CHAPTER 10 -- WAVE MOTION

QUESTION SOLUTIONS

10.1) What, exactly, is a wave?

Solution: A wave is a disturbance that moves through a medium.

10.2) In Star Wars, a giant gallactic battle cruiser is attacked in space by a host of little single engine fighters. Cannons from the battle cruiser blaze as the tiny fighters swarm around it. You see a fighter hit, blowing up spectacularly with a ferocious outpouring of light and fire and a caucaphanous kaboom. What's wrong with this picture?

Solution: Because there is only about 1 atom per cubic centimeter in space, there is no medium through which the compression waves associated with sound can propogate. In other words, *there is no sound in space!*

10.3) How many wavelengths are there between a crest and its adjacent trough? Between every third crests? Between every two successive troughs?

Solution: From crest to trough is a half wavelength. From crest to crest is one wavelength. From crest to second crest is two wavelengths. From crest to third crest is three wavelengths. It runs the same with trough to trough.

10.4) You are on a pier watching ocean waves pass by. Six crests pass you in 25 seconds. If there is approximately 20 meters between crests:

a.) Will the wavelength of the wave train be a whole number?

Solution: If there is 20 between crests, the wavelength is 20 . . . which is a whole number.

b.) If the number of waves passing by every 25 seconds was halved, what would the period do?

Solution: If 3 crests pass by over the same amount of time (i.e., the 25 second interval), the wavelengths must be twice as big as they were which means the frequency must be half as big as it was which means the period (this is the inverse of the frequency) must be the inverse of a half, or double.

c.) How is the wave velocity related to the time it takes for a full wave to pass by?

Solution: The time it takes a wave to pass by is the period T. The period is the inverse of the frequency (i.e., ν). The wave velocity is proportional to the frequency (i.e., $v = \lambda\nu$). In other words, the wave velocity must be inversely proportional to the period ($v = \frac{\lambda}{T}$).

d.) What is the wave frequency?

Solution: Frequency is the number of cycles that pass by per unit time. In this case, that is 6 wavelengths per 25 seconds, or .24 cycles per second.

e.) How is the wave velocity related to the frequency of the wave train?

Solution: As was pointed out in *part c*, the wave velocity is directly proportional to the frequency (i.e., $v = \lambda\nu$).

f.) A student is given the following scenario: *A closed, empty plastic water bottle is thrown off the end of the pier (bad form!) and into the waves. If the pier is 100 meters long, how long will it take the bottle to hit the beach?* The student complains that this is a trick question. Explain why he or she might think so.

Solution: Unless the bottom of the ocean is so shallow at the end of the pier (remember, it's 100 meters from shore) that it retards the wave's motion, thereby making the waves break, the waves will do nothing to the bottle but lift it up, then put it back down essentially where it began. In other words, water waves displace objects vertically as they pass by, but they don't pick objects up and force them to travel with them. In short, the kids could argue that the bottle would *never* reach the shore.

10.5) What is the difference between a *wave* and a *standing wave*?

Solution: A wave is a disturbance that moves through a medium. Waves are usually produced by structures that vibrate in a periodic manner (guitar produced sound waves, for instance, come from vibrating strings). A standing wave is the superposition of two waves moving in the same medium but going in different directions. Additionally, there needs to be a resonance condition set up between the "natural frequency of the system" and the frequency of the periodic force that drives the system. The easiest example is a stretched out string that is attached at both ends to a fixed support. If you wiggle one end of the string in a periodic manner (i.e, at a given freqeuncy), a wave train will proceed down the string, bounce off the far fixed end, and come back toward you. If your wiggle frequency matches one of the natural frequencies of the string system (this frequency will, I might add, be related to the density and length of the string), you will get resonance and a standing wave will become evident on the string. In short, standing waves are a whole lot more complicated than a simple wave even though may look alike if captured in a snapshot.

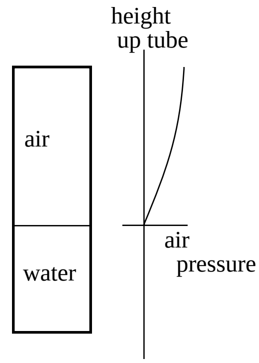
10.6) What *kind* of waves are your ears sensitive to? What makes these waves this type?

