CHAPTER 4 -- KINEMATICS

QUESTION SOLUTIONS

4.1) Without using a formal presentation of formulas, determine the following in your head:

a.) The units you get when you multiply velocity and time.

Solution: This is an example of what is called *dimensional analysis*. The idea is that you can treat units the same way you treat numbers. So just as you can cancel out the *3's* in the expression (5/3)(3) leaving you with 5 as the solution, you can cancel out the *seconds* in the expression (meters/second)(second) leaving the units solution of *meters*. Noting that *meters/second* are the units for velocity and *seconds* are the units for time, the product of the two yields *meters* . . . or the units of *distance traveled*.

b.) The distance an object travels in 8 seconds when moving with a velocity magnitude of 6 m/s.

Solution: (6 meters/second) times (8 seconds) yields (48 meters).

c.) The units you get when you multiply acceleration and time.

Solution: Again, a *dimensional analysis* situation. Acceleration's units are (meters/second^2), so multiplying that by time yields (meters/second^2)(second) = (meters/second) . . . or the units of *velocity*.

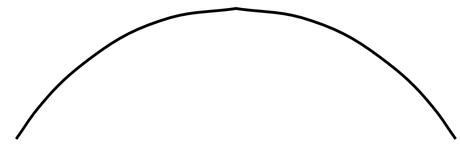
d.) The velocity an object will *pick up* in 7 seconds when moving under an acceleration magnitude of 5 m/s^2, assuming the velocity and acceleration are in the same direction.

Solution: (5 meters/second^2) times (7 seconds) yields (35 meters/second).

4.2) True or False: An object that negatively accelerates slows down.

Solution: The direction of an acceleration actually identifies for you the direction of the *change of velocity* of an object. The meaning of this is not intuitively obvious, at least as far as most people are concerned. The easiest way to get a handle on it is to notice that acceleration and net force are directly proportional to one another. The idea of a negative force isn't mysterious. If an object is moving in the negative direction and a force (hence acceleration) in the negative direction is applied to it, the body will speed up in the negative direction. By the same token, if an object is moving in the negative direction and a force (hence acceleration) in the positive direction is applied to it, the body will slow down. The rule of thumb is: if the net force (hence acceleration) is in the *same direction* as the velocity vector (i.e., they have the same sign), the body will speed up. If the net force (hence acceleration) is in the *opposite* direction of the velocity vector (i.e., they have different signs), the body will slow down. In short, a negative acceleration does NOT necessarily mean slowing down.

4.3) Think about a two dimensional projectile situation (someone throws a baseball in from the outfield). Once the ball has become free, and ignoring friction:



a.) Is there a point in the flight where the acceleration is perpendicular to the velocity? Explain.

Solution: Once thrown, the body freefalls. Ignoring friction, the only acceleration in that case is that of gravity, which is downward in the vertical. The direction of the velocity of an object is always tangent to its path. That tangent (i.e., the direction of the velocity) is perpendicular to the vertical (the direction of the gravitational acceleration) at the top of its flight.

b.) Is there a point in the flight where the velocity is zero but the acceleration is non-zero? Explain.

Solution: If students have dealt with one dimensional freefall problems, they might be tempted to say *yes* because the *y component* of velocity *is* zero at the top of the flight. What has to be remembered, at least in this case, is that velocity is a vector--it can have more than one component. For this situation, the *x component* is non-zero at the top. In fact, with no friction, the *x component* is ALWAYS the same value throughout the flight.

c.) Is there a point in the flight where a *component* of the flight's motion has zero velocity with a non-zero acceleration? Explain.

Solution: At the top of the arc the body's *y component* of velocity is zero while its *y component* of acceleration, being gravity, is non-zero.

d.) Is there a point in the flight where a *component* of the flight's motion has non-zero velocity with zero acceleration? Explain.

Solution: This is kinda tricky . . . and fun. Without friction, the *x component* of the velocity ALL THROUGHOUT THE MOTION is a constant while the *x component* of the acceleration is zero (not surprising--if the acceleration was non-zero, the velocity would be changing) . . . so there are an infinite number of points that fit the bill.

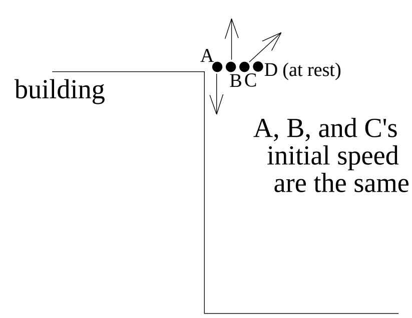
e.) Is there more than one point that fits the description outlined in *Part d*? Explain.

Solution: Yup. See above.

f.) Is there anywhere in the flight where the ratio of the acceleration in the *x direction* to the acceleration in the *y direction* is zero? Explain.

Solution: As you can't have a zero in the denominator, this situation can occur only when the acceleration in the *x direction* is zero. Of course, that is everywhere, so the answer is *yes, everywhere*.

4.4) *Rock A* is thrown vertically downward from a rooftop (see sketch). *Rock B* is thrown vertically upward. *Rock C* is thrown at an angle relative to the horizontal. *Rock D* drops from rest. All four are released from the same spot with those initially moving having the same velocity magnitude v_o. Assume we can neglect friction.



a.) Considering *Rock A* and *Rock B*, which of the following quantities is the same for both rocks:

i.) The time of flight to the ground;

Solution: *B* has farther to travel, net, to get to the ground, so its time of flight will be greater.

ii.) The velocity just before hitting the ground;

Solution: Velocity is a vector, so we would have to be careful if we were dealing with a two dimensional situation. In this case, the motion is one dimensional. As there is no friction, gravity will act on *B* slowing it down, then speeding it up after it makes the top and finally begins to move back toward the ground. As it passes through its initial position, *B's* velocity will, due to symmetry, be the same as its initial velocity (i.e., v_o). From that point on, its flight and *A's* flight will be exactly the same. In other words, they will have the same velocity just before hitting the ground.

iii.) The magnitude of the velocity just before hitting the ground;

Solution: Again, for a one dimensional situation without friction, the *magnitude* of the two velocities should be the same when they get to the ground.

iv.) The acceleration during the flight;

Solution: The acceleration in both cases is that of gravity, so it will be the same for both.

v.) The net displacement;

Solution: Displacement is defined as the net distance between the *start point* and the *finish point*, regardless of the path taken to get to that final point. Both start at the same point and end at the same point, so the net displacement will be the same for both.

vi.) The average speed to the ground;

Solution: The average speed is defined as the net distance traveled (*not* the net displacement) divided by the time it takes to get to the final point. The net distance traveled will be greater for *B* than for *A*, but the time will be greater, also. In other words, this isn't as clear cut a question as one might think. (The relationship wasn't obvious to me when I first looked at it, so I did the problem numerically and found that the average speeds are different.) To do the problem from a purely conceptual perspective, the following needs to be noticed: 1.) The average speed for the bottom part of *B's* flight will be the same as the average speed for all of *A's* flight because the two flights will be duplicates of one another (both move downward from the ledge with velocity v_o, etc.). 2.) IF the average speed for the upper part of *B's* flight (i.e., going up and coming back down to the start position) is the same as *B's* average speed for the bottom of its flight, the two average speeds will be the same and will equal the average speed for the whole flight. 3.) But that isn't what happens. The average speed for the bottom part will be *greater than* the initial speed of v_o (this just makes sense--the body is starting at v_o and picking up speed throughout that part of the flight--the average *has* to be greater than the initial speed v_o). By the same token, the average speed for the upper part will be *less than* v_o (this also makes sense as *B's* upper motion starts at v_o, then gets smaller and smaller until it hits the top where it is zero, then increases back to v_o--during that part of the motion, the average speed will be *less than* v_o). 4.) As *B's* average speed for the up/down part is different than its speed in the lower section, and as the average for the lower section must be the same for both *A* and *B*, the overall average speed for the two rocks must be different.

