Chapter 14

CAPACITORS IN *AC* AND *DC* CIRCUITS

So far, all we have discussed have been electrical elements in which the voltage across the element is proportional to the current through the element (i.e., elements like the resistor that obey Ohm's Law). There are electrical elements that do not follow this pattern. One of these elements is the capacitor--a critter that has very different characteristics when found in an AC circuit as opposed to a DC circuit. This chapter is devoted to that lowly creature.

A.) Capacitors in General:

1.) The circuit symbol for the capacitor (see Figures 14.1a and 14.1b) evokes a feeling for what a capacitor really is. Physically, it is no more than two plates (the symbol depicts the side view) *that do not touch* (there is normally insulation placed between the two plates to insure no contact). In other words, a capacitor in a circuit technically effects a break in the circuit.

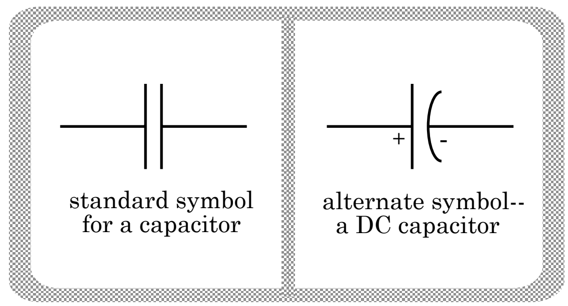


FIGURE 14.1a FIGURE 14.1b

Note: Although there are AC capacitors made to take high voltage at either terminal, DC capacitors have definite high and low voltage sides. When a designer of circuitry wants to specify a DC capacitor, he or she uses the symbol shown in Figure 14.1b. The *straight side* of that symbol is designated the *high voltage side* (the positive terminal) while the curved side is designated the *low voltage side*. We will use either symbol in DC situations.

2.) A circuit element that does not allow charge to freely flow through it probably sounds like a fairly useless device. In fact, capacitors do allow current to flow in the circuit under the right conditions.

3.) Consider a circuit in which there is an initially uncharged capacitor, a DC power supply, a resistor, and an initially open switch (this is commonly called an *RC circuit*).

a.) When the switch is first closed, neither plate has charge on it. This means there is no voltage difference between the two. As the right-hand plate is connected to the *ground terminal* of the battery, both plates must have an initial *electrical potential* of zero (see Figure 14.2a).

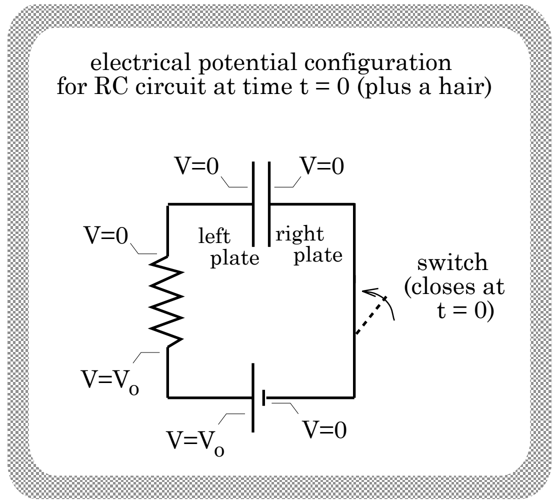


FIGURE 14.2a

b.) Just after the switch is closed, a voltage difference exists across the resistor (again, see Figure 14.2a) and, hence, current flows through the circuit. (Remember, the voltage across a resistor is proportional to the current through it--if the voltage is relatively large, the current will be relatively large, if the voltage is relatively small, the current will be relatively small.)

c.) As time proceeds, *positive charge* accumulates on the capacitor's left plate (this is looking at the circuit from a *conventional current* perspective in which *positive charge* moves).

d.) As it does, two things happen:

i.) Electrostatic repulsion from the *positive charge* accumulated on the left plate forces an equal amount of *positive charge* off the right plate. That leaves the right plate *electrically negative*.

Note: The amount of *negative charge* on the right plate is always equal to the amount of *positive charge* on the left plate. That means that current *appears* to be passing through a capacitor even though the capacitor's plates are not connected.

ii.) The second consequence is that the *left plate's voltage* begins to *increase* and a *voltage difference* begins to form across the capacitor's plates.

e.) As the voltage of the capacitor's *left plate* increases, the voltage on the resistor's *low voltage side* also begins to increase (that point and the capacitor's *left plate* are the same point). This decreases the *voltage difference* across the resistor.

f.) Figure 14.2b (next page) shows the voltage distribution around the circuit midway through the capacitor's charge-up cycle. This, in turn, *decreases* the *current* in the circuit.

g.) Figure 14.3 (below) shows the *Current vs. Time* graph for a circuit in which a capacitor is charging.

h.) In looking back at Figure 14.2b, it should be obvious that current will flow until the *voltage* of the *capacitor's left plate* equals the *voltage* of the *power supply's high voltage terminal* and the voltage difference across the resistor is zero. Put another way, once the voltage across the capacitor equals the voltage across the power supply, current ceases.

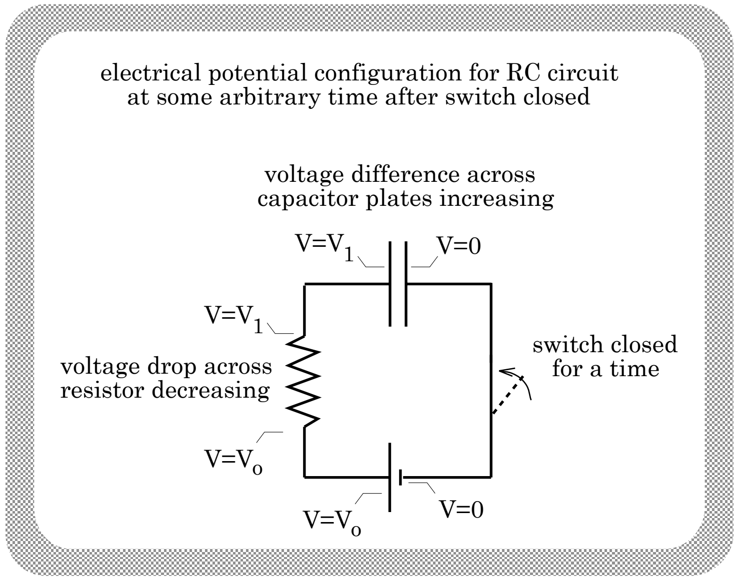


FIGURE 14.2b

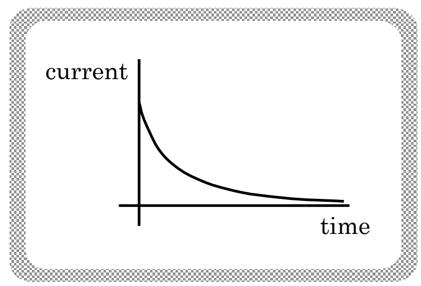


FIGURE 14.3

Note 1: In a little different light, current will flow until the left plate holds as much charge as it can, given the size of the *power source* to which it is attached.

Note 2: Does this analysis hold in theory if we switch the positions of the capacitor and resistor? Figure 14.4 shows the situation along with the circuit's voltage distribution after the switch has been closed for a long time. Notice that the voltage drop across the capacitor is still equal to the voltage across the power supply when the current in the circuit along with the voltage across the resistor goes to zero.