Solution: If you think about the way a speaker works, the speaker cone pushes outward compressing air thereby creating a high pressure zone, then pulls back thereby creating a low pressure zone. When this periodic high pressure, low pressure, high pressure, low pressure disturbance moves out in air and passes you by, little hairs in your ears are are motivated to move. That movement produces the electrical responses that your brain interprets as sound. As the force that produces the disturbance in the air pressure is along the line of the direction of the subsequent wave, the wave is a longitudinal wave. Of course, if the wave is produced by dropping a book on a table top, then the force that produced the wave is perpendicular to the direction of wave propogation and you might argue that the wave was transverse. Usually, though, sound is treated as a longitudinal wave.

10.7) What has to be true for resonance to occur?

Solution: The frequency of the force that drives the system must be the same as one of the natural frequencies of the system being disturbed. In that way, the force is in tune with the system in the sense that it is constantly helping the system to oscillate with greater and greater energy and amplitude.

10.8) The waveform for the lowest frequency that will "stand" in a tube of length L is shown to the right. Calling the speed of sound in air v_{air}:



a.) What is the wavelength of the waveform?

Solution: If the quarter wavelength shown has a length L, then the wave's full wavelength must be $4L$.

b.) What is the frequency of the waveform?

Solution: The speed of sound in air is approximately 335 m/s. As the frequency and wavelength are related to the wave velocity as $v = \lambda \nu$, it would appear that the frequency is $\nu = \frac{v}{\lambda} = \frac{335\ m/s}{4L}$.

c.) A frequency that is twice the frequency calculated in *part b* is projected down at the tube. Will the tube howl the way tubes do when resonance is present? Explain.

Solution: It is possible to find other frequencies that will stand in the tube. The question is whether doubling the frequency associated with the standing wave shown will do the job. An in fact, the answer is *no*. How so? If you double the frequency, you halve the wavelength (they are inversely proportional). Halving the wavelength means there will not be a quarter wavelength standing in the tube, as shown in the pressure graph provided, but rather a half wavelength. A half wavelength will means there must be a node at the water interface (just as is the case in the graph shown) *and* a node at the mouth of the tube. The problem is, tube mouths are open ends. That is, they must have anitnodes at them--regions where the wave intensity can fluctuate. In short, the wave we would be dealing with would not fit the criteria for a standing wave, given the constraints imposed by the tube itself.

10.9) Two waves moving in the same medium meet and pass by one another. What single word describes how the waves will interact while they are coexisting in the medium?

Solution: The word is *superposition*.

10.10) When a guitarist is using beats to tune a guitar, does the player want the beat frequency to increase or decrease as he/she gets the instrument closer to being tuned.

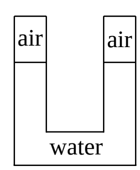
Solution: Beat frequencies are numerically calculated by taking the difference between the frequencies of the two sources that are being superimposed upon one another. As the two strings are plucked to produce the beat frequency, the closer their frequencies get, the lower the beat frequency. In fact, when the two frequencies are dead on, the beat frequency goes to zero and no beats are heard at all.

10.11) It's night time. You are lying on a train track, minding your own business, just looking at the stars with your pals. All of a sudden, you hear a train whistle. Half of your friends yell, "Oh, s_ _t, a train is coming," and jump up off the track. The other half chide, "Ah, you big sissies, the train is going the other way," and stay put. You happen to have perfect pitch, and because you are an officianado of trains, you know what a train whistle's frequency sounds like when you are actually traveling with the train. If your "I'm not going to move" friends are correct, how will the frequency you actually hear compare to the whistle frequency you hear when traveling with the train?

Solution: This is a Doppler Effects problem. If you have ever been at a train crossing as a train has passed by with its whistle blowing, you will know that the frequency drops as the train and whistle approaches. In other words, you will hear a train's whistle to be higher than it would be if it was sitting still next to you as the train approaches, and you will hear it to be lower as the train recedes. The answer, therefore, is your hoping the frequency is *lower* than usual. That will mean the train is moving away from you.

10.12) You have a motor that runs at a constant speed. You attach to the motor's shaft a gear and plunger assembly that displaces water in a very large pond (in fact, the pond is so large that you can ignore any returning wave that have bounced off the pond's edges, so big is the pond). As night falls, the water temperature drops. As a consequence, will the wavelength the traveling waves produced by the constant frequency plunger increase, decrease, or stay the same. Explain.