vii.) The average velocity to the ground.
Solution: The *average velocity* is the single constant velocity that will move the body through the appointed displacement in the appointed time. As such, it is defined as the *net displacement divided by time* (in this case, we're talking *time of flight*). Remembering that the displacement is the shortest distance between the beginning and ending points regardless of *how* you get from the one to the other, both rocks have the same displacement. The time of flight for each is different, though, so the two average velocities will be different.

b.) Considering *Rock B* and *Rock C*, which of the following quantities is the same for both situations:

i.) The time of flight to the ground;
Solution: In a two dimensional situation, the *x component* of motion occurs at a constant velocity. The time of flight is determined by the *y motion*--specifically, by *how high the object goes*. Because *C's* initial velocity is oriented at an angle with *B's* vertically upward, *B* will be in the air the longest.

ii.) The velocity just before hitting the ground;
Solution: This is a little tricky because it might be assumed that the *magnitude* of the velocities is what is being looked at (in fact, that is the next question). The two vectors, being directional creatures, are *not* the same for the two velocities--they are going in different directions just before they hit the ground--so at the very least the *direction* of their velocities will be different.

iii.) The magnitude of the velocity just before hitting the ground;
Solution: It is common for people to believe that because *C* isn't in flight for as long as *B* and, as a consequence, wouldn't have had as much time to pick up speed via gravity, the two final velocities will *not* be the same. In fact, it turns out that that is wrong--the two magnitudes will be the same. What is being missed is that *B* has no *x component* of velocity, whereas *C* does. So although *C's* vertical speed isn't as great as *B's*, its vertical speed coupled vectorially with its horizontal speed combine to give the two the same net velocity magnitude just before touchdown (the math bears this out as is shown in the special case outlined at the end of *Problem 4d*). When we get to energy conservation, this will be more easily seen.

iv.) The acceleration during the flight;
Solution: Acceleration is a vector. For the two accelerations to be the same, their *x* and *y components* must be the same. In this case, both experience a vertical acceleration component due to gravity, and neither has an acceleration component in the *x direction*. In other words, the acceleration vector is the same for both rocks.

v.) The net displacement;

Solution: A displacement vector starts at the beginning of the motion and ends at the conclusion of the motion. If two objects start at the same spot but don't have a common finish spot, their displacement can't be the same.

vi.) The average speed to the ground;

Solution: As before, this is probably the hardest question of the bunch. And as before, the average speeds will not be the same (same reasoning as used in the previous query of the same concept).

vii.) The average velocity to the ground.

Solution: The temptation is to go through an analysis like the one for the similar question in *Part 4a-vii*. In fact, that isn't necessary. Remember, the *average velocity* is the single constant velocity that will move the body through the appointed displacement in the appointed time. It is defined as the net displacement (as a vector) divided by time (the time of flight in this case) . The important thing to catch here is that the net displacement in these two cases are different (one body moves directly downward, the other moves downward and to the right). That means the *direction* of the average velocities will be different, which means the average velocities as vectors will be different.

c.) What is common to the flight of *Rock A* and *Rock D*?

Solution: Both rocks have no acceleration in the horizontal and gravitational acceleration in the vertical, and both will experience the same displacement.

d.) If rock *C's* angle had been zero degrees (that is , if it had been thrown horizontally), what would have been common to the flight of *Rock C* and *Rock D*?

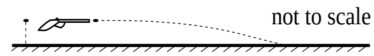
Solution: As with all frictionless freefall situations, both rocks have no acceleration in the horizontal and gravitational acceleration in the vertical. Unlike all freefall situations, both rocks in this case will take the same amount of time to hit the ground (this is surprising until you think about the force that is accelerating each rock toward the ground--in both cases, it's gravity . . . as there is no initial vertical velocity in either case, it'll take the same amount of time for each to cover that same vertical distance). Both rocks will have the same velocity magnitude just before touch down. This last one isn't really intuitively obvious because the vectors are different (one is purely vertical while the other has vertical and horizontal components), but if you VECTORIALLY add the constant horizontal velocity v_o of rock *C* to its final vertical velocity $(2gd)^{1/2}$, you'll get the same final velocity magnitude as for rock *D*.

4.5) An object accelerates from rest at a constant rate *a*. In time *t*, it travels *d* units. If the acceleration is doubled, how much time will it take to travel the same distance *d*?

Solution: Most people would probably analyze this by using the relationship between the acceleration and displacement of an object whose initial velocity was zero (i.e., $d = .5at_a^2$). Using that expression with acceleration a and then with $2a$ while letting d stay the same in both cases, the time relationship is found to be $t_{2a} = .707t_a$. This is not a straightforward, linear relationship. The problem is that in a time sense, acceleration is twice removed from displacement (i.e., it's the second derivative of displacement). If we had simply doubled a constant velocity, then it would take half the time to travel the distance *d*. But that's not what we're doing. We are doubling the rate of change of an already changing velocity, so the time relationship isn't so straightforward.

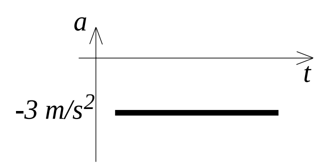
4.6) What's a jerk? (No, it's not the guy sitting next to you.)

Solution: Just as the change of velocity with time is called *acceleration*, the change of acceleration with time is called a *jerk*. It's actually a very descriptive term. When you are in an accelerating car, you feel a force pushing you back into the seat. If the acceleration increases abruptly, you will be jerked back further into the seat (or jerked forward if the acceleration decreases suddenly).



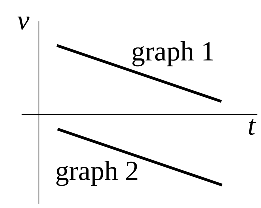
4.7) The muzzle velocity of a gun is 100 m/s. A bullet is fired horizontally from the gun when it is 2 meters off the ground. At the same time, a second bullet held next to the gun is dropped from rest. It takes the dropped bullet .64 seconds to hit the ground. Ignoring friction and assuming the terrain is flat, how far will the fired bullet travel before hitting the ground? (This is almost all conceptual--use your head a lot with only a little bit of math).

Solution: The key here is in the fact that the time it takes the second bullet to freefall 2 meters is the same time it takes the traveling bullet to freefall that same distance (remember, the traveling bullet has no vertical component of initial velocity). That means the traveling bullet moving at 100 m/s in the horizontal will continue moving with that velocity (with no friction, there is no horizontal acceleration to slow it down) for .64 seconds, moving a horizontal distance of (100 meters/second)(.64 second) = 64 meters.



4.8) A graph of the negative acceleration applied to two equal masses is shown. Mass A moves in the *+x direction* while mass B moves in the *-x direction.*

a.) Are either of the *velocity versus time* graphs shown associated with either particle? Explain.



Solution: The temptation is to assume that because you have a body moving in the positive direction, hence having a positive velocity, that *graph 1* should be matched up with that body (that is what *positive velocity* means). A similar argument could be made for the body with negative velocity. The problem with this is that we haven't determined whether the acceleration in each case is motivating the bodies to act as the velocity graph suggests. That is, is *mass A*'s velocity decreasing toward zero (i.e., is it slowing down) as is denoted in *graph 1*, or is *mass B's* velocity increasing away from zero (i.e., is it speeding up) as is denoted in *graph 2*? To determine this, the trick is to realize that the acceleration and the net force acting on an object are directly proportional. In other words, if the net force is positive and to the right, the acceleration will be positive and to the right. With that in mind, the first part of the question could be restated as: *A graph of the negative force applied to two equal masses is shown.* So think about it. If *mass A* is moving in the *+x direction* (that was given) and a negative acceleration (read that force) is applied to it, what is the body's velocity going to do? It is going to slow down--its velocity is going to approach *zero*. That is what *graph 1* depicts, so it must reflect the motion of *mass A*. By the same token, a negative acceleration (read that force) applied to *mass B* moving in the negative direction (it's direction of motion was given) is going to motivate the body to speed up in the negative direction--its velocity is going to proceed away from zero. That is the situation in *graph 2*. In short, both graphs are associated with one or the other mass.