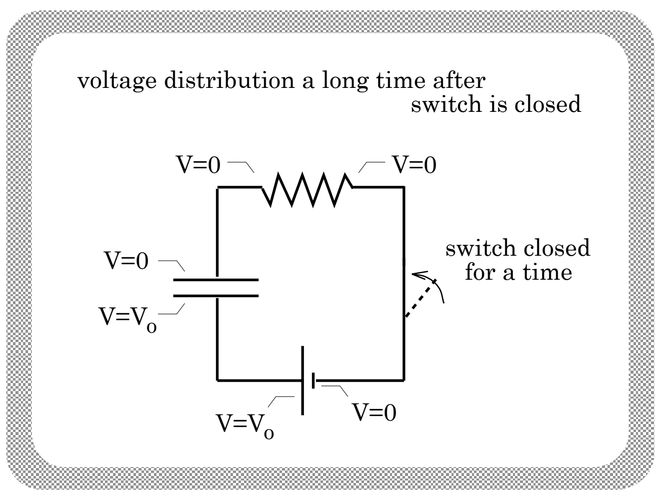


FIGURE 14.4

4.) Bottom Line:

a.) A capacitor stores charge and, in doing so, stores *energy* in the form of an *electric field* between its plates (see Figure 14.5).

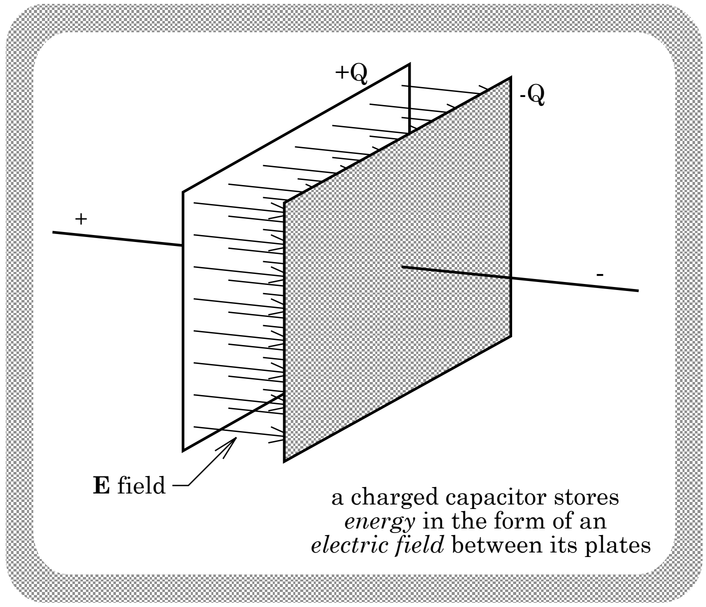


FIGURE 14.5

b.) If a capacitor has *Q's* worth of *positive charge* on one plate, it must by its very nature have *Q's* worth of *negative charge* on its other plate.

c.) If the magnitude of the charge on ONE PLATE is Q when the magnitude of the voltage drop across the capacitor's plates is V_c, then the *capacitance* of the capacitor is defined as:

$$C = Q/V_c.$$

i.) Put another way, the magnitude of the voltage V_c across the plates of a capacitor is proportional to the charge Q on one plate. The proportionality constant is called the capacitance C, and the relationship between the variables is:

$$Q = CV_c.$$

d.) By the definition of capacitance (i.e., $C = Q/V$), the MKS unit is *coulombs per volt*. The name given to this unit is the *farad*.

One farad is an enormous amount of capacitance. It is common to use capacitor values that are much smaller. The following are the ranges most often encountered (you should know not only their prefixes and definitions *but also their symbols*):

i.) A *millifarad* is symbolized as *mf* and is equal to 10^{-3} farads;

ii.) A *microfarad* is symbolized as μf (sometimes *Mf*) and is equal to 10^{-6} farads;

iii.) A *nanofarad* is symbolized as *nf* and is equal to 10^{-9} farads;

iv.) A *picofarad* is symbolized as *pf* and is equal to 10^{-12} farads.

5.) Example of a Capacitor In Action: Consider the *camera-flash* circuit shown in Figure 14.6.

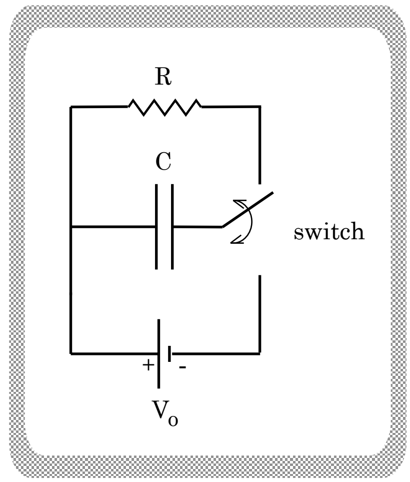


FIGURE 14.6

a.) The switch is initially connected in the *down position* so that the capacitor is hooked across the power supply. This allows the capacitor's plates to charge up.

b.) When the flash is activated, the switch flips to the *up position*. The capacitor discharges across the resistor (i.e., charge flows from one plate to the other, passing through the resistor/lightbulb in the process) with the large, momentary charge-flow lighting the flashbulb.

c.) Once fired, the switch automatically flips down allowing the capacitor to once again charge itself off the power supply.

B.) Equivalent Capacitance of Parallel and Series Combinations:

1.) The Equivalent Capacitance for Capacitors in Series:

a.) Just as *current* is common for all *resistors connected in series*, *charge accumulation on capacitor plates* is the common quantity for *capacitors in series*.

i.) Examining Figure 14.7, the *positive charge* electrically forced off the right plate of the first capacitor must go somewhere. Where? It accumulates on the left plate of the second capacitor.

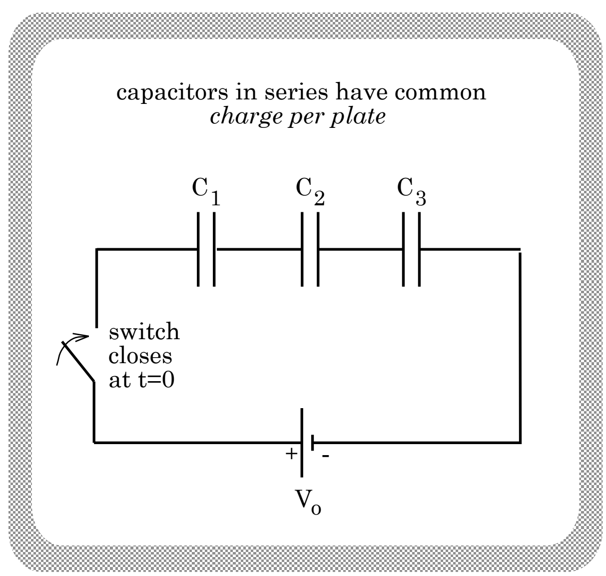


FIGURE 14.7

ii.) Conclusion: The *amount of charge* associated with each series capacitor must be the same.

b.) At a given instant, the sum of the voltage drops across the three capacitors must equal the voltage drop across the power supply, or:

$$V_o = V_1 + V_2 + V_3 + \ldots$$

c.) As the *voltage across* a capacitor is related to the *charge on* and *capacitance of* a capacitor ($V = Q/C$), we can write:

$$\begin{aligned} V_o &= V_1 + V_2 + V_3 + \ldots \\ Q/C_{eq} &= Q/C_1 + Q/C_2 + Q/C_3 + \ldots \end{aligned}$$

d.) With the *Q's* canceling nicely, we end up with:

$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3.$$

e.) In other words, the *equivalent capacitance* for a *series combination of capacitors* has the same mathematical form as that of a *parallel combination for resistors*.