Solution: This is more of a logic problem than anything else. The wavelength is inversely proportional to the frequency and directly proportional to the wave velocity. The frequency of the source is constant, so the frequency of the wave will not change. That means the behavior of the wavelength is determined by the wave velocity. As water gets colder, its molecules slow down and compress into a smaller volume. Although water is weird in the sense that frozen water is less dense (i.e., more spread out) than water that is just above freezing (this is why ice cubes float), at higher temperatures the molecules do get closer to one another as temperature drops. It takes more time for a wave to propogate through a "denser" medium than a less dense material, so in this case the wave velocity should decrease as the water cools. As such, the wavelength should become shorter.



10.13) A hollow, U shaped tube is partially filled with water. When sound is projected at the system, the system resonances when the sound source's frequency is ν_1. If you pour water into the system, thereby raising the water level, will the new resonance frequency be greater, smaller, or the same as the old resonance frequency? Explain.

Solution: As $\nu \propto 1/\lambda$, less air in the column means a shorter wavelength will stand in the column which means the resonant frequency will be higher.

PROBLEM SOLUTIONS

10.14) The relationship between a wave's *frequency* ν, its *wavelength* λ, and its *wave velocity* v is $v = \lambda\ \nu$. For sound in air, the wave velocity is approximately $v = 330\ m/s$. To get the wavelength:

a.) For $\nu = 20\ hz$:

$$\begin{aligned}\lambda &= v/\nu \\ &= (330\ \text{m/s})/(20\ \text{hz}) \\ &= 16.5\ \text{meters} \qquad \text{(around 50 feet).}\end{aligned}$$

Note: Technically, the units of *frequency* are *seconds*$^{-1}$ and of *wavelength* are *meters*. The *cycles* term in the frequency units is a label, being the same for MKS, CGS, and the English system of units. This means that by dividing frequency into velocity we get the units *(m/s)/(1/s) = meters*. If you had included the cycles label, that division would have yielded units of *(m/s)/(cycles/s) = meters/cycle*. There is really nothing wrong with this--it is a literal description of what the wavelength is (the number of meters there is in one wave--one cycle), but using it could potentially get you in trouble later. Best go with *seconds*$^{-1}$ for simplicity.

b.) For $\nu = 20{,}000\ hz$:

$$\begin{aligned}\lambda &= v/\nu \\ &= (330\ \text{m/s})/(20{,}000\ \text{hz}) \\ &= .0165\ \text{meters} \qquad \text{(a little over half an inch).}\end{aligned}$$

10.15)

a.) The sketch is shown on the next page.

b.) If the first wave has an amplitude of $A_1 = 1\ meter$, the second largest amplitude wave will have an amplitude of approximately $A_2 = .33\ meters$ and the third approximately $A_3 = .2\ meters$.

Note that the largest wave (the *first wave* as defined above) has one half-wavelength in the same space that the *second wave* has 3 half-wavelengths and the *third wave* has 5 half-wavelengths. That means that if the first wave has a frequency of $\nu_1 = 1\nu$, the second wave will have a frequency of $\nu_2 = 3\nu$ and the third wave a frequency of $\nu_3 = 5\nu$.