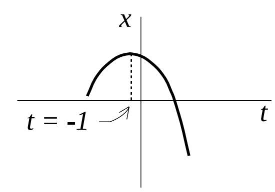
b.) How would things change if the acceleration had been positive?
Solution: For the two graphs to work, their slopes would have had to have been positive rather than negative.

4.9) A body moves along the *x axis* as depicted by the graph

a.) In what direction is the body moving at $t = -1$ *seconds*?
Solution: The slope of the tangent to an object's *position versus time* graph at a particular point yields the object's velocity (i.e., its *change of position with time*) at that point in time. At $t = -1$ *second*, the slope of the *position versus time* graph is zero. In other words, the body isn't moving in *any* direction at that time.

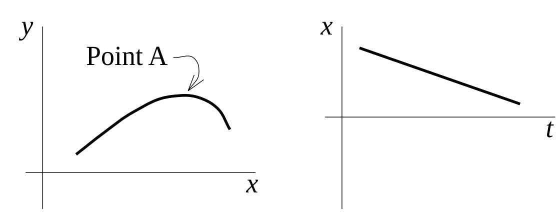
b.) In what direction is it moving at $t = +1$ *seconds*?
Solution: The direction of an object's velocity vector tells you the direction the object is moving at a given instant. The slope of the *position versus time* graph yields velocity. At $t = +1$ *seconds*, the slope (velocity) is negative . . . so the body must be moving in the negative direction.

c.) Is this a constant velocity situation? Explain.
Solution: As the slope of the *position versus time* graph is changing, the velocity is changing and it is not a constant velocity situation.



d.) Is this a constant acceleration situation? Explain.
Solution: The slope of the tangent to an object's *velocity versus time* graph at a particular point yields the acceleration of the object at that particular point in time (remember, acceleration is the measure of an object's *change* of velocity with time). For an acceleration to be constant, therefore, the slope of the *velocity versus time* graph must be constant (i.e., the graph must be linear). In other words, the velocity must be changing, but it must be doing it at a constant rate. So what kind of a *position versus time* graph yields a slope function (i.e., a velocity function) that changes constantly? It's a parabola. And as our *position versus time* graph looks like a parabola, it's likely that this is a constant acceleration situation.

4.10) The two graphs depict different characteristics of the motion of a mass. In what direction is the mass's velocity when at Point A? In what direction is the motion's acceleration?

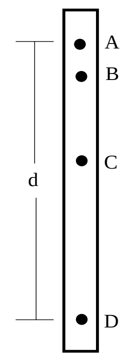


Solution: This is tricky because it isn't obvious what is and isn't important. Also, it turns out that some might think that we haven't enough information to answer the question (this isn't really the case, but one might be misled to believe so). The first thing to note is that the direction of motion at a given point must be tangent to the trace of the path at that point. Looking at *Point A* as depicted in the first graph, it is obvious that that tangent will be in the horizontal (i.e., in the *x direction*). At *Point A*, the body is *not* moving in the *y direction* at all! But is its *x motion* positive or negative? That is, is it moving to the right or the left in the first graph? The answer to that can be seen in the second graph.

The slope of the *position versus time* graph yields the velocity function for the motion. Notice that that slope is negative, which means the velocity is negative, which means the direction of motion is in the *-x direction*. (Wasn't that fun?) As for acceleration, if the path (shown in the first sketch) is bending downward at *Point A*, the change of velocity (if in no other way, in at least its direction) is downward which means its acceleration at *Point A* is directly downward.

4.11) Make up a conceptually based graphical question for a friend. Make it a real stinker, but give enough information so the solution *can* be had (no fair giving an impossible problem).

Solution: Whatever!



4.12) There is a classic experiment in which a tape freefalls through a timer that impresses a mark on the tape every 60^{th} of a second (see sketch). As the tape picks up speed, the marks become farther apart (note that the sketch is not necessarily to scale). Assume you can ignore friction.

a.) What is the ratio between the distance AB and the distance AD?

Solution: The distance traveled during a freefall from rest is $d = .5gt^2$. That means that distance is proportional to *time squared*. The time between any two consecutive dots is the same. For the points we are looking at, the ratio of times is 1:3 which means the ratio of distances must be 1:9.

b.) You measure the total distance between the four dots and call it d. What is the time duration over this interval? If you divide d by that time, what kind of quantity will it give you (think about its units . . .)?

Solution: The time interval over four dots (three intervals) is 3/60 of a second, or .05 seconds. Dividing that number into d will give you something whose units are centimeters/second, or a velocity. In fact, if you think about it, you are taking the total distance and dividing it by the total time. That yields the *average velocity* over the three-interval section.

c.) At what point in the AC interval is the average velocity and instantaneous velocity the same?

Solution: Some might think that the average velocity over an interval will be the same as the instantaneous velocity at the geometric center of the interval (i.e. measure the distance between *A* and *C* and divide it by 2). This can't be the case, though, as the velocity is getting bigger and bigger with time (therefore, you'd expect the average velocity to be larger than the velocity at the geometric center). It turns out that the average velocity over any interval is the same as the instantaneous velocity at the interval's *halfway time point*. For the AC interval, this is at *Point B*.

4.13) Two buildings stand side by side. The taller is 20 meters higher than the shorter. Rocks are dropped from rest from both roofs at the same time. When the rock from the taller building passes the top of the shorter building, the rock from the shorter building will be (a.) 20 meters below its start point; (b.) less than 20 meters below its start point; (c.) farther than 20 meters below its start point.

Solution: The answer is *a*. Both objects are falling with the same acceleration (gravity), and as both are accelerating without friction and with the same initial velocity, the two ought to stay the same distance apart throughout the motion.

4.14) A brick is thrown upward with velocity v_1. Two bricks stuck together are thrown upward with three times that velocity. If the first brick reaches a maximum height of H, how high will the two bricks go?

Solution: What is relevant here is the initial velocity (the fact that the masses are different is inconsequential as all objects in a frictionless setting will experience THE SAME gravitational acceleration regardless of their mass). So assuming everything is happening in the vertical, how is an object's initial velocity related to its maximum height when time of flight is not known? The relationship is generally governed by $v_2^2 = v_1^2 + 2ad$, where d is the distance traveled between the times t_1 and t_2. For a *maximum height* situation, $d = H$ and the velocity at the top is zero (that's what it means to be *at the top of the flight*--it's where the body pauses to stop moving upward and start moving downward). Putting $a = -g$, $d = H$, and $v_2 = 0$ into the general expression, we get $H = v_1^2/2g$. Conclusion: the maximum height is proportional to the square of the initial velocity in the *y direction*. As such, if the velocity of the second block is three times that of the first, it should go nine times as high.

4.15) An idiot drops a coke bottle out of the window of a Cesna aircraft flying in the horizontal. Ignoring air friction, what will determine how long it takes for the bottle to hit the ground? That is, what parameters (i.e., mass, height, velocity, what?) would you need to calculate the time of freefall?

Solution: Ignoring friction, time of flight is a function of height above the ground, initial velocity in the *y direction*, and acceleration in the *y direction* (i.e., gravity).

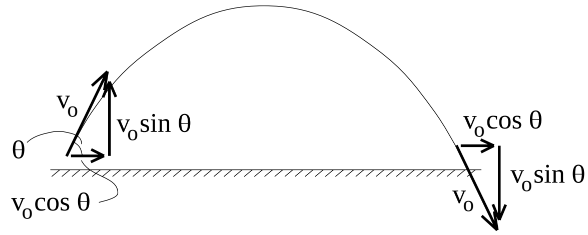
4.16) Two identical guns are fired from the same place at ground level on a horizontal range. One is angled at 20^o whereas the second is angled at 40^o. Ignoring friction:

a.) Which bullet would you expect to be in the air the longest?

