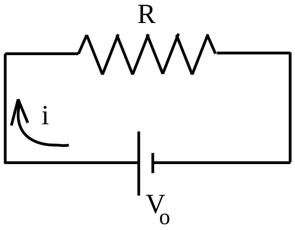
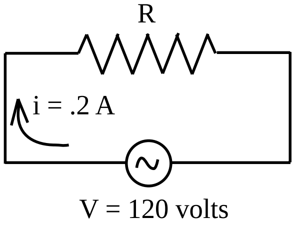
A.C. Circuits -- Conceptual Solutions

1.) Charge carriers in a DC circuit move in one direction only. What do charge carriers do in an AC circuit?

Solution: The voltage difference between the terminals of a DC power supply is fixed, the electric field generated by that potential difference is oriented in one direction only, and charge carriers respond to that field by moving in one direction only. On the other hand, the voltage difference between the terminals on an AC power supply is constantly changing with one terminal being the *high voltage side*, then the other terminal taking up that chore. This alternating voltage produces an alternating electric field and, as a consequence, the charge carriers in such a circuit respond to that field by jiggling back and forth (versus *giggling* back and forth as I originally wrote).

2.) The idea of *current through* and *voltage across* a resistor in a DC circuit is fairly straightforward. Current measures the amount of charge that passes through the resistor *per unit time*, and voltage measures the unchanging voltage difference between the two sides of the resistor. The idea of a resistor's current and voltage in an AC circuit is a little more complex, given that fact the charge carriers in AC circuits don't really go anywhere. So how do we deal with the idea of current and voltage in an AC circuit. That is, when someone says that your home wall socket is providing 110 volts AC, and that a light bulb plugged into that socket draws .2 amps of current, what are those numbers really telling you?





Solution: Power is the key here. Hook up an AC power supply to a resistor. Assume the power *provided to* and *dissipated by* the resistor is 100 watts. *If you wanted to hook the same resistor to a DC source, how large would the DC voltage have to be if you wanted the resistor to continue to dissipate 100 watts*? In other words, what would the *DC equivalent* voltage be? The answer to the question--the DC equivalent to the AC voltage--is called the *RMS* voltage. Although the relationship is derived in the book, the bottom line is that $V_{RMS} = .707V_o$, where V_o is the amplitude of the voltage function associated with the AC circuit. Likewise, the DC equivalent current would be $i_{RMS} = .707i_o$, where i_o is the amplitude of the current function associated with the AC circuit. So what does it mean when we are told that the wall socket voltage in your home is 110 volts? It means that if you put an AC voltmeter in the socket, it would read 110 volts. More important, though, it would mean that from the perspective of power, you would need a 110 volt DC source to provide the same power to a 100 watt light bulb as does the AC wall socket. And how is this voltage related to the alternating voltage produced by the wall socket? The *maximum* voltage (i.e., the amplitude of the voltage function) is, according to the RMS relationship quoted above, $V_o = V_{RMS}/.707 = (110 \text{ volts})/.707 = 155 \text{ volts}$. Likewise, an AC ammeter would read .2 amps RMS through the circuit. This would NOT be the maximum current to flow through the circuit. Rather, it would be the DC equivalent current.

3.) I used a transformer to step up the wall-socket voltage to 5000 volts so that I could charge up a 6000 volt capacitor. After making the voltage into DC (I used a rectifier to do this), I managed to charge the capacitor . . . to the point of blowing it up (well, it didn't blow, but it did die). What went wrong? What was I NOT taking into consideration when I bought the transformer?

Solution: When the capacitor said *6000 volts* on it, it meant that the largest voltage I could put across the plates was 6000 volts. When the transformer said *5000 volts*, though, it was telling me the RMS value of the voltage the transformer would provide if I plugged it into the wall. That wasn't the *maximum* voltage the transformer would supply, it was the DC equivalent voltage the transformer would supply. The problem was during the charging cycle, the actual voltage across the transformer went considerably higher than 5000 volts. In fact, the value went over 7000 volts (to calculate that, noting that $V_{RMS} = .707V_o$, so $V_o = V_{max} = 5000/.707$).

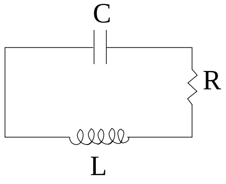
4.) Why won't an inductor allow high frequency AC current to flow through it?