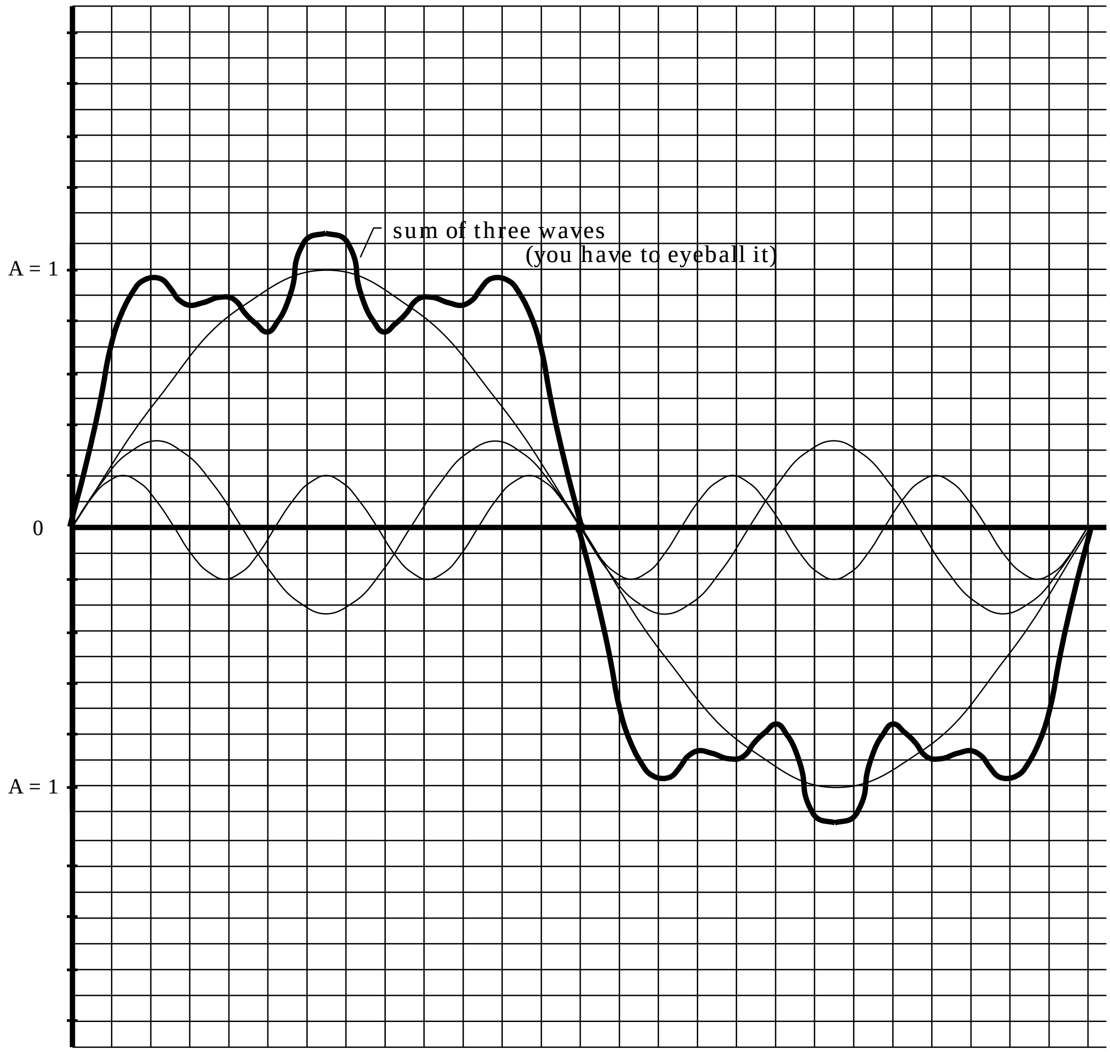


FIGURE I

Frequency is proportional to the *angular frequency*. That means that if the *angular frequency* of the first wave is $\omega_1 = 1\ rad/sec$, the second wave's angular frequency will be $\omega_2 = 3\ rad/sec$ and the third wave's angular frequency will be $\omega_3 = 5\ rad/sec$.

Putting it all together, remembering that the general algebraic expression for a sine wave with no phase shift is $A \sin \omega t$, we get:

$$y_{tot} = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t + A_3 \sin \omega_3 t$$
$$= 1 \sin t + .33 \sin 3t + .2 \sin 5t.$$
$$= (1/1) \sin t + (1/3) \sin 3t + (1/5) \sin 5t.$$

c.) The first six terms of the series are:

$$y_t = 1\sin 1t + (1/3)\sin 3t + (1/5)\sin 5t + (1/7)\sin 7t + (1/9)\sin 9t + (1/11)\sin 11t.$$

d.) The waveform is shown to the right. It is called a *square wave*.

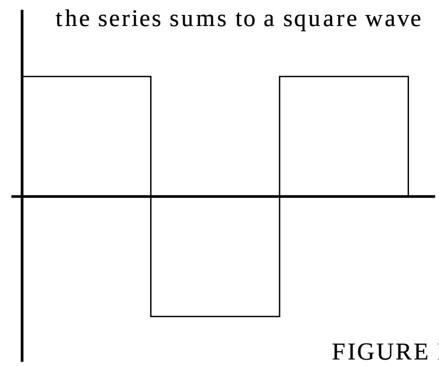


FIGURE II

10.16)

a.) At $t = 0$:

$$y = 12 \sin (25x - .67(0))$$
$$= 12 \sin 25x.$$

This function is graphed in Figure III to the right.

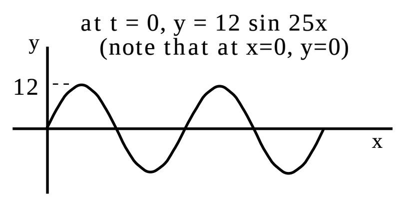


FIGURE III

At $t = 1$ *second*:

$$y = 12 \sin (25x - .67(1))$$
$$= 12 \sin (25x - .67).$$

This function is graphed in Figure IV to the right.