Solution: The bullet with the largest initial *y component* of velocity will be in the air the longest. That is the second bullet.

b.) Which would go the farthest?

Solution: Distance traveled is a function of initial velocity in the *x direction* and time of flight. We know the 40^o bullet is in the air the longest, but that's because its initial *y component* of velocity is larger. Unfortunately, because that's true, its *x component* of velocity must be *smaller* than that of the 20^o bullet. It is possible, therefore, that the 20^o bullet, with its shorter time in the air but larger *x component* of velocity, might go farther than the 40^o bullet. The only way to really know which will prevail is to actually do the problem (the fact that things aren't obvious is OK--remember, we are trying to get people to THINK . . . if, after thinking, the conclusion drawn is that *this is convoluted*, that's fine). As for the math: What do we know? We know that we are assuming a frictionless set-up. We know that, due to the symmetry of the frictionless situation, each bullet will follow a parabolic arc. It also follows that the magnitude of the *y component* of the initial and final velocities *will be equal* (see the sketch) but will have opposite directions (i.e., $v_{f,y} = -v_{o,y}$). Finally, we know that the acceleration in the *x direction* is zero and in the *y direction* is *g*. As for relationships, $v_{f,y} = v_{o,y} + a_y t$ will allow us to determine the *time of flight* (in that expression, $a_y = -g$ and we could use any number for v_o --I used 100 m/s so that $v_{o,y} = 100\ sin\theta$). Approximating gravity to be equal to 10 m/s^2, the numbers for the 20^o time of flight are $t = (v_{f,y} - v_{o,y})/a_y = ((-v_{o,y}) - v_{o,y})/a_y = ((-v_{o,y}) - v_{o,y})/a_y = 2(-100\ sin\ 20^o)/(-10) = 6.84$ *seconds*. Likewise, the time for the 40^o flight is 12.86 seconds (note that though the angles are doubled, the flight times aren't . . . Why? Because the time of flight isn't a function of the angle, it's a function of the sine of the angle . . . the sine function isn't linear). With $a_x = 0$, the distance traveled is determined using $\Delta x = v_{o,x}t = v_o\ (cos\theta)t$. For the 20^o flight, this comes out to be 640 meters. For the 40^o flight, it's 984 meters. In other words, the 40^o flight will go the farthest . . . in this case, a lot farther.



Note that in a frictionless situation, the *x component* of the velocity doesn't change throughout the motion, and the initial *y component* of velocity is equal in magnitude and opposite in direction to the final *y component* of velocity.

c.) Which would go the highest?

Solution: You'd pretty much expect that the projectile with the largest initial *y component* of velocity will be in the air the longest and, as such, will travel the highest. In fact, that's true, so it's the 40^o flight.

d.) Which would be traveling the fastest as it hits the ground?

Solution: If air friction is neglected, which it is in this problem, the flight should follow a parabola. If the initial and final heights are the same, due to the symmetry of the problem, the velocity magnitudes (the speeds) should be the same at the beginning and end. As both projectiles started with the same speed, both should hit the ground with the same speed.

e.) Which would have experienced the greatest acceleration during the flight?

Solution: DURING THE FLIGHT, there will be no acceleration in the *x direction*. As for the *y direction*, gravity will accelerate both objects equally. In short, the two will experience exactly the same acceleration.

4.17) Answer all of *question 16* assuming air friction exists.

a.) Which bullet would you expect to be in the air the longest?

Solution: Even though friction is going to most affect the bullet that would have been in the air the longest if friction hadn't existed, the presence of friction in this case shouldn't be great enough to overcome the considerable disparity in the two flights (as determined in the previous problem). My guess is that the 40^o bullet will still stay in the air the longest.

b.) Which would go the farthest?

Solution: Again, friction will most affect the flight of the bullet that would have been in the air the longest, assuming no friction, but the 40^o bullet would still be expected to go the farthest (the difference in actual distances traveled, as determined in *Problem 16b*, is large enough to allow us to conclude this). If we tried to do this problem with numbers, we would have to add an acceleration due to friction into the equations used in *Problem 16b*. This would be very dicey to do as the *direction* of that acceleration would change (frictional forces are always opposite the direction of motion and, hence, would have components that change depending upon the direction of the projectile at a particular point in time). Assuming, though, that we could do a good job of approximating an average acceleration fudge factor for each component of motion, the procedure used to do the math would be similar to that used in the previous problem. On the whole, though, it would be a real pain in the arse to do.

c.) Which would go the highest?

Solution: You'd still expect the 40^o flight to go the highest.

d.) Which would be traveling the fastest as it hits the ground?

Solution: Finally, something different. With no air friction, due to the symmetry and the fact that their initial speeds were the same, the projectile that should slow down the most will be the one that is impinged upon by friction for the longest time. That, in this case, will be the 40^o bullet.

e.) Which would have experienced the greatest maximum acceleration during the flight?

Solution: If there was no air friction, the acceleration components would not change with time and would be the same for both cases. When air friction exists, though, it has been observed to be a function of the *bullet's velocity* (i.e., the faster the bullet is moving, the more air it runs into). In the *x direction* where there is no acceleration except that due to friction, the greatest acceleration will occur for the bullet with the greatest *x component* of velocity. That will occur for the 20^o bullet at the beginning of its motion (later in its motion, friction will have slowed the bullet down so that the velocity dependent retarding effect is lessened). As for the *y direction*, even though gravity will accelerate both bullets downward, the maximum acceleration will occur for the bullet that reaches the greatest velocity upward (in that way, the frictional force will oppose the upward motion producing a downward acceleration). This will happen at the beginning of the motion of the 40^o bullet.

4.18) As a projectile passes through its maximum height, little Mr. Know-It-All says, "Right now, the dot product of the velocity and the acceleration is zero." What do you think about that statement (aside from the possibility that little Mr. Know-It-All needs to get a life)?

Solutions: At the top of the flight, the velocity vector is tangent to the path (i.e., in the horizontal) and the acceleration is, assuming a frictionless situation, gravity in the vertical. The *dot product* between two vectors that are perpendicular to one another is, indeed, zero (the cosine of the angle between the two vectors is zero), so little Mr. Know-it-all is right . . . assuming there is no friction. If there is friction, then there will be an acceleration component in the horizontal and the little twit is wrong.

<u>PROBLEM SOLUTIONS</u>

4.19)

a.) The *total* distance traversed (versus the net displacement) divided by the elapsed time. That scalar is:

$$\begin{aligned} s &= \text{dist/time} \\ &= (440\text{ m})/(49\text{ sec}) \\ &= 8.98\text{ m/s.} \end{aligned}$$

b.) The magnitude of the *average* <u>*velocity*</u> is the *net displacement* divided by the elapsed time. That is:

$$\begin{aligned} \mathbf{v} &= \text{(net disp)/time} \\ &= (0\text{ m})/(49\text{ sec}) \\ &= 0\text{ m/s.} \end{aligned}$$

Making sense of this: The woman finished where she started, so her net displacement is zero. The average velocity tells us the constant velocity she would have to travel to effect that displacement in 49 seconds. That velocity is zero!

c.) You know nothing about her instantaneous velocity at any point in the motion--not even at the beginning (for all you know, she may have had a running start).

4.20) The only options we have are:

a.) The curve could be an *acceleration versus time* graph. For a given *time interval*, the area under such a curve yields the *velocity change* over the interval. As we have *not* been told about *time intervals* or *velocity changes*, we don't have enough information to say *yea* or *nay*.

b.) The curve could be a *velocity versus time* curve. If that be so, the velocity magnitude at $t = 1$ *second* should be $v = -1$ *m/s*. Unfortunately, the graph reads +1 at $t = 1$ *second*. The graph is not a *velocity versus time* graph.

c.) The slope of the tangent to a *distance versus time* graph at any given point equals the *instantaneous velocity* at that point in time. Taking the curve's slope at $t = 1$ *second* we get -1. The curve must, therefore, be a *position versus time* graph.