Solution: A one cycle per second alternating current changes direction twice per second (i.e., once per half cycle). A high frequency signal means the current in the circuit is changing directions very fast (current from a 15,000 hertz power supply changes direction 30,000 times a second--again, twice per cycle). So what? When the current changes very fast in an RL circuit (i.e., a circuit in which exists a resistor and an inductor), the magnetic field down the axis of an inductor changes fast. This changes the magnetic flux through the coil which produces a really big EMF in the coil (remember, the magnitude of the induced EMF in a coil is equal to $L di/dt$). With almost all the voltage drop across the inductor, there will be very little voltage drop across the resistor in the circuit (at any given instant, the sum of the voltage drops across the inductor and resistor must equal the voltage difference across the power supply terminals at that instant). A small voltage across the resistor means little current in the circuit (current through a resistor is directly proportional to the voltage across the resistor--measuring the voltage across a resistor is a great indicator of how big the current is going to be, relatively speaking). At low frequency, on the other hand, there is very little change of current, hence little change of magnetic flux, hence a small induced EMF across the inductor. That means there is a larger voltage, relatively speaking, across the resistor, which means there is a larger current in the circuit. In short, inductors are great low-pass filters, but they won't let high frequency pass through a circuit at all.

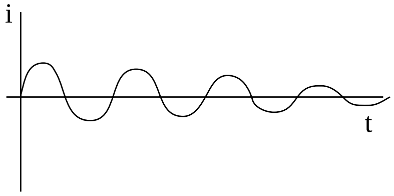
5.) Why won't a capacitor allow low frequency AC current to flow through it?

Solution: The voltage across a capacitor is proportional to the charge on one plate. At low frequency, there is, on average, a lot of charge on the plate most of the time. That means the voltage across the cap is large at low frequency. With large voltage across the cap, there must be little voltage across the resistor . . . which means little current in the circuit. Only at high frequency does the average charge on the plate diminish (see the book if this isn't clear), hence a small voltage across the cap with large *voltage across* and *current through* the resistor. In short, capacitors are great high-pass filters, but they won't let low frequency pass through a circuit at all.

6.) What happens when a capacitor and an inductor are put in the same AC circuit. That is, if a low frequency signal (i.e., current) is blocked by the cap and a high frequency signal is blocked by the inductor, is there *any* frequency at which current will flow?



Solution: An interesting thing happens when an inductor and capacitor are coupled in the same circuit. The easiest way to see this is to consider a charged capacitor that is placed across an inductor (see circuit). As the capacitor begins to discharge across the inductor, an induced EMF is set up in the inductor that fights the increase in current. The current will, nevertheless, reach some maximum value after which it will begin to decrease. During the decrease, the induced EMF across the coil will fight the decrease forcing charge flow to diminish more slowly than would normally be the case if the inductor were not in the circuit. What's interesting is that once the capacitor has completely discharged, the induced EMF across the inductor will continue to push current through the circuit. That current will recharge the capacitor going the other way, so to speak (that is, the previously positive side of the capacitor becomes negative with the previously negative side becoming positive). Once the current finally ceases, the capacitor discharges again going the other way, and the process starts up all over again. With time, resistance in the circuit (even if only due to the wires) will dissipate energy so that the energy content of the system decreases (a graph of the current as a function of time is shown in the sketch), but the *frequency* of the oscillation remains the same through the charging/discharging/charging, etc., process. This natural frequency is called the *resonant frequency* of the RLC circuit. If a power supply is in the RLC circuit, and if the frequency of the power supply matches this natural resonance frequency, the power supply will feed the oscillation and the current will not dampen out as shown in the sketch. In other words, if you push an AC signal (i.e., current) through an RLC circuit, the signal will be absorbed by the capacitor if the frequency is low; the signal will be absorbed by the inductor if the frequency is high; and the signal will flourish if its frequency just happens to match the resonance frequency of the circuit (i.e., $[1/(LC)]^{1/2}/2\pi$).



7.) What does the phase shift tell you? Is the phase shift ever truly a quarter cycle (i.e., $\pi/2$ radians)?