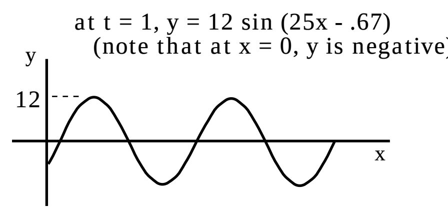


FIGURE IV

b.) The wave is moving to the right (note that its peaks are further to the right at $t = 1$ *second* than they are at $t = 0$ *seconds*).

c.) *Positive* values for the time dependent part of this equation yield wave motion (in time) to the left (in the negative *x direction*); *negative* values for the time dependent part of this equation yield wave motion to the right (i.e., in the *+x direction*), as was pointed out in *Part b*.

d.) The *angular frequency* is .67 rad/sec. As:

$$\omega = 2\pi\nu$$
$$\Rightarrow \quad \nu = \omega/2\pi$$
$$= (.67 \text{ rad/sec})/2\pi$$
$$= .107 \text{ Hz.}$$

e.) The period is:

$$T = 1/\nu$$
$$= 1/(.107 \text{ Hz})$$
$$= 9.35 \text{ sec/cycle.}$$

f.) We know that the wave number is $k = 25\ m^{-1}$. The wavelength is related to the wave number by:

$$k = 2\pi/\lambda$$
$$\Rightarrow \quad \lambda = 2\pi/k$$
$$= 2\pi/(25 \text{ m}^{-1})$$
$$= .25 \text{ meters.}$$

Note: Just as *angular frequency* tells you how many radians the wave sweeps through per unit time at a given point, the *wave number* tells you how many radians of wave there are per meter of wave.

Look at the units if this isn't clear. The equation states that there are (*2π radians/wavelength*) divided by (λ *meters* of wave per wavelength), or $2\pi/\lambda$ *radians per meter* of wave. Put another way, if $k = 2\pi$ *rad/m*, we are being told that one full cycle of wave (i.e., 2π radians worth) spans *one meter*.

g.) Wave velocity:

$$v = \lambda\nu.$$
$$= (.25 \text{ m})(.107 \text{ Hz})$$
$$= .02675 \text{ m/s.}$$

h.) The amplitude is 12 meters (by inspection).

10.17) We know that the frequency is 225 Hz, the amplitude is .7 meters, and the wave velocity is 140 m/s. Knowing the wave velocity, we can write:

$$\begin{aligned} v &= \lambda \nu \\ \Rightarrow \quad \lambda &= v/\nu \\ &= (140 \text{ m/s})/(225 \text{ Hz}) \\ &= .622 \text{ meters.} \end{aligned}$$

Traveling waves have a general algebraic expression of:

$$\begin{aligned} y &= A \sin (kx + \omega t) \\ &= A \sin [(2\pi/\lambda)x + 2\pi \nu t] \\ &= .7 \sin \big[[2\pi/(.622 \text{ m})]x + 2\pi(225 \text{ Hz})t\big] \\ &= .7 \sin (10.1x + 1413.7t). \end{aligned}$$

10.18) Dividing out the coefficient of the α term to get this equation in the right form (i.e., the standard *simple harmonic motion* equation), we get:

$$\alpha + (3g/2L)\ \theta = 0.$$

a.) The angular frequency for this motion is:

$$\begin{aligned} \omega &= (3g/2L)^{1/2} \\ &= [3(9.8 \text{ m/s}^2)/2(.8 \text{ m})]^{1/2} \\ &= 4.29 \text{ rad/sec.} \end{aligned}$$

Knowing this, we can find the natural frequency-of-oscillation for this system:

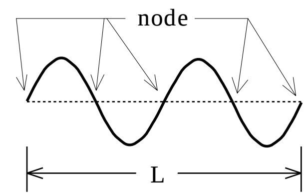
$$\begin{aligned} \nu &= \omega/2\pi \\ &= (4.29 \text{ rad/sec})/2\pi \\ &= .683 \text{ Hz.} \end{aligned}$$

b.) The period is:

$$\begin{aligned} T &= 1/\nu \\ &= 1/(.683 \text{ Hz}) \\ &= 1.46 \text{ sec/cycle.} \end{aligned}$$

If the frequency (hence period) of the applied force is close to the natural frequency (hence period) of the system, resonance will occur and the amplitude of the motion will grow immensely. If not, the applied force will fight the natural motion and the net effect will be small amplitude motion. The period in *Part i* (1.31 seconds) fits into the latter category; the period in *Part ii* (1.47 seconds) fits into the former category.

