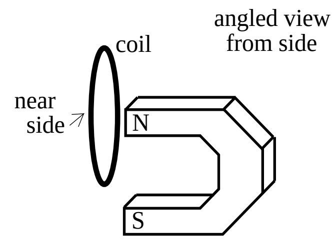
# Faraday's Law -- Conceptual Solutions

1.) As you found when we dealt with Gauss's Law, it is possible to have an electric flux. Is it possible to have a magnetic flux? If so, how would it be defined?

Solution: Any vector field that passes through a surface will produce a flux through that surface. If a constant magnetic field passes through the face of a coil of area $A$, for instance, the magnetic flux through the coil will equal $\boldsymbol{B}\cdot\boldsymbol{A}$, where $\boldsymbol{A}$ is an area vector whose magnitude is equal to the area of the coil's face and whose direction is perpendicularly outward from the face. If anything varies (i.e., the magnitude or direction of $\boldsymbol{B}$ or $\boldsymbol{A}$, or the angle between the two vectors), then a differential approach must be used to determine the net flux. Mathematically, that would be $\int \boldsymbol{B}\cdot d\boldsymbol{A}$.

2.) A coil is placed in the vicinity of a horseshoe magnet.




a.) Once in place, is there a flux through the coil?

Solution: The magnetic field lines associated with a horseshoe magnet billow out leaving the *north pole* and entering the *south pole*. That means that magnetic field lines are passing through the coil's face and there is a magnetic flux through the coil.

b.) Once in place, is there a current in the coil? If so, why? Also, if so, in what direction will the current flow?

Solution: There is nothing that would motivate charge to flow in this situation (remember, magnetic fields don't act like electric fields), so there would be no current in this situation.

3.) The coil alluded to in *Problem 2* is placed in the vicinity of the same horseshoe magnet, but this time the coil is rapidly pulled away from the magnet.

a.) Is there an initial flux through the coil?

Solution: As was the case in *Problem 2a*, there is an initial magnetic flux through the coil.

b.) What happens to the flux as the coil is pulled away?

Solution: Pulling the coil away from the magnet decreases the magnetic field through the coil's face. This, in turn, decreases the magnetic flux through the coil.

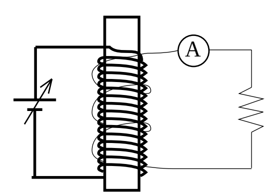
c.) From the standard perspective associated with magnetic fields and charges moving in magnetic fields, would you expect a current to flow in the coil as the coil is pulled away from the magnet? If so, why? Also, in what direction would the current flow?

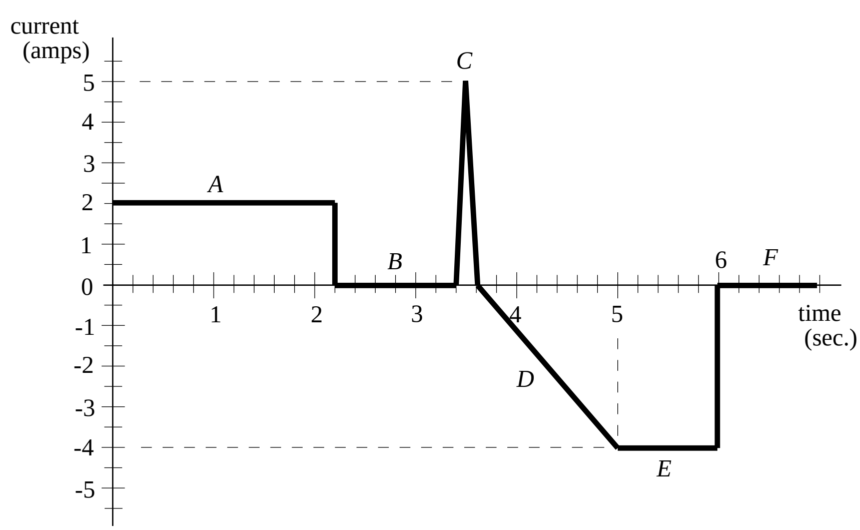
Solution: As far as the charges carried in the wire are concerned, they are moving across magnetic field lines as the coil is pulled away from the magnet. When this happens, they will experience a force (i.e., $q\mathbf{v}\mathbf{x}\mathbf{B}$) resulting in a current in the coil. The direction of the current can be determined using the right-hand rule. It will be clockwise as viewed from the perspective shown in the sketch.

d.) From Faraday's perspective, would you expect a current to flow in the coil as the coil is pulled away from the magnet? If so, how would Faraday explain the current? Also, how would he determine the direction of current flow?

Solution: According to Faraday, a changing magnetic flux will induce an EMF in the coil. That EMF will produce a current whose magnetic field will either *add to* or *subtract from* the external field (i.e., the field produced by the magnet). What determines which happens depends upon how the magnetic flux is changing. If it is increasing, the current will flow in such a way as to produce an induced *B field* which will *fight the increase* by subtracting from the external field. If the magnetic flux is decreasing, the current will flow so as to create an induced *B field* that adds to the external field, thereby *diminishing the decrease*. In all cases, the current's B field will produce a magnetic flux of its own that will OPPOSE the *change of flux* that started the process in the first place. In this case, the magnetic flux is decreasing. A clockwise current will produce a *B field* that adds to the external field. That will be the direction of the induced current. Note that this may seem a lot more complex than the explanation given in *Part c*, but there will be situations in which this approach/perspective is much cleaner and easier to deal with than the classical view.