2.) The Equivalent Capacitance for Capacitors in Parallel:

a.) Just as *voltage* is common for all *resistors connected in parallel*, *voltage across capacitor plates* is the common quantity for capacitors in *parallel* (see Figure 14.8).

b.) Over time, the charge that accumulates on the various capacitors has to equal the total charge Q_o drawn from the power supply, or:

$$Q_o = Q_1 + Q_2 + Q_3 + \ldots$$

As each capacitor's charge is related to the voltage across its plates by $Q = CV$, we can write:

$$\begin{aligned} Q_o &= Q_1 + Q_2 + Q_3 + \ldots \\ C_{eq}V_o &= C_1V_o + C_2V_o + C_3V_o + \ldots \end{aligned}$$

With the V_o's canceling nicely, we end up with:

$$C_{eq} = C_1 + C_2 + C_3.$$

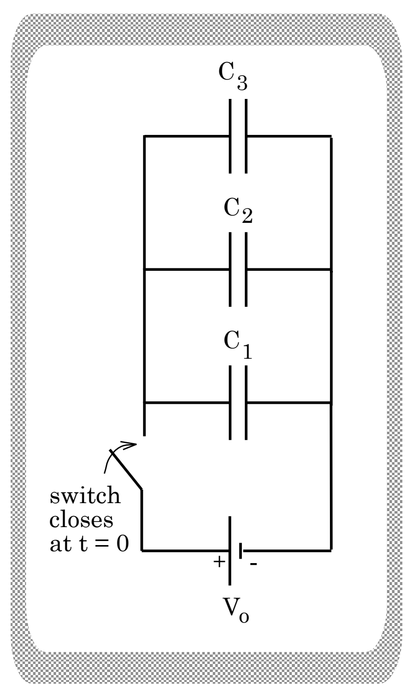


FIGURE 14.8

c.) In other words, the *equivalent capacitance for a <u>parallel</u> combination of capacitors* has the same mathematical form as that of the *<u>series</u> combination for resistors*.

C.) The Current Characteristics of a Charging Capacitor in a DC Circuit:

1.) Because there is no charge on the plates of an uncharged capacitor, a capacitor will initially provide no resistance to charge flow in an *RC* circuit.

a.) This means all of the initial voltage drop in the circuit is across the resistor, which means the initial current i_o in the circuit is

$$V_o = i_o R,$$

or

$$i_o = V_o/R.$$

2.) As the capacitor charges up, it will become increasingly more difficult for additional charge to be forced onto the capacitor's plates. As such, the current in the circuit will decrease.

a.) We would like to derive an expression for the current in a *DC-RC circuit* as a function of time, but we really don't need the derivation to get at the good stuff. All we need now are the basics.

b.) Figure 14.9 shows the circuit. Remembering that the voltage drop across a capacitor will be q/C, we can use Kirchoff's Laws to write:

$$V_o - (q_{plate}/C) - iR = 0,$$

where q_{plate} is the charge on the capacitor and i is the conventional current in the circuit.

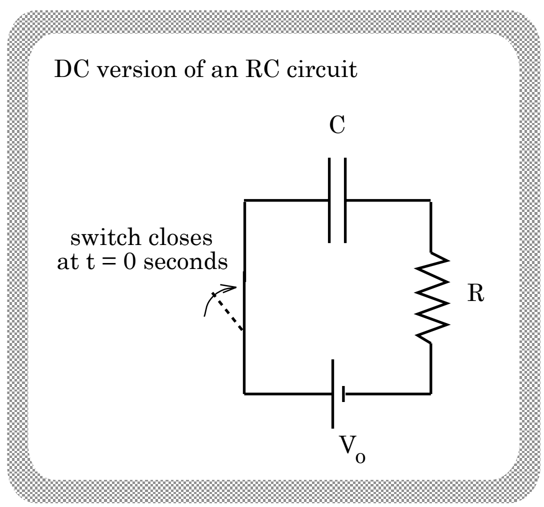


FIGURE 14.9

c.) Noticing that the rate at which charge flows onto the capacitor plates (i.e., dq_{plate}/dt) is, in this case, equal to the charge flow in the circuit (i.e., the current i, or dq_{flow}/dt), we can divide through by R and re-rewrite this as:

$$\frac{dq_{flow}}{dt} + \left[\frac{1}{RC}\right] q_{plate} = \frac{V_o}{R},$$

or

$$\frac{dq_{plate}}{dt} + \left[\frac{1}{RC}\right] q_{plate} = \frac{V_o}{R}.$$

Minor Note: If the capacitor had been discharging, dq_{plate}/dt would be *NEGATIVE* and the q_{flow}/q_{plate} relationship would be $i = dq_{flow}/dt = -dq_{plate}/dt$. If this bit of whimsy is missed, you will end up with mush for a solution.

d.) This differential equation essentially states that we are looking for a function q_{plate} such that when we take its derivative (i.e., dq_{plate}/dt) and add to it *a constant times itself* (i.e., $(1/RC)q_{plate}$), we will always get the same number (in this case, V_o/R).

e.) Bottom line #1: Solving our differential equation yields a solution that defines how much charge there will be on the capacitor *as a function of time*. That function is

$$q_{flow}(t) = Q_{max}(1 - e^{-t/RC}),$$

where $Q_{max} = CV_o$.

f.) Bottom line #2: The function that defines the current in the circuit *as a function of time* is the derivative of our q_{flow} function, or

$$i(t) = i_o e^{-t/RC},$$

where the initial current i_o in the circuit is $i_o = V_o/R$.

g.) See Figure 14.10.

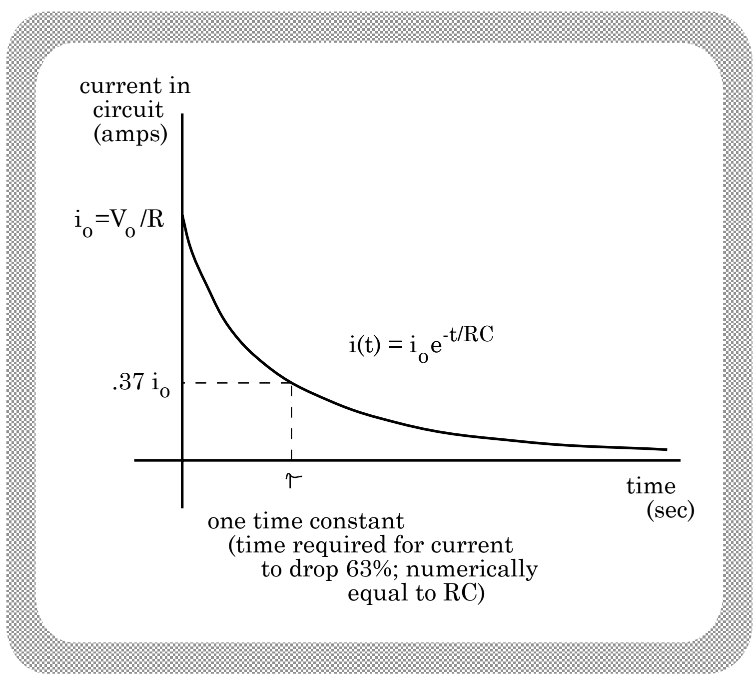


FIGURE 14.10

3.) The graph of the *current as a function of time* for a charging capacitor visually points out several important things.