10.19) We know $\nu = 800\ Hz;\ L = .3\ meters$; and there are 5 nodes with one at each end (that is, the string is split into four sections by the three remaining nodes). A sketch of the system is shown to the right.



To get the velocity, we will use $v = \lambda\,\nu$. We know ν; we need λ. To get it, notice that there are TWO full wavelengths in the length L. Mathematically:

$$\begin{aligned} 2\lambda &= L \\ &= .3\ \text{m} \\ \Rightarrow \quad \lambda &= .15\ \text{m}. \end{aligned}$$

Putting it all together, we get:

$$\begin{aligned} v &= \lambda\,\nu \\ &= (.15\ \text{m})(800\ \text{Hz}) \\ &= 120\ \text{m/s}. \end{aligned}$$

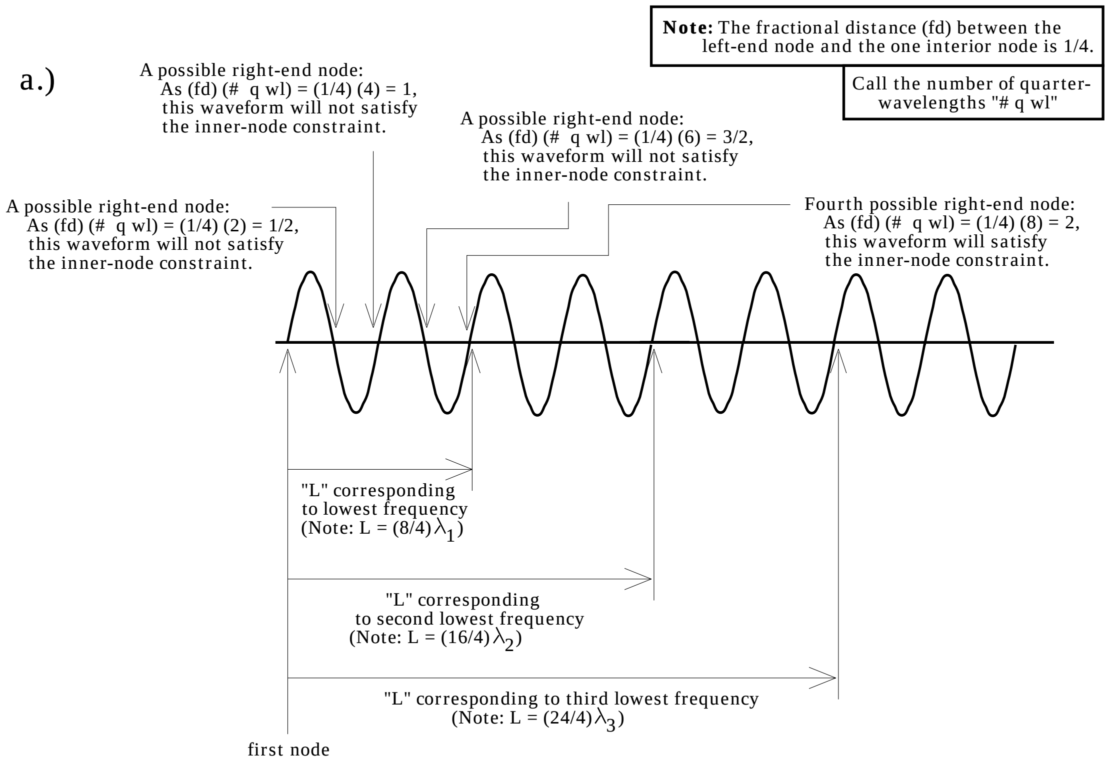
10.20) Calculations for all parts follow the sketches on the next few pages.

a.) We need a node at both ends and one *L/4* units from the left end. The *sine wave* on the next page depicts the various possibilities.

b.) We need a node at the ceiling, an antinode at the free end, and a node *(2/5)L* of the way down from the ceiling. The *sine wave* on the next page depicts the various possibilities.