4.21)

a.) During a given time interval, the *net displacement* of a body is equal to the area under the *velocity versus time* curve. The *area under the curve* between $t = .5$ *seconds* and $t = 3$ *seconds* has two parts: one above the axis and one below the axis.

Noting that one square on the graph is equal to 1/8 meter, the *area above the axis* is eyeballed at approximately (+2.3 squares)(1/8 meter/square), or +.29 meters. The *area below the axis* is eyeballed at approximately (-22 squ)(1/8 meter/squ), or -2.75 meters.

The net distance traveled is approximately (.29 m) + (-2.75 m) = -2.46 meters. That is, the ant travels 2.46 meters to the left of its starting point.

Note 1: This number is *not* the ant's final position-coordinate. It is only the net distance the ant traveled *from its original position* during the time interval.

Note 2: Written out fully, $\Delta \boldsymbol{x} = -2.46\boldsymbol{i}$ *meters*.

b.) *Average velocity* is defined as the *net displacement per unit time* over a time interval. In *Part a*, we determined the ant's *net displacement* between the .5 second and 3 second mark as -2.46 meters. As the *time interval* is 2.5 seconds:

$$
\begin{aligned}
v_{avg} &= \text{(net disp)/(time interval)} \\
&= (-2.46\mathbf{i} \text{ meters})/(2.5 \text{ seconds}) \\
&= -.98\mathbf{i} \text{ m/s.}
\end{aligned}
$$

c.) Taking the information directly off the graph, the ant's velocity:

i.) At $t = .5$ *seconds* is approximately 1.3**i** m/s.

ii.) At $t = 3$ *seconds* is approximately -2.1**i** m/s.

d.) At a given point in time, the acceleration (i.e., the change of velocity with time) is the *slope of the tangent to the velocity curve*. Eyeballing it (watch the graph scaling), the slope of the tangent at:

i.) $t = .5$ *seconds* is approximately (-2)/(.5), or $\boldsymbol{a} = -4\boldsymbol{i}\ m/s^2$.

ii.) $t = 3$ *seconds* is approximately (1)/(2.25), or $\boldsymbol{a} = +.44\boldsymbol{i}\ m/s^2$.

e.) When the velocity is positive, the ant is moving in the *+x direction* (the direction of motion is the direction of the instantaneous velocity). This occurs between $t = .3$ *seconds* and $t = 1$ *second*.

f.) When the velocity is zero, the ant is standing still. This occurs at $t = 1$ *second*.

g.) The acceleration is zero when the slope of the tangent to the curve is zero (that is, when the velocity is not changing). This occurs at times $t = 2.9$ *seconds* and $t = 3.6$ *seconds*.

h.) When the *slope* of the velocity graph *changes*, we have what are called *inflection points*. To determine one, we need to observe two things:

i.) When the acceleration is *not changing*, the acceleration is constant.

ii.) A constant acceleration generates a *velocity function* that *changes linearly* (i.e., the velocity changes at a constant rate).

The interval over which the velocity seems to be *changing* linearly is between $t = .3$ *seconds* and $t = .5$ *seconds*, at *3.25 seconds*, and maybe between $t = 4$ *seconds* and $t = 4.3$ *seconds* (though this latter suggestion is debatable).

4.22) The velocity function is $\boldsymbol{v}(t) = (3k_1t^2\boldsymbol{i} - 4k_2t\boldsymbol{j})\ m/s$.

a.) The x and y velocity quantities both have to end up with units of m/s. If, in the case of the $\boldsymbol{i}$ component, there was no time dependence (i.e., no t^2 term in the expression), the k_1 term would simply have the units m/s. With the presence of the t^2 term, though, that has to be altered to m/s^3. How do you know? Because if you multiply out the units of k_1t^2, you get $(m/sec^3)(sec)^2 = m/s$.

By the same token, because the y component is a function of t, the units of k_2 must be m/s^2.

b.) Plugging $t = 2$ *seconds* into the *velocity function* yielding:

$$\begin{aligned}\mathbf{v}(t) &= (3k_1t^2\mathbf{i} - 4k_2t\mathbf{j})\ .\\ &= [3(1\ \text{m/sec}^3)(2\ \text{sec})^2\mathbf{i} - 4(1\ \text{m/sec}^2)(2\ \text{sec})\mathbf{j}]\\ &= [12\mathbf{i} - 8\mathbf{j}]\ \text{m/s}.\end{aligned}$$

c-i.) Kinematics can be used only when the acceleration is constant. That corresponds to a situation in which the velocity is linear in time. That is the case in the *y* direction, but not in the *x* direction.

c-ii.) You have been asked to use kinematics in the one direction in which kinematics is actually justified (clever, eh?). In that direction, the velocity at time $t = 3$ *seconds* is $4k_2t = 4(1\ m/sec^2)(3\ sec) = 12\ m/s$, and the velocity at time $t = 5$ *seconds* is $20\ m/s$. The acceleration is $a = (v_2 - v_1)/t = [(20\ m/s) - (12\ m/s)]/(2\ sec) = 4\ m/s^2$, and the distance traveled over that period of time is $\Delta x = v_1 \Delta t + (1/2)a(\Delta t)^2 = (12\ m/s)(2\ sec) + .5(4\ m/s^2)(2\ sec)^2 = 32\ meters$.

4.23) We know the bats' initial vertical velocity $v_1 = 0$, their initial height $h_1 = 100$ *meters*, their pull-out height $h_2 = 3$ *meters*, and their acceleration a is minus the magnitude of the acceleration of gravity, or $a = -g = -9.8\ m/s^2$:

a.) We want v_2. Scanning the kinematic equations (shown below),

$$(x_2 - x_1) = v_1 \Delta t + (1/2)a(\Delta t)^2$$
$$(x_2 - x_1) = v_{avg}\Delta t \quad \text{or} \quad v_{avg} = (x_2 - x_1)/\Delta t$$
$$v_{avg} = (v_2 + v_1)/2$$
$$a = (v_2 - v_1)/\Delta t \quad \text{or} \quad v_2 = v_1 + a\Delta t$$
$$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1),$$

we decide to use:

$$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$$
$$\Rightarrow \quad v_2 = [(0)^2 + 2(-9.8\ m/s^2)[(3\ m) - (100\ m)]]^{1/2}$$
$$= 43.6\ m/s.$$

Note: This equation yields the *magnitude* of the velocity only (the quantity v_2 is *squared* in the equation). If we use this value in subsequent problems, we must make it negative (the body is moving downward in the *-**j** direction*).

$$\Rightarrow \quad \mathbf{v}_2 = -43.6\mathbf{j}\ m/s.$$

b.) We know that at 3 meters above the ground, $y_2 = 3$ *meters* and $v_2 = -43.6$ *m/s* (we got the latter from *Part a*). We want the pull-out acceleration a_p executed between $y_2 = 3$ *meters* and $y_3 = 1$ *meter*. Scanning the kinematic equations, we find that the same equation used in *Part a* will do the job:

$$(v_3)^2 = (v_2)^2 + 2a_p(x_3 - x_2)$$
$$\Rightarrow \quad (0)^2 = (-43.6 \text{ m/s})^2 + 2a_p[(1 \text{ m}) - (3 \text{ m})]$$
$$\Rightarrow \quad a_p = 475 \text{ m/s}^2.$$

Note 1: We used the same equation in *Parts a* and *b* even though the two situations are related to entirely different sections of motion. The moral of the story: these equations work between *ANY* two points for which you have information.