a.) Initially, the current through an *RC circuit* in which the capacitor is initially uncharged is at a *maximum*. That is, the capacitor initially acts like it is a short (i.e., not even there). It isn't until charge begins to accumulate that charge flow begins to diminish.

b.) The graph identifies a particular *point in time* that has been deemed important. It is the amount of time associated with what is called *one time constant.*

i.) One time constant is defined as $\tau = RC$, where the symbol τ is a lower case tau.

ii.) Putting *one time constant* into our current expression (i.e., letting $t = RC$) yields:

$$
\begin{aligned}
i &= i_o e^{-RC/RC} \\
&= i_o(e^{-1}) \\
&= .37 i_o.
\end{aligned}
$$

iii.) Bottom line: One time constant is the amount of time it takes the circuit's current to diminish to 37% of its initial value.

iv.) It is also the amount of time it takes for the capacitor to charge up to 63% of its initial charge (to see this, put one time constant in the charge expression).

c.) What does this tell us? It tells us that if we multiply the value of the capacitance and resistance together (i.e., RC), the number we end up with will:

i.) Have the units of seconds (this has to be the case if the *exponent* is to be unitless);

ii.) Be the amount of time required for the capacitor to charge to 63% of its maximum; and

iii.) Be the amount of time required for the current to drop to 37% of its maximum.

Note 1: In doing the math, the time interval *2t* will give us approximately 87% charge-up for the capacitor and a current that will have dropped to approximately 13% of its initial value.

Note 2: The charge/discharge characteristics of a capacitor in an RC circuit are symmetric. That is, the time it takes to *charge* a capacitor to 63% of its maximum is the same amount of time required for the charged capacitor to discharge 63% of its charge (leaving 37% on the cap).

d.) Why is *t* important? It would be idiotic to build a camera flash using a resistor and capacitor whose *time constant* was, say, ten seconds. Waiting *twenty seconds* for 87% of your charge to dump through the resistor would never do. A system's *time constant* is a very useful quantity to know.

D.) Dielectrics:

1.) Consider the situation in which a piece of insulating material, called a *dielectric*, is placed between the plates of the capacitor (see Figure 14.11). The capacitor is charged, then isolated (that is, once charged it is disconnected from the power supply). What must be true?

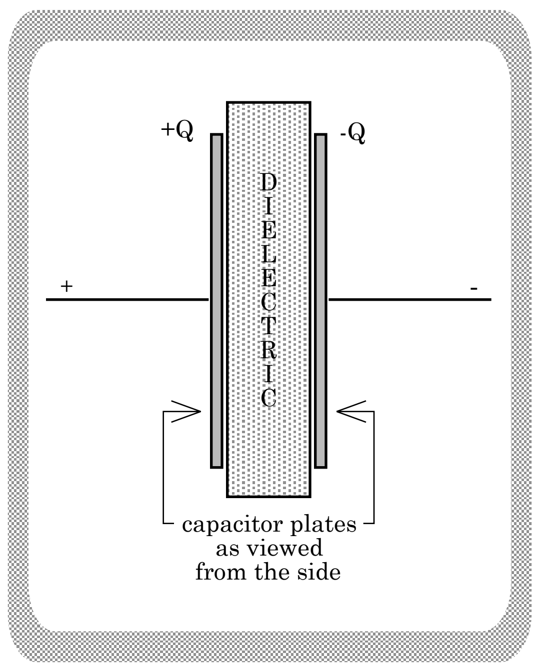


FIGURE 14.11

a.) Let E_o be the electric field *without the dielectric between the capacitor's plates*.

b.) When the insulator is placed between the plates, the surface of the insulator facing the *positive plate* of the capacitor will experience a Van der Waal-type charge separation that makes that face appear *negative*. A similar effect will be found on the other face making it appear *positive*.

c.) The charge separation in the dielectric creates a second electric field E_d between the plates (again, see Figure 14.12) in the *opposite direction of* E_o. Although E_d is considerably smaller than E_o, the effect is to *decrease the net electric field between the plates*.

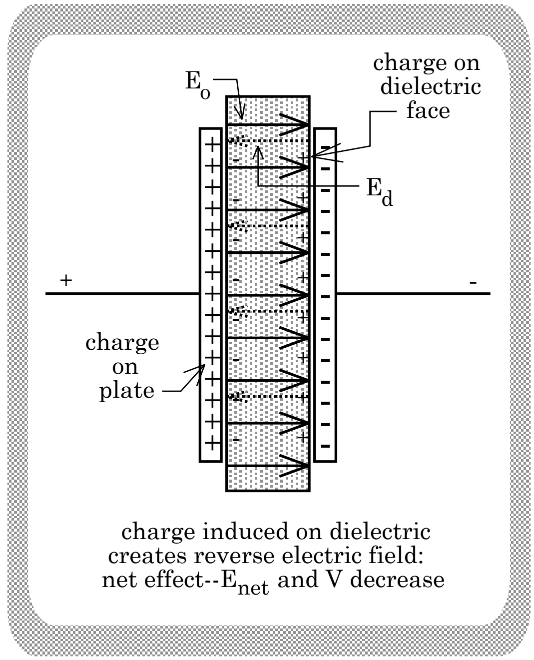


FIGURE 14.12

d.) As the *electric field* between the plates is proportional to the *voltage difference* across the plates, decreasing the *electric field* by inserting the dielectric effectively *decreases the voltage* across the plates.

e.) We know that $C = Q/V_c$. If the *charge on the plates* stays the same while the *voltage across the plates* goes *down*, the capacitance *C increases*.

f.) Bottom Line: Inserting a dielectric between the plates of a capacitor INCREASES THE CAPACITANCE.

g.) If the ratio of the capacitance *with dielectric* to capacitance *without dielectric* is defined as the *dielectric constant* κ_d (i.e., the *dielectric constant* for a material is simply a number that tells you how much the *capacitance* of an *air-filled capacitor* will be boosted when the dielectric is placed between its plates), we can write

$$C_d = \kappa_d C_{w/o}.$$

2.) Dielectrics used in conjunction with capacitors are useful for three reasons:

a.) As explained above, the presence of a dielectric between a capacitor's plates inherently increases the capacitance of the capacitor.

b.) A piece of insulating material (a dielectric) placed between the plates acts like a gap-jumping barrier for electricity. That means much larger *voltages*, hence much larger *electric fields*, can be dealt with using a capacitor that would not otherwise have been able to handle the situation. Put another way, more charge can be stored on the plates without fear of breakdown that would otherwise have been the case (breakdown occurs when the electric field between the plates is so large that charge leaps the gap--once breakdown is achieved in a dielectric-filled capacitor, the capacitor is ruined).

c.) Due to their insulating properties, dielectrics allow plates to be brought very close to one another. As the capacitance is inversely proportional to the distance d between the plates, this allows for both the miniaturization of capacitors as well as the increasing of a capacitor's *capacitance per unit of plate area.*

3.) It is possible to derive an expression for the capacitance of a parallel plate capacitor in terms of its geometric parameters (i.e., its plate area, the distance between its plates, etc.). It requires the use of the definition of capacitance (i.e., $C = q/V_c$), the relationship between a voltage difference and the electric field that is set up as a consequence of the charge on the plates (i.e., $\Delta V = -\int \boldsymbol{E} \cdot d\boldsymbol{r}$), the fact that $\Delta V = (V_- - V_+) = -V_c$, and the electric field function for charged parallel plates.

