two things: the capacitor's capacitance and the amount of charge on one of the capacitor's plates. If there is a lot of charge on the plates *most of the time* (this is the case with low frequency), the short-term voltage of the capacitor will be relatively high. If the voltage across the capacitor is relatively large, the voltage across the resistor in the circuit will be relatively small and there will be very little current flowing through the circuit (again, current mimics resistor voltage). That is why capacitors don't pass low frequency. Their plates are charged too much of the time at low frequency.

14.22) What is the measure of a capacitor's net resistive nature? That is, what is it called, what are its units, and how is it calculated?

Solution: A capacitor's net resistive nature is called the *capacitive reactance*. It's symbol is X_C. It's units are *ohms*, and it is numerically determined using the relationship $X_C = \frac{1}{2\pi \nu C}$, where ν is the frequency of the signal and C is the capacitor's capacitance in *farads*.