Solution: In normal resistor-only circuits, the voltage and current are lockstep with one another. That is, when the voltage is zero, the current is zero; when the voltage is a maximum, the current is a maximum; when the voltage decreases, the current decreases, etc. When an element acts like this (i.e., when the voltage across the element mirrors the current through the element), the voltage and current are said to be *in phase* with one another. In capacitor or inductor circuits, the voltage and current will not be *in phase* with one another. Voltage *leads* the current in inductor circuits whereas voltage *lags* the current in capacitor circuits. IF there was no resistance in such circuits, the lead/lag phase shift between the voltage across the power supply and the current drawn from the power supply would be a quarter of a cycle (i.e., $\pi/2$ radians). With resistance, which always exists within a circuit, the net phase shift is less than $\pi/2$ radians. The actual angle is determined using the impedance relationship $\tan^{-1}[(X_L - X_C)/R_{net}]$, where X_L --the inductive reactance-- is equal to $2\pi \nu L$, X_C--the capacitive reactance-- is equal to $1/(2\pi \nu C)$, and R_{net} is the total resistor-like resistance in the circuit. Please note that a positive phase shift corresponds to a situation in which the voltage *leads* the current, and a negative phase shift corresponds to a situation in which the voltage *lags* the current.

8.) What is the measure of a capacitor's net resistive nature? That is, what is it called, what are its units, and how is it calculated?

Solution: The net resistive nature of a capacitor is measured in *ohms*, as is always the case when measuring resistance to current flow, and it is frequency dependent. The quantity is formally called *the capacitive reactance*. The symbol used to denote the capacitive reactance is X_C. It is numerically equal to $1/(2\pi \nu C)$, where the capacitance C must be in *farads* and ν is the frequency of the AC source. As would be expected, given the discussion in *Question 5*, the relationship suggests that capacitors produce large resistance in AC circuits when the frequency of the source is low (a small number in the denominator of the capacitive reactance expression produces a large resistance). High frequency signals, on the other hand, produce very little resistance to charge flow. As a consequence, capacitors are often referred to as *high pass filters*.

9.) What is the measure of an inductor's net resistive nature? That is, what is it called, what are its units, and how is it calculated?

Solution: There are two aspects to an inductor's net resistive nature, both of which are measured in *ohms*. The first comes from the fact that inductors are physically made up of wire which has resistor-like resistance associated with it. The symbol usually used to take this into account is r_L. The frequency-dependent resistive nature of inductors is formally called *the inductive reactance*. The symbol used to denote the inductive reactance is X_L. It is numerically equal to $2\pi \nu L$, where the inductance L must be in *henrys* and ν is the frequency of the AC source. As would be expected, given the discussion in *Question 4*, the relationship suggests that inductors produce large resistance in AC circuits when the frequency of the source is high (a large number in the numerator of the inductive reactance expression produces a large resistance). Low frequency signals, on the other hand, produce very little resistance to charge flow. As a consequence, inductors are often referred to as *low pass filters*. The net resistive nature of the inductor, therefore, is a combination of the inductive reactance and the resistor-like resistance r_L. To determine a number for this overall quantity, the impedance expression $Z = [R_{net}^2 + X_L^2]^{1/2}$ does the job.

10.) What does the impedance of a circuit tell you? Also, what are its units and how is it calculated?

Solution: Impedance is the measure of the net resistive nature of a circuit, and its units are ohms. For circuits in which there are no capacitors or inductors, it is simply the resistance R_{net} of the circuit. For circuits in which capacitors, inductors, or both exist, it is numerically equal to $Z = [R_{net}^2 + (X_L - X_C)^2]^{1/2}$.

11.) Is the impedance of a circuit frequency dependent? If so, how so?

Solution: Impedance is defined as $Z = [R_{net}{}^2 + (X_L - X_C)^2]^{1/2}$, where $X_C = 1/(2\pi \nu C)$ and $X_L = 2\pi \nu L$. As can be seen, the impedance is a frequency dependent quantity.

12.) Tweeters and woofers are types of speakers found in most speaker systems. Let's assume you have split the signal from a radio station and want only the high frequency part of a signal to be fed to the tweeters while only the low frequency part of a signal is fed to the woofers. What additional bit of circuit wizardry would you have to create to ensure that that would happen?

Solution: Inductors are low pass filters. Make the signal going to the woofer pass through an inductor and the inductor will allow only the low frequency to pass through. A capacitor in the second branch will do a similar service for the tweeter, allowing only the high frequency component of the signal to pass.