Sound nasty? It is.

Fortunately for you, all you need is the bottom line. Sooo . . .

a.) Assuming the distance between the plates is *d meters* (see Figure 14.13), the area of *one* plate is A_o *square meters*, and the dielectric constant of the dielectric between the plates is κ_d, the capacitance of a parallel plate capacitor is

$$C = \kappa_d \varepsilon_o \frac{A_o}{d},$$

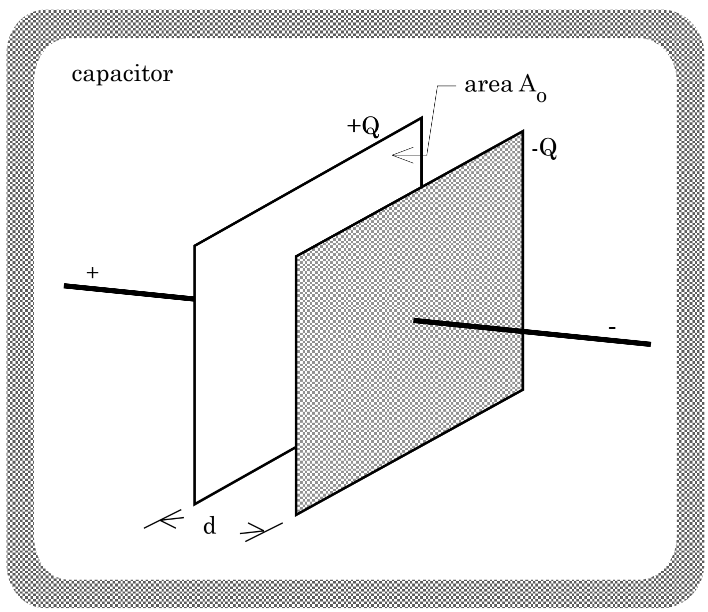


FIGURE 14.13

where ε_o is called the *permittivity of free space* and is numerically equal to $8.85\text{x}10^{-12}$ farads/meter.

Note: Coaxial cables are used in industry for TV and VCR hook-ups. A dielectric-filled coaxial cable with inside radius R_1 and outside radius R_2 has a *capacitance per unit length* of

$$\frac{C}{L} = \frac{2\pi\kappa_d\varepsilon_o}{\ln\left(\frac{R_2}{R_1}\right)}.$$

E.) Energy Stored in a Capacitor:

1.) *Work* must be done to charge a capacitor. The energy associated with that work is stored as electrical potential energy in the electric field created between the capacitor's plates. In other words, we can determine the amount of energy stored in a capacitor by determining the amount of work required to charge the capacitor. Because you have been deprived of some of the more interesting (translation: diabolic) derivations, I'll let you see how this plays out mathematically.

a.) To be as general as possible, assume a capacitor of capacitance C initially has charge q on its high voltage plate and $-q$ on its low voltage plate. If the voltage across the plates is initially V_c, how much work must be done to add an additional *dq's* worth of charge to the positive plate?

b.) The amount of work we are looking for will equal the amount of work required to move the charge *dq* from one plate to the other (that is effectively what is happening as electrostatic repulsion pushes *dq's* worth of positive charge off the capacitor's low voltage plate).

c.) The relationship between the differential work *dW* done on the differential charge *dq* as it moves through a potential difference $(V_- - V_+) = -V_c$ is:

$$\begin{aligned} dW/dq &= -\Delta V \\ &= +V_c. \end{aligned}$$

d.) Remembering that $V_c = q/C$, where q is the charge already on the plates, we can rewrite this as:

$$\begin{aligned} dW &= (V_c)dq \\ &= (q/C)dq. \end{aligned}$$

e.) The total amount of energy required to place a net charge Q on the capacitor's plates will be the sum (i.e., integral) of all the differential work quantities evaluated from $q = 0$ to $q = Q$. Doing that operation yields:

$$\begin{aligned} W &= \int dW \\ &= \int_{q=0}^{Q} \left[\frac{q}{C}\right] dq \\ &= \left(\frac{1}{C}\right)\left[\frac{q^2}{2}\right]_{q=0}^{Q} \\ &= \left(\frac{1}{C}\right)\left[\frac{Q^2}{2}\right]. \end{aligned}$$

f.) As $Q = CV_c$, the work expression can be re-written as:

$$W = \frac{1}{2} C V_c^{\,2}.$$

This is the amount of ENERGY wrapped up in a capacitor whose capacitance is C and across whose plates a voltage V_c is impressed.

2.) What's interesting about all of this is that if we are clever, we might be able to charge up a capacitor, then discharge it through a motor making the motor run. Attach the motor to wheels and we have a robot (OK, a very simple robot, but a robot nevertheless).

F.) Capacitors in AC Circuits:

1.) So far, all we have dealt with have been capacitors as they act in DC circuits. They charge up. When given the chance, they discharge.

In AC circuits, capacitors are constantly charging up and discharging. This makes for some very fun times.

2.) Consider the *RC circuit* shown in Figure 14.14. Unless it is "leaky," the capacitor in the circuit will have no resistor-like resistance inherent within it. As such, we will assume there is no *ir* voltage drop across the capacitor.

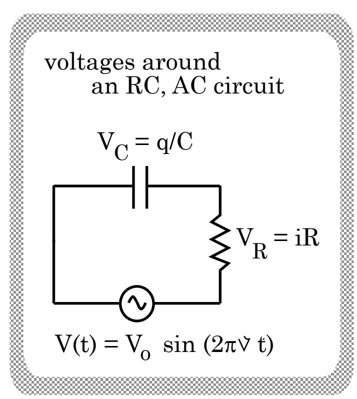


FIGURE 14.14

Though there is, in theory, no *resistor-like* resistance to charge flow associated with the capacitor, capacitors do have a *frequency-dependent* resistive nature. Not obvious? Consider the following:

a.) The voltage drop across a capacitor is defined as:

$$V_C = q/C,$$

where q is the magnitude of charge on *one* capacitor plate and C is the capacitor's capacitance.

b.) To make the evaluation easier later on, let's assume the power supply's voltage is characterized as a *sine function* (a *cosine function* would also work--it would just be a little messier to deal with). With that assumption, a Kirchoff's Loop Equation for this circuit (see Figure 14.14) becomes:

$$- q/C - iR + V_o \sin (2\pi \nu t) = 0.$$

Manipulating, we get:

$$q/C + iR = V_o \sin (2\pi \nu t),$$

where q is a time varying quantity in the expression (we could denote it $q(t)$ but, for simplicity, we will leave it as presented).

c.) Though you will never have to derive this on a test, we need an expression for the resistive nature of the capacitor. To do this:

i.) Assume the resistor-like resistance in the circuit is negligible (i.e., that $R = 0$). In that case, Kirchoff's Law becomes:

$$\begin{aligned} q/C &= V_o \sin (2\pi \nu t) \\ \Rightarrow \quad q &= CV_o \sin (2\pi \nu t). \end{aligned}$$

ii.) Remembering that $i = dq/dt$, we can write:

$$i = \frac{dq}{dt}$$

$$= \frac{d\left[CV_o \sin(2\pi\nu t)\right]}{dt}$$

$$= CV_o (2\pi\nu)\cos(2\pi\nu t)$$

$$= \frac{V_o \cos(2\pi\nu t)}{\left(\frac{1}{2\pi\nu C}\right)} \qquad \text{(Equation B).}$$

3.) Ohm's Law maintains that the *current* through an element must equal the *voltage* across the element divided by a quantity that reflects the *resistive nature* of the element. In the above expression, the voltage across the element is $V_o\cos(2\pi\nu t)$. That means the *resistive nature* of the capacitor must be $1/(2\pi\nu C)$.

a.) In fact, this is the *frequency-dependent* resistive nature of a capacitor. It is called the *capacitive reactance*, its symbol is X_C, and its units are *ohms*. Summarizing, we can write:

$$X_C = \frac{1}{2\pi\nu C} \quad \text{(ohms),}$$

where the capacitance *C* must be written in terms of *farads* (versus leaving it in *microfarads* or whatever).