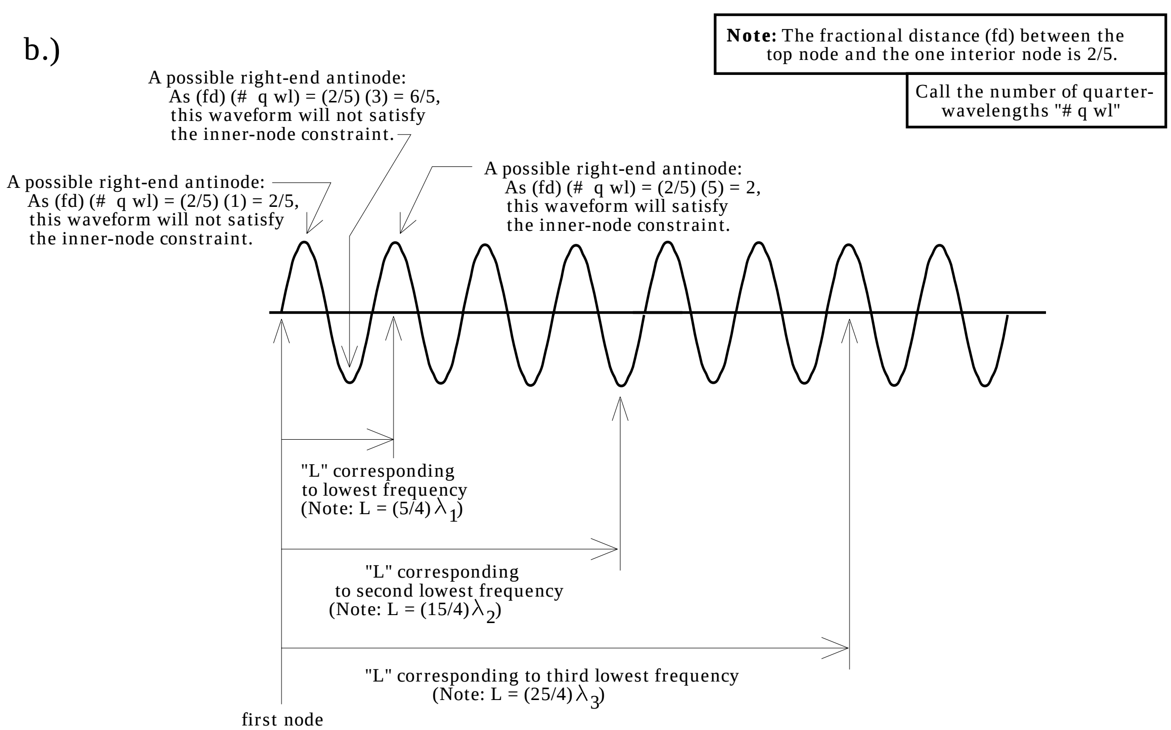
c.) We need an antinode at the top, a node at the bottom, and a node at *L/3* from the top. The *sine waves* on the next two pages depict the various possibilities (I've pictured the sine wave horizontally for simplicity).

a.)