Note 2: That the acceleration is positive should not be surprising. The net force and the acceleration are proportional; it will take a positive force (i.e., a force upward in the ***+j** direction*) to stop the bats' freefall.

c.) We know the velocity at the beginning of the pull-out ($v_2 = -43.6$ *m/s*), the velocity at the end of the pull-out ($v_3 = 0$), and the acceleration through the pull-out ($a_p = 475$ *m/s*^{2}). To determine the time:

$$a = (v_3 - v_2)/\Delta t$$
$$\Rightarrow \quad \Delta t = (v_3 - v_2)/a$$
$$= [(0) - (-43.6 \text{ m/s})]/(475 \text{ m/s}^2)$$
$$= .092 \text{ seconds.}$$

4.24) The stunt woman's velocity at *Point A* is $v_A = -25$ *m/s*. The acceleration is still $a = -g$.

a.) After 2 seconds, she has moved a distance:

$$(y_B - y_A) = v_A \Delta t + (1/2)a(\Delta t)^2$$
$$= (-25 \text{ m/s})(2 \text{ sec}) + .5(-9.8 \text{ m/s}^2)(2 \text{ sec})^2$$
$$= -69.6 \text{ meters.}$$

b.) Her velocity at *Point B*:

$$\begin{aligned} v_B &= v_A + a\Delta t \\ &= (-25 \text{ m/s}) + (-9.8 \text{ m/s}^2)(2 \text{ sec}) \\ &= -44.6 \text{ m/s}. \end{aligned}$$

4.25) Because there are two cars in the system, we have two sets of kinematic equations available to us (one for the motion of each car). There are two common quantities that will link the two sets of equations during the time interval between the first pass and the second pass. They are: 1.) both cars will travel for the same amount of time during the interval; and 2.) their last-pass coordinate will be the same (we'll call this the "final" coordinate for simplicity).

If we define the origin (i.e., $x_1 = 0$) at the point where the two cars *first pass* and define the *last pass* position as x_2:

a.) For the *first car* whose initial velocity is 18 m/s and whose acceleration is zero:

$$\begin{aligned} & (x_2 - x_1) = v_{fst,1}\Delta t + (1/2)a_{fst}(\Delta t)^2 \\ \Rightarrow\quad & (x_2 - 0) = (18 \text{ m/s})t + .5(0)t^2 \\ \Rightarrow\quad & x_2 = 18t \qquad\qquad \text{(Equation 1).} \end{aligned}$$

--For the *second car* whose initial velocity is 4 m/s and whose acceleration is 6 m/s^2:

$$\begin{aligned} & (x_2 - x_1) = v_{sec,1}\Delta t + (1/2)a_{sec}(\Delta t)^2 \\ \Rightarrow\quad & (x_2 - 0) = (4 \text{ m/s})t + .5(6\text{m/s}^2)t^2 \\ \Rightarrow\quad & x_2 = 4t + 3t^2 \qquad\qquad \text{(Equation 2).} \end{aligned}$$

Equating the two independent expressions for x_2 (i.e., Equations 1 and 2) yields:

$$18t = 4t + 3t^2.$$

Dividing by t yields:

$$\begin{aligned} & 18 = 4 + 3t \\ \Rightarrow\quad & t = 4.67 \text{ seconds.} \end{aligned}$$

b.) Using Equation 1 yields:

$$\begin{aligned} x_2 &= 18t \\ &= 18(4.67 \text{ seconds}) \end{aligned}$$

$$= 84.1 \text{ meters.}$$

Using Equation 2 yields:

$$\begin{aligned} x_2 &= 4t + 3t^2 \\ &= 4(4.67 \text{ sec}) + 3(4.67 \text{ sec})^2 \\ &= 84.1 \text{ meters.} \end{aligned}$$

EITHER EQUATION WILL DO!

c.) The *second car's* velocity as it passes the first car:

$$\begin{aligned} v_{sec,2} &= v_{sec,1} + a_{sec}\Delta t \\ &= (4 \text{ m/s}) + (6 \text{ m/s}^2)(4.67 \text{ sec}) \\ &= 32.02 \text{ m/s.} \end{aligned}$$

An alternative approach would be:

$$\begin{aligned} (v_{sec,2})^2 &= (v_{sec,1})^2 + 2a_{fst}(x_p - x_o) \\ &= (4 \text{ m/s})^2 + 2(6 \text{ m/s}^2)[(84.1 \text{ m}) - 0] \\ &= 32.02 \text{ m/s.} \end{aligned}$$

EITHER APPROACH WORKS!

d.) The *second car's* average velocity:

$$\begin{aligned} v_{avg} &= (v_{sec,2} + v_{sec,1})/2 \\ &= (32.02 \text{ m/s} + 4 \text{ m/s})/2 \\ &= 18.01 \text{ m/s.} \end{aligned}$$

e.) Time for the *second car* to travel to a velocity of 100 m/s:

$$\begin{aligned} a_{sec} &= (v_{100} - v_{sec,1})/\Delta t \\ \Rightarrow \quad Dt &= (100 \text{ m/s} - 4 \text{ m/s})/(6 \text{ m/s}^2) \\ &= 16 \text{ seconds.} \end{aligned}$$

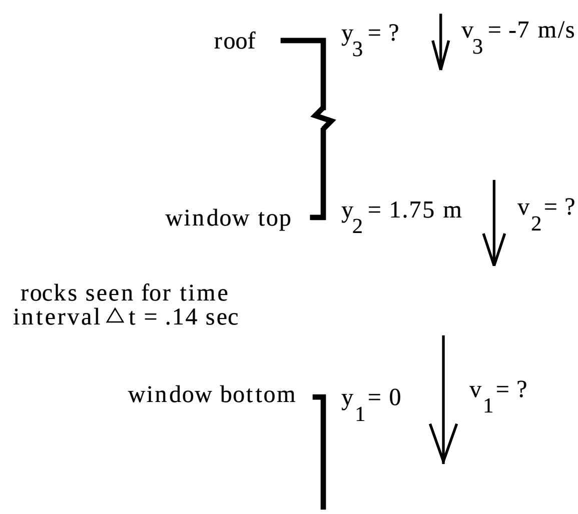
4.26)

a.) All the known information has been inserted into the sketch to the right. The coordinate axis has been placed so that $y_1 = 0$ at the bottom of the window. We will proceed by writing out kinematic relationships that will allow us to solve for information we need.

To determine the velocity of the rock when it is at the *top of the window*, we will define the "final" position to be at y_1 with an "initial" position at y_2. With that, we can write:

$$(y_1 - y_2) = v_2 \Delta t + (1/2)a(\Delta t)^2$$
$$(0) - (1.75 \text{ m}) = v_2(.14 \text{ s}) + (1/2)(-9.8 \text{ m/s}^2)(.14 \text{ s})^2$$
$$\Rightarrow \quad v_2 = [(-1.75 \text{ m}) - .5(-9.8\text{m/s}^2)(.14 \text{ s})^2]/ (.14 \text{ s})$$
$$= -11.8$$

m/s.



Note: The negative sign makes sense considering the fact that the rock is moving *downward.*

b.) We want the distance between the *top of the building* and the *bottom of the window* (this will numerically equal y_3). An equation with known values and y_3 is (note that the "final" position here is y_2):

$$(v_2)^2 = (v_3)^2 + 2 \quad a \quad (y_2 - y_3)$$
$$(-11.8 \text{ m/s})^2 = (-7 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)[(1.75 \text{ m}) - y_3)]$$
$$\Rightarrow \quad y_3 = 6.35 \text{ meters.}$$

c.) A change in the initial velocity of the rock does nothing to the acceleration of the rock. Once it becomes free, the rock picks up velocity due to gravity at a rate of *9.8 m/s* <u>every second</u>, no matter what. Tricky, eh?