4.) Does the frequency-dependent expression for the *resistive nature* of a capacitor (i.e., its capacitive reactance) make sense? Consider:

a.) Assume the voltage of a power supply runs at low frequency.

i.) Examining the low frequency signal shown in Figure 14.15, it is evident that the signal is changing very slowly and that there is a respectable amount of charge on the capacitor a fair portion of the time. In other words, the capacitor has plenty of time to charge up and, on the average, the voltage (q/C)

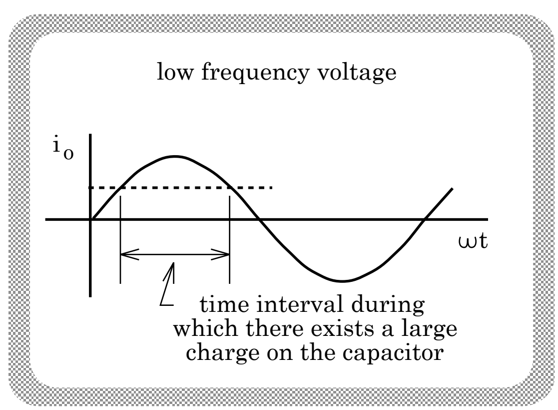


FIGURE 14.15

across the capacitor is relatively large.

ii.) Because the capacitor's voltage is relatively large on average, the voltage across the resistor will be relatively small. This implies a *small current* in the circuit.

b.) Bottom line #1: The current in an RC circuit will be relatively small (i.e., edging toward a readable zero) when a *low frequency signal* passes through the circuit. That means we would expect the capacitive reactance (the *resistive nature* of the capacitor) to be large at low frequencies. This is exactly what our expression predicts (i.e., when ν is small, $X_C = 1/(2\pi \nu C)$ is large).

c.) Assume the voltage of a power supply now runs at high frequency.

i.) Examining the high frequency signal shown in Figure 14.16, it is evident that the signal is changing very fast. There are not great spans of time during which the capacitor is charged, hence there are not great spans of time during which the voltage across the capacitor is high. In fact, the voltage (on the average) is low (remember, the time average of a high frequency sine wave is zero even over relatively small time intervals).

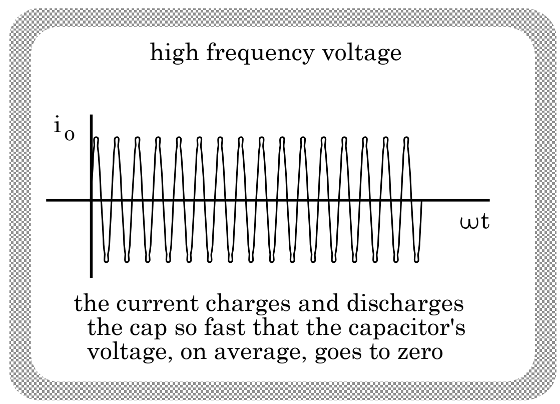


FIGURE 14.16

ii.) A small voltage across the capacitor (on average) means a large voltage across the resistor. This implies a large current in the circuit.

d.) Bottom line #2: The current in an RC circuit will be relatively large when a *high frequency signal* passes through the circuit. That means we would expect the capacitive reactance to be small at high frequencies. This is exactly what our expression predicts (i.e., when ν is large, X_C is small).

e.) Summary: A *capacitor* in an AC circuit *passes high frequency signals* while damping out *low frequency signals*. As such, capacitors are sometimes referred to as *high pass filters*.

5.) The second point to note about the time dependent current expression we derived above is its form. By assuming a power supply voltage that is proportional to *sin* $(2\pi\nu t)$, and assuming that *the net resistance in the circuit is zero*, we find that the circuit's current is proportional to *cos* $(2\pi\nu t)$. Examining the graph of these two functions allows us to conclude that <u>in this situation</u> the *voltage* across the capacitor *lags* the *current* through the capacitor (i.e., the circuit's current) by $\pi/2$ radians.

Does this make sense?

a.) The voltage across a capacitor is proportional to the charge on the capacitor (i.e., $V_C = q/C$). Figure 14.17 depicts a graphical representation of this.

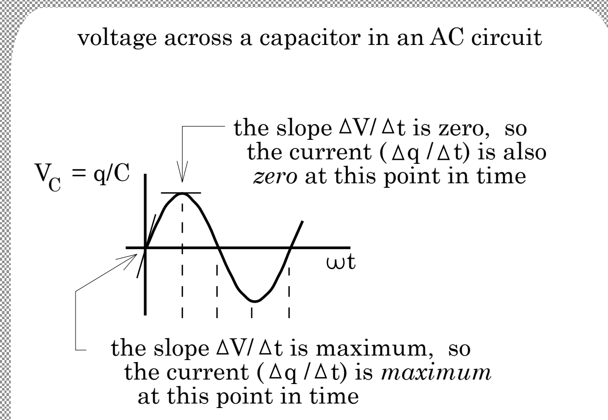


FIGURE 14.17

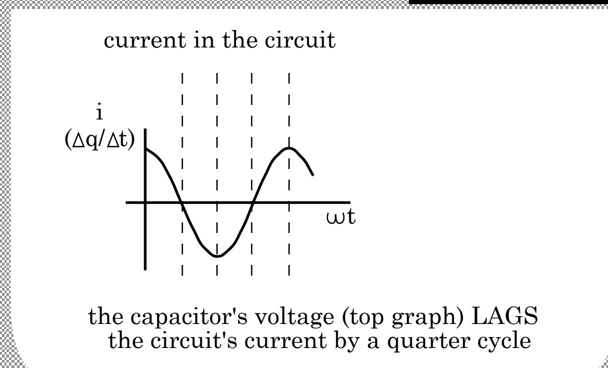


FIGURE 14.18

b.) Current is defined as the amount of charge that passes a particular point per unit time (i.e., $i = dq/dt$).

c.) The *slope* of the capacitor's *voltage function* is

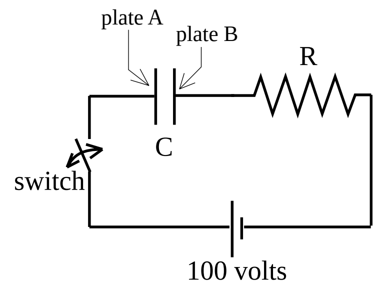
$$\begin{aligned} dV_C/dt &= (1/C)(dq/dt) \\ &= i/C. \end{aligned}$$

d.) In other words, a graph of the slope of the capacitor's voltage function gives us a modified *current function*. Figure 14.18 shows this.

e.) In comparing the graphs, it is evident that the voltage across the capacitor LAGS the current in the circuit by one quarter of a cycle, or $\pi/2$ radians.