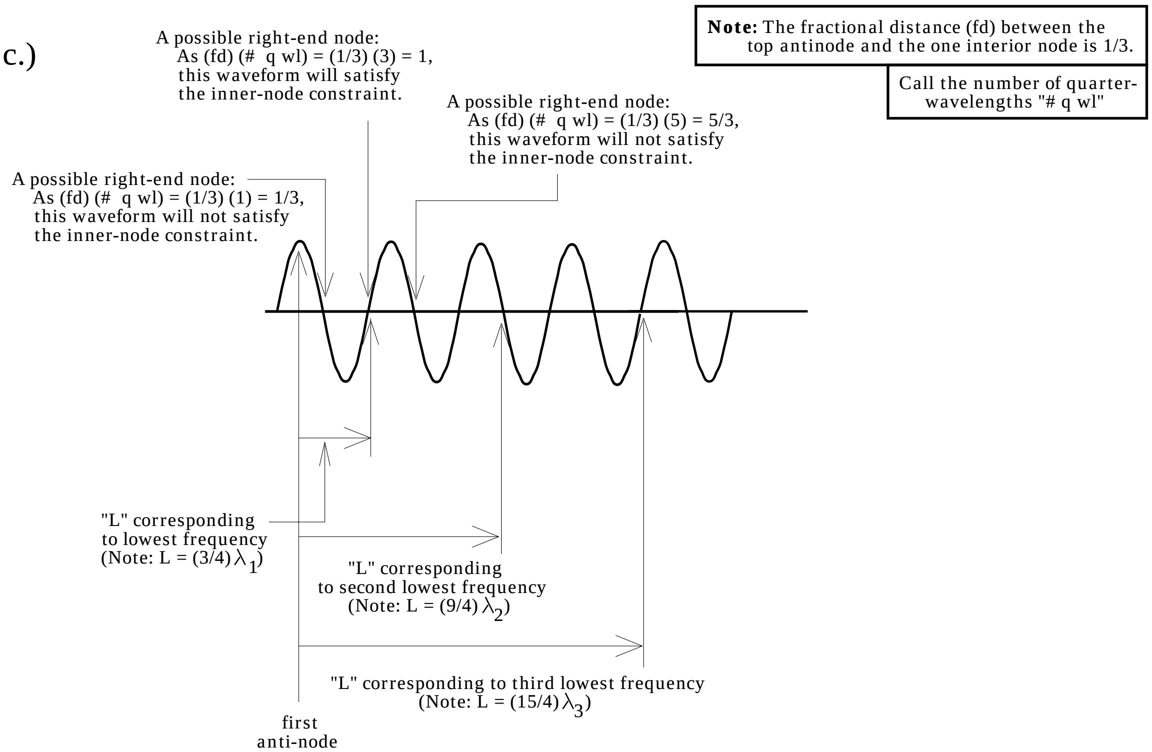
Note: The fractional distance (fd) between the left-end node and the one interior node is 1/4.
Call the number of quarter-wavelengths "# q wl"
A possible right-end node: As (fd) (# q wl) = (1/4) (4) = 1, this waveform will not satisfy the inner-node constraint.
A possible right-end node: As (fd) (# q wl) = (1/4) (6) = 3/2, this waveform will not satisfy the inner-node constraint.
A possible right-end node: As (fd) (# q wl) = (1/4) (2) = 1/2, this waveform will not satisfy the inner-node constraint.
Fourth possible right-end node: As (fd) (# q wl) = (1/4) (8) = 2, this waveform will satisfy the inner-node constraint.
"L" corresponding to lowest frequency (Note: L = (8/4)λ1)
"L" corresponding to second lowest frequency (Note: L = (16/4)λ2)
"L" corresponding to third lowest frequency (Note: L = (24/4)λ3)
first node

b.)



Note: The fractional distance (fd) between the top node and the one interior node is 2/5.
Call the number of quarter-wavelengths "# q wl"
A possible right-end antinode: As (fd) (# q wl) = (2/5) (3) = 6/5, this waveform will not satisfy the inner-node constraint.
A possible right-end antinode: As (fd) (# q wl) = (2/5) (1) = 2/5, this waveform will not satisfy the inner-node constraint.
A possible right-end antinode: As (fd) (# q wl) = (2/5) (5) = 2, this waveform will satisfy the inner-node constraint.
"L" corresponding to lowest frequency (Note: L = (5/4)λ1)
"L" corresponding to second lowest frequency (Note: L = (15/4)λ2)
"L" corresponding to third lowest frequency (Note: L = (25/4)λ3)
first node

c.)



AS FOR THE NUMBERS:

a.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/beam length ratio that leaves:

$$L = 6\lambda_3$$
$$\Rightarrow \quad \lambda_3 = L/6.$$

Using this with $v_{beam} = \lambda \nu$, we get:

$$\begin{aligned}\nu_3 &= v_{beam}/\lambda_3 \\ &= v_{beam}/(L/6) \\ &= 6v_{beam}/L.\end{aligned}$$

b.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/string length ratio that leaves:

$$L = (25/4)\lambda_3$$
$$\Rightarrow \quad \lambda_3 = 4L/25.$$

Using this with $v_{str} = \lambda\ \nu$, we get:

$$\nu_3 = v_{str}/\lambda_3$$
$$= v_{str}/(4L/25)$$
$$= 25v_{str}/4L.$$

c.) From the sketch it can be seen that the *third lowest frequency* corresponds to a wavelength/air-column-length ratio that leaves:

$$L = (15/4)\lambda_3$$
$$\Rightarrow \quad \lambda_3 = 4L/15.$$

Using this with $v_{air} = \lambda\ \nu$, we get:

$$\nu_3 = v_{air}/\lambda_3$$
$$= v_{air}/(4L/15)$$
$$= 15v_{air}/4L.$$

We can go a little further with this problem because we know that the velocity of sound in air is approximately 330 m/s. Putting that in yields:

$$\nu_3 = 15v_{air}/4L$$
$$= 15(330\text{ m/s})/4L$$
$$= 1237.5/L.$$

NOTE: The hard part of these problems is finding the appropriate *piece of sine-wave* (relating its wavelength to *L* isn't hard at all). Make sure you understand how to do this. If you are confused, come see me!