4.27) The car's initial velocity is $v_1 = 40$ *m/s*; its pedal-to-the-metal acceleration is $a_{go} = +3$ *m/s*^{2} and its stopping acceleration is $a_{stop} = -3$ *m/s*^{2}. The light stays yellow for 1.2 seconds before turning. We need to consider two situations: a.) what is the maximum distance the car can be from the restraining line if it is to successfully accelerate all the way through the intersection in 1.2 seconds; and b.) what is the minimum distance the car can be from the restraining line if it is to brake successfully?

a.) Pedal-to-the-metal: Taking Δx_{go} to be the distance from the restraining line to the point at which the car begins its acceleration, the car must go Δx_{go} *+ 18 meters* to make it *through the intersection* without incident (remember, the intersection extends 18 meters beyond the restraining line). With 1.2 seconds to accomplish the feat, we can write:

$$
\begin{aligned}
(\Delta x_{go} + 18) &= v_1 \Delta t + (1/2)a_{go}(\Delta t)^2 \\
\Rightarrow \quad \Delta x_{go} &= (-18 \text{ m}) + (40 \text{ m/s})(1.2 \text{ sec}) + .5(+3 \text{ m/s}^2)(1.2 \text{ sec})^2 \\
&= 32.16 \text{ meters.}
\end{aligned}
$$

If the car is closer than 32.16 meters, it can accelerate and still make it across the intersection. If the car is farther than 32.16 meters, it will not be able to accelerate completely through the intersection.

b.) Braking: The 1.2 seconds is useless in this section. It doesn't matter whether the car is sliding while the light is red or not. All that matters is that the car stop just behind the restraining line. Putting the origin at the restraining line (i.e., $x_{r.l.} = 0$), and defining x_{brake} to be the position at which the brakes must be hit to effect the stop right at the restraining line, we can write:

$$
\begin{aligned}
v_{r.l.}^2 &= v_{brk}^2 + 2a(x_{r.l.} - x_{brk}) \\
\Rightarrow \quad x_{brk} &= -(v_{r.l.}^2 - v_{brk}^2)/(2a) \\
&= -[0^2 - (40 \text{ m/s})^2]/[2(-3 \text{ m/s}^2)] \\
&= -267 \text{ meters.}
\end{aligned}
$$

Note: The negative sign simply means that a car moving to the right in the positive direction must begin to stop to the *left* of the origin (the restraining line).

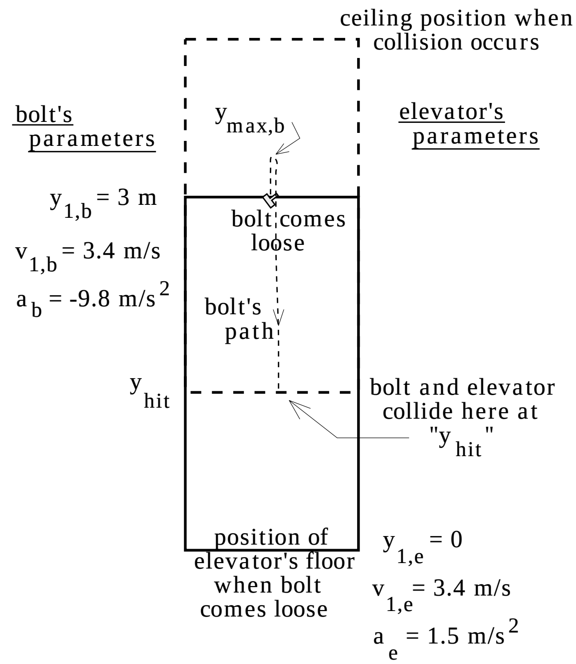
c.) Bottom Line: The car eats it no matter what if it is between 267 meters and 32 meters of the restraining line.

4.28)

a.) We need a *fixed* coordinate axis from which to make our measurements. We could take ground level as our $y = 0$ level, but instead we will take the position in space of the *elevator's floor* just as the bolt releases (this will be useful because it is the elevator's floor that will come in contact with the bolt at the *collide* position).

The bolt's initial velocity is the same as that of the elevator at the moment it becomes free. That velocity is upward, so the bolt will move above its *suddenly-free* position to some maximum position y_{max} before beginning to descend. WHENEVER YOU ARE LOOKING FOR y_{max}, ALWAYS USE:

$$v_{top}{}^2 = v_1{}^2 + 2a(y_{top} - y_1).$$



As we know the bolt's velocity at the top of its motion is $v_{top} = 0$, we can write:

$$v_{top}{}^2 = v_{1,b}{}^2 + 2a_b(y_{top} - y_{1,b}).$$

$$\Rightarrow \quad (0)^2 = (3.4\ \text{m/s})^2 + 2(-9.8\ \text{m/s}^2)[y_{top} - (3\ \text{m})]$$

$$\Rightarrow \quad y_{top} = 3.59\ \text{meters}$$

$$\Rightarrow \quad y_{max\ above\ ground} = y_{top} + 4 = 7.59\ \text{meters}.$$

b. and c.) Finding the *time of flight* Δt and the *final coordinate position* y_{hit} requires two equations solved simultaneously. As such, we will do both *Parts b* and *c* in this section.

The first thing to notice is that this is really two problems happening at the same time. During the bolt's freefall, the elevator is accelerating upward by its motor while the bolt is accelerated downward by gravity. Treating the two entities as individuals:

i.) For the elevator's motion (or, at least, the motion of the elevator's floor), assuming $y = 0$ is the floor's position when the bolt releases:

$$y_{hit} = y_{1,e} + \quad v_{1,e}\,\Delta t \quad + (1/2) \quad a_e \quad (\Delta t)^2$$

$$= (0) + (3.4\text{ m/s})t + .5\ (1.5\text{ m/s}^2)\ t^2 \qquad \text{(Equ. A).}$$

ii.) For the bolt's motion, assuming that we are using the same coordinate axis as above so that the bolt's initial position is *y = 3 meters*:

$$y_{hit} = y_{1,b} + v_{1,b}\,\Delta t + (1/2)\ a_b\ (\Delta t)^2$$
$$= (3\text{ m}) + (3.4\text{ m/s})t + .5\ (-9.8\text{ m/s}^2)\ t^2 \qquad \text{(Equ. B).}$$

iii.) Equating *Equations A* and *B* (y_{hit} is the same in both equations):

$$(3.4\text{ m/s})t + .5(1.5\text{ m/s}^2)\ t^2 = 3 + (3.4\text{ m/s})t + .5(-9.8\text{ m/s}^2)t^2$$
$$\Rightarrow \quad 5.65t^2 = 3$$
$$\Rightarrow \quad t = .73\text{ seconds.}$$

iv.) Using *Equation A* to determine y_{hit}:

$$y_{hit} = y_{1,e} + v_{1,e}\ \Delta t + (1/2)\ a_e\ (\Delta t)^2$$
$$= (0) + (3.4\text{ m/s})(.73\text{ s}) + .5\ (1.5\text{ m/s}^2)\ (.73\text{ s})^2$$
$$= 2.88\text{ meters.}$$

v.) The bolt was initially at *y = 3 meters*. When it hits the floor, its coordinate is $y_{hit} = 2.88$ *meters*. The distance the bolt actually falls is:

$$\Delta y = y_{final} - y_{initial}$$
$$= (2.88\text{ m}) - (3.0\text{ m})$$
$$= -.12\text{ meters.}$$

vi.) For the amusement of it, let's check our y_{hit} value by solving *Equation B*:

$$y_{hit} = y_{1,b} + v_{1,b}\ \Delta t + (1/2)\ a_b\ (\Delta t)^2$$
$$= (3\text{ m}) + (3.4\text{ m/s})(.73\text{ s}) + .5\ (-9.8\text{ m/s}^2)(.73\text{ s})^2$$
$$= 2.87\text{ meters.}$$

. . . close enough for government work.

d.) The magnitude of the velocity of the bolt, relative to a fixed frame of reference (i.e., *not* relative to the elevator's floor) is determined as follows:

$$v_{b,bot}{}^2 = v_{1,b}{}^2 + 2 a_b [y_{b,bot} - y_{1,b}]$$
$$= (3.4 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)[(2.87 \text{ m}) - (3 \text{ m})]$$
$$\Rightarrow \quad v_{b,bot} = 3.76 \text{ m/s}.$$

As a vector, this would be $\mathbf{v}_{b,bot} = -(3.76 \text{ m/s})\mathbf{j}$.