Big Note: This $\pi/2$ phase shift exists ONLY if there is no resistor-like resistance in the circuit. As there will never be a case in which there is absolutely no resistor-like resistance in a circuit, the phase shift in a real *RC circuit* will never be $\pi/2$. Calculating what it actually is in a given case is something we may do later, but not now.

QUESTIONS & PROBLEMS



14.1) You have a power supply whose low voltage "ground" terminal is attached to a resistor whose resistance is $R = 10^4$ *ohms*. The resistor is attached to a plate (we'll call it *plate B*) which is next to, but not connected to, a second plate (we'll call it *plate A*). Reiterating, THERE IS NO CONNECTION between *plate A* and *plate B*. There is, additionally, no initial charge on either of the plates. Attached to *Plate A* is a switch. On the other side of the switch is the high voltage "hot" terminal of the power supply. A sketch of the situation is shown. At $t = 0$, the switch is closed.

a.) Current initially flows between the high voltage terminal and *Plate A*. Why? That is, what's going on?

b.) Current initially flows from *Plate B* through the resistor, and back to the ground of the power supply. Why? That is, what's going on?

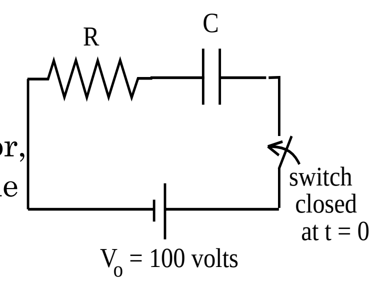
c.) What is the two-plate device called?

d.) After a while, there is a voltage $V = 10$ *volts* across the plates. At that point in time, there is 10^{-10} coulombs of charge on *plate A*. The ratio of the *charge to voltage* is 10^{-9}.

i.) How much charge is on Plate B?

ii.) What is this ratio called?

iii.) At some later point in time, the voltage across the plates is doubled. What is the ratio of *charge to voltage* in that case? Explain.



14.2) What do capacitors (often referred to as *caps*) generally do in DC circuits? Give an example.

14.3) A 10^{-6} *farad* capacitor is in series with a 10^4 *ohm* resistor, a battery whose voltage is $V_o = 100$ *volts*, and a switch. Assume the capacitor is initially uncharged and the switch is thrown at $t = 0$.

a.) The capacitance value tells you something that is always true no matter what the voltage across the capacitor happens to be. What does it tell you?

b.) What is the initial current in the circuit?

c.) What is the circuit's current after a long period of time?

d.) How much charge will the capacitor hold when fully charged?

e.) How much energy is wrapped up in the capacitor when fully charged?

f.) Where is the energy stored in the capacitor?

g.) You are told that the time constant for the system is 10^{-2} seconds.

i.) What does that tell you about the system?

ii.) How much charge will be associated with the capacitor after at time equal to one time constant?

iii.) Where will the charge alluded to in *Part g-ii* be found?

h.) After a very long time, the switch is opened. What happens to the capacitor? Will it hold its charge forever?

i.) At $t = 1$ *second*, the current is i_1. At $t = 2$ *seconds*, the current is i_2. At $t = 4$ *seconds*, the current is i_4, and at $t = 8$ *seconds*, the current is i_8. Is i_2/i_1 going to give you the same ratio as i_8/i_4?

14.4) Can you have capacitance if you have only one plate?

14.5) You have a series combination of capacitors.

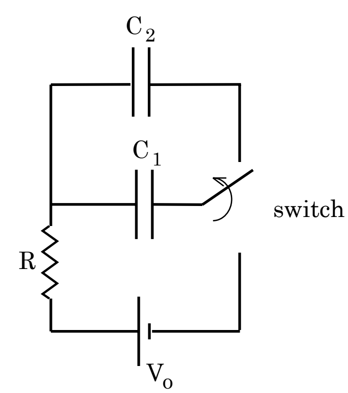
a.) What happens to the equivalent capacitance when you add another capacitor?

b.) What is common to all the capacitors in the series combination?

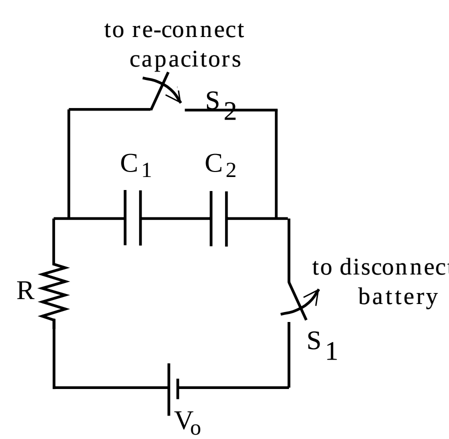
14.6) You have a parallel combination of capacitors.

a.) What happens to the equivalent capacitance when you add another capacitor?

b.) What is common to all the capacitors in the parallel combination?



14.7) You charge up two single capacitors that are in parallel. You disconnect the battery. What happens to the current in the system when you do this?



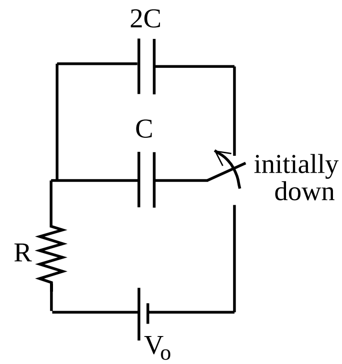
14.8) You charge up two unequal capacitors that are in series. You disconnect the battery by opening S_1, then reconnect the two capacitors by closing S_2.

a.) What happens to the current in the system when you do this?

b.) Out of curiosity, why was the resistor included in the circuit?

c.) What kind of circuit do you have after both switches are thrown? That is, what kind of relationship will exist between the capacitors after the throw?

14.9) You use a battery whose voltage is V_o to charge up a capacitor C. When fully charged, there is q's worth of charge on the cap. You then disconnect the capacitor from the battery and reconnect it to a second uncharged capacitor whose capacitance is $2C$ (in the sketch, this disconnection, then reconnection, is done with the switch). After the switch is thrown:



a.) Before the charge on C can redistribute, what is the voltage across the second capacitor?

b.) How will the charge redistribute itself? That is, how much charge ends up on the second capacitor?

14.10) You charge up a parallel plate capacitor that has air between its plates. Once charged, you *disconnect it* from the battery, then insert a piece of plastic (an insulator) between the plates. The amount of charge on the capacitor does not change (being disconnected from the circuit, it has no place to go), but the voltage across the capacitor does change.

a.) What is the insulator usually called in these situations?

b.) How and why does the voltage change (up, down, what?)?

c.) What happens to the capacitance of the capacitor?

d.) What happens to the energy content of the capacitor? If it goes up, from whence did the new energy come? If it goes down, where did it go?