4.29)

a.) In projectile motion, questions like, "How far (Δx)" and "How long (Δt)" are usually approached simultaneously. In this case, the *time* variable is most easily determined by looking at the motion in the *y direction* only. Noting that the ball started at $y_1 = 1.3$ *meters*, ended at $y_2 = 0$ (i.e., at ground level), and had a *y component* velocity of $v_y = 41 \sin 50^o$, we can write:

$$(y_2 - y_1) = v_{1,y}t + (1/2)a_y t^2$$
$$(0 \text{ m} - 1.3 \text{ m}) = (41 \sin 50^o)t + .5(-9.8 \text{ m/s}^2)t^2$$
$$-1.3 = 31.4t - 4.9t^2$$
$$\Rightarrow \quad 4.9t^2 - 31.4t - 1.3 = 0.$$

Using the Quadratic Formula, we get:

$$t = [-b \pm [b^2 - 4ac]^{1/2}]/2a$$
$$= [-(-31.4) \pm [(-31.4)^2 - 4(4.9)(-1.3)]^{1/2}]/[2(4.9)]$$
$$= [31.4 \pm (31.8)]/(9.8)$$
$$= 6.45 \text{ seconds}.$$

b.) Assuming $x_1 = 0$ is the *x coordinate* at which the ball is struck, the net horizontal distance traveled will be:

$$(x_2 - x_1) = v_{1,x}t + (1/2)a_x t^2$$
$$(x_2 - 0) = (v_1 \cos q)t + .5(0 \text{ m/s}^2)t^2$$
$$\Rightarrow \quad x_2 = (41 \text{ m/s})(\cos 50^o)(6.45 \text{ sec}) + 0$$
$$= 170 \text{ meters}.$$

c.) Height is a *y* related quantity. Knowing that the *y component* of velocity at the top of the arc (i.e., at y_{max}) will be zero, we can write:

$$(v_{max,y})^2 = (v_{1,y})^2 + 2a_y(y_{max} - y_1)$$

$$\Rightarrow \quad y_{max} = [(v_{max,y})^2 - (v_{1,y})^2 + 2a_y(y_1)]/2a_y$$
$$= [(0)^2 - (41 \sin 50^o)^2 + 2(-9.8 \text{ m/s}^2)(1.3 \text{ m})]/[(2)(-9.8 \text{ m/s}^2)]$$
$$= 51.6 \text{ meters.}$$

d.) At the end of the flight, the ball has an *x component* of velocity that has not changed throughout the motion (we are neglecting air friction and there are no other natural forces out there to make the *x motion* change). That value will be:

$$v_{2,x} = (41 \text{ m/s})(\cos 50^o)$$
$$= 26.4 \text{ m/s.}$$

The *y motion* velocity has changed because gravity has been accelerating the ball throughout the motion. We can get the *y* velocity using:

$$(v_{2,y})^2 = (v_{1,y})^2 + 2a_y(y_2 - y_1)$$
$$\Rightarrow \quad v_{2,y} = [(v_{1,y})^2 + 2a_y(y_2 - y_1)]^{1/2}$$
$$= \left[[(41 \text{ m/s})(\sin 50^o)]^2 + 2(-9.8 \text{ m/s}^2)(0 - 1.3 \text{ m})\right]^{1/2}$$
$$= 31.8 \text{ m/s.}$$

Note: This equation yields <u>magnitudes only</u>. The *y component* of the "final" velocity is in the ***-j** direction.* That means $v_{2,y} = -31.8$ *m/s.*

Putting it all together:

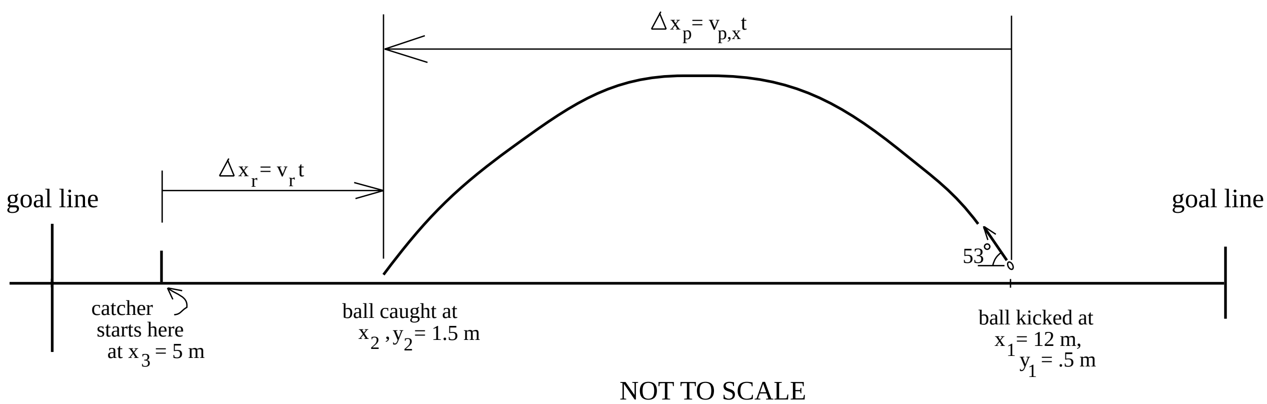
$$\mathbf{v}_2 = (26.4\mathbf{i} - 31.8\mathbf{j}) \text{ m/s.}$$

4.30) Whenever you do a problem in which two bodies are moving independently, you have to find common parameters that will allow you to link the kinematic equations you write for one body to the kinematic equations you write for the other body. In this particular problem, what is common to both the runner and the punted ball is time and position. Specifically, the time the ball is in the air is the same as the time the runner runs, and the ball's "final" *x coordinate* is the same as the "final" coordinate of the runner as he catches the ball.

Having said that, this is one of those problems that is best done by simply playing with the information you have. Look to see what is given, then just tinker with the information and accumulate whatever other information you can.

Bottom line, though, is that we need to determine where the ball comes down (i.e., what its *x coordinate* is when its *y* coordinate is 1.5 meters), and how long it takes for the runner to get to that coordinate.

The sketch should help.



To get the time of flight (i.e., how long the ball is in the air before it gets to the *y = 1.5 meter* mark--this is also the time the runner runs), we use

$$(y_2 - y_1) = v_{1,y}t + (1/2)a_y t^2$$
$$(1.5\text{ m} - .5\text{ m}) = (20 \sin 53^o\text{ m/s})t + .5(-9.8\text{ m/s}^2)t^2$$
$$1 = 16t - 4.9t^2$$
$$\Rightarrow \quad 4.9t^2 - 16t + 1 = 0.$$

Using the Quadratic Formula, we get:

$$t = [-b \pm [b^2 - 4ac]^{1/2}]/2a$$
$$= [-(-16) \pm [(-16)^2 - 4(4.9)(1)]^{1/2}]/[2(4.9)]$$
$$= [16 \pm (15.4)]/(9.8)$$
$$= 3.2\text{ seconds.}$$

Noting that the *x component* of the ball's initial velocity is *negative*, relative to our coordinate axis, and that the acceleration in the *x direction* is zero, we can determine the ball's *x coordinate* when caught using:

$$(x_2 - x_1) = v_{1,x}t + (1/2)a_x t^2$$
$$(x_2 - 88\text{ m}) = (-20 \cos 53^o)t + .5(0)t^2$$
$$\Rightarrow \quad x_2 = 88 + (-20 \cos 53^o\text{m/s})(3.2\text{ sec})$$
$$= 49.5\text{ meters.}$$

With the coordinate at which the ball is caught, and noting that the catcher will be running for the same amount of time as the ball is in the air (i.e., 3.2 seconds), we can determine the catcher's required velocity v_c using:

$$(x_2 - x_3) = v_c t + (1/2)a_c t^2$$
$$(49.5 - 5 \text{ m}) = v_c(3.2 \text{ sec})$$
$$\Rightarrow \quad v_c = (44.5 \text{ m})/(3.2 \text{ sec})$$
$$= 13.9 \text{ m/s.}$$

Given the fact that this speed suggests a 100 meter dash time that is 2.5 seconds *under* the world record, this kid would be something to see!