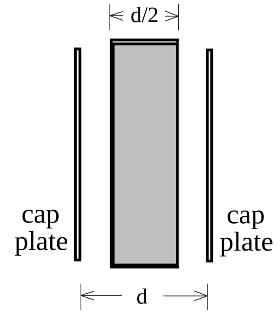
14.11) You have a parallel plate capacitor with air between its plates hooked up to a power supply whose voltage is V_o. *Without disconnecting the battery*, you carefully insert a piece of plastic between the plates.

a.) What happens to the voltage across the capacitor?

b.) What happens to the capacitor's capacitance?

c.) What happens to the charge on the capacitor's plates?

14.12) Between the plates of one air-filled capacitor, you insert a dielectric whose dielectric constant is k and whose thickness is half the plate separation. Between the plates of a second cap, you insert a piece of metal whose thickness is also half the plate separation. (Both situations look like the sketch.) After some nasty Calculus, the capacitance expression for the dielectric situation is found to be $C = \left(2\varepsilon_o \frac{A}{d}\right)\left(\frac{2k}{1+k}\right)$.



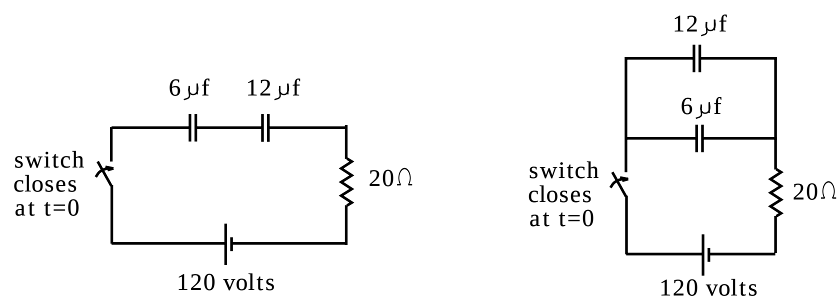
a.) Which modified capacitor will end up with the greater capacitance?
b.) What is the ratio of the two capacitances?

14.13) You have a capacitor in series with a switch, a resistor, and a power supply. At $t = 0$, you throw the switch and current begins to flow.

a.) For the amusement of it, draw the circuit.

b.) If the capacitor had been half as big, how would current flow? That is, would the cap have charged faster or slower? Justify your response.

14.14) Assuming there is no charge initially on any capacitor, answer all the following questions for the capacitor circuit in *sketch a*. When done, repeat the process for the circuit shown in *sketch b*:



(a) (b)

FIGURE I

a.) Determine the initial current in the circuit when the switch is first thrown.

b.) A long time after the switch is thrown (i.e., by the time the caps are charged up fully), how much charge is there on each plate?

c.) What is the voltage across the 6 μf capacitor when fully charged?

d.) How much energy does the 6 μf capacitor hold when completely charged?

e.) Determine the RC circuit's *time constant*. What does this information tell you?

f.) How much charge is there on the 6 μf capacitor after a time interval equal to one time constant passes?

14.15) Three identical capacitors are connected in several ways as shown in Figure II.

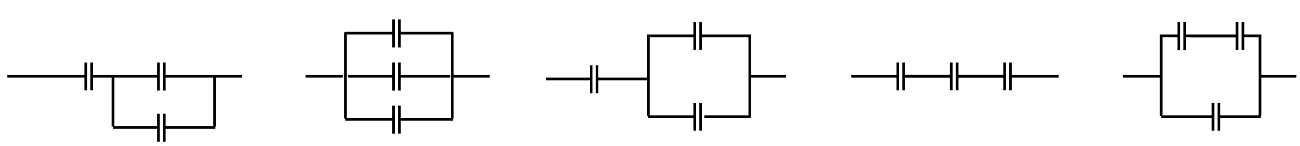

(a) (b) (c) (d) (e)

FIGURE II

a.) Order the combinations from the smallest *equivalent capacitance* to the largest; and

b.) Which combination has the potential of storing the most energy?

14.16) A *parallel plate capacitor* is connected to a 20 volt power supply. Once charged to its maximum possible *Q*, the capacitor's plates are separated by a factor of four (that is, the distance between the plates is quadrupled) while the capacitor is *kept hooked to the power supply*. As a consequence of this change in geometry:

a.) How will the capacitor's *capacitance change*?

b.) How will the *charge* on the capacitor *change*?

c.) How will the *energy stored* in the capacitor *change*?

d.) If a dielectric ($\kappa_d = 1.6$) had been placed between the plates of the *original setup*, what would the *new capacitance* have been?

14.17) Determine:

a.) The equivalent capacitance of the circuit shown in Figure III.

b.) Assuming each capacitor's capacitance is 25 mf, how much energy can this system store if it is hooked across a 120 volt battery?

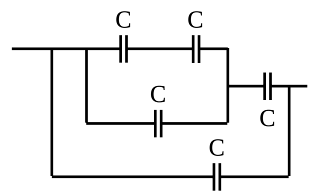


FIGURE III

14.18) The capacitors in the circuit shown in Figure IV are initially uncharged. At $t = 0$, the switch is closed. Knowing the resistor and capacitor values:

a.) Determine all three *initial* currents in the circuit (i.e., the currents just after the switch is closed).

b.) Determine all three currents in the circuit after a long period of time (i.e., at the theoretical point $t = \infty$).

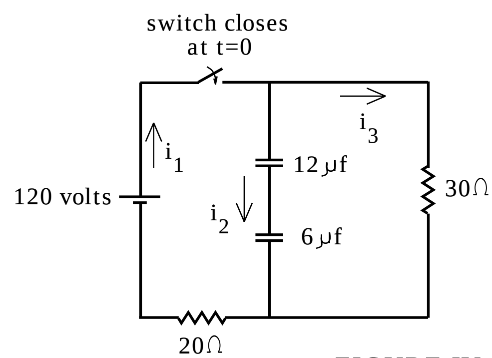


FIGURE IV

c.) Without solving them, write out the equations you would need to solve if you wanted to determine the currents in the circuit at any arbitrary point in time. Be sure you are complete.

d.) Determine the *total charge* the 6 mf capacitor will accumulate (i.e., the amount of charge on its plates at $t = \infty$).

e.) Once totally charged, how much energy do the capacitors hold?

f.) After a very long time (i.e., long after the capacitors have fully charged), the switch is opened. How long will it take for the two capacitors to dump 87% of their charge across the 30 Ω resistor?

14.19) An AC voltage source is found to produce a 12 volt peak to peak signal at 2500 hertz.

a.) Characterize this voltage as a *sine function.*

b.) Determine the *RMS voltage* of the source.

c.) It is found that when a *capacitor* and *resistor* are placed across the source as characterized above, an ammeter in the circuit reads 1.2 amps. What is the *maximum current* drawn from the source?

14.20) An RC circuit is hooked across an AC power supply. Which of the following statements are true (there can be more than one)? Explain each response.

a.) The RMS voltage across the resistor is the same as the average voltage across the resistor.

b.) The RMS voltage across the resistor is equal to R times the RMS current through the resistor.

c.) The RMS voltage across the resistor will be very large if the capacitive reactance is very large.

d.) The RMS current in the circuit will be very large if the capacitive reactance is very small.

e.) A decrease in frequency will increase the voltage across the capacitor.

f.) An increase in the capacitance will increase the current in the circuit for a given frequency.

g.) A decrease in frequency will increase the voltage across the resistor.

14.21) Why won't a capacitor allow low frequency AC current to flow through it?

14.22) What is the measure of a capacitor's net resistive nature? That is, what is it called, what are its units, and how is it calculated?