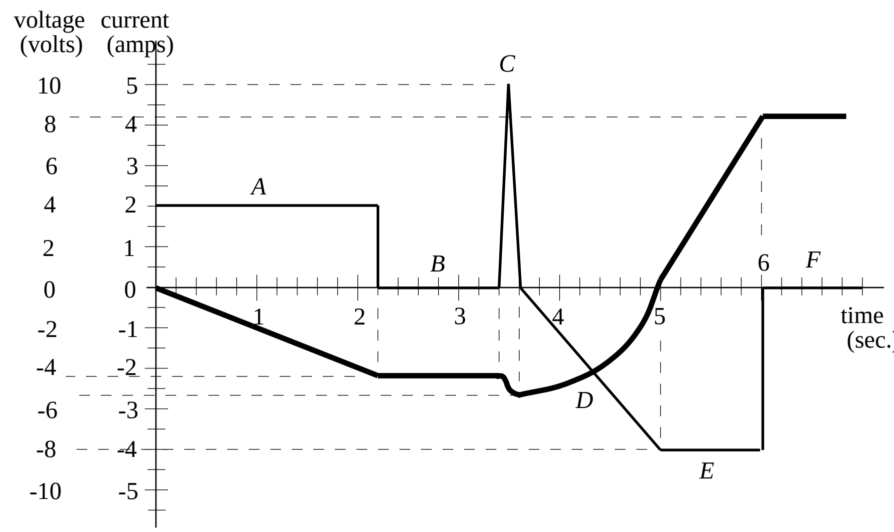
4.) Two coils share a common axis having been wrapped around a common steel rod, but are electrically isolated from one another (that is, they aren't electrically connected). The coil on the left is attached to a variable power supply (we'll call this *the primary circuit*). The coil on the right is attached to a resistor and ammeter (we'll call this *the secondary circuit*). One of the more hyperactive students in the crowd begins to play with the voltage across the primary coil power supply while a



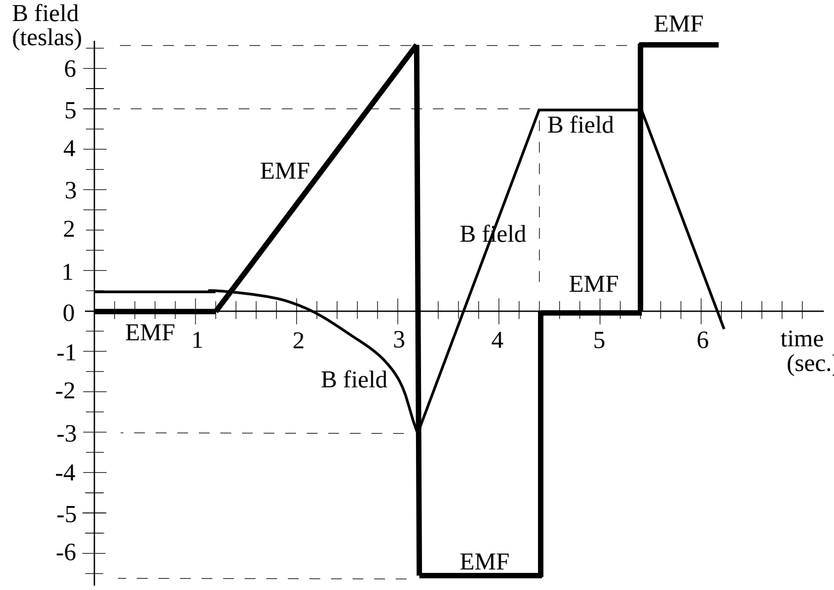
second student records, then graphs the current in the SECONDARY coil. That graph is shown in the sketch. There are six time intervals identified by letters on the graph (i.e., *A* corresponds to the current during the period between $t = 0$ and $t = 2.2$ *seconds*, etc.). Explain what must be happening to the power supply in the *primary circuit* during each of those time periods.

Solution: Every time the power supply voltage changes, there is an increase or decrease of current in the primary circuit (that is, $di_{prim}/dt$ is non-zero). That change of current through the primary coil produces a changing magnetic flux through both coils via the steel rod. With the changing flux comes an induced EMF across the secondary coil $\varepsilon_{sec}$ which motivates current to flow in the secondary circuit (note that $\varepsilon_{sec} = i_{sec}R$, and that our graph is that of $i_{sec}$). We know from Faraday's Law that $\varepsilon_{sec} = -L(di_{prim}/dt)$, which further implies that $di_{prim} = -\int \varepsilon_{sec} dt$. Evidently, what is happening in the primary coil is related to the area under the *EMF*$_{sec}$ *versus time* graph, where the *EMF*$_{sec}$ *versus time* graph is proportional to the graph we have been given (i.e., $i_{sec}$ *vs. time*). Using what we know, then, yields the final graph as shown. Note that conceptually, this all follows nicely. Think about it. If the change in the primary voltage occurs for only a moment, we will see a spike of induced current in the secondary, then nothing (where do you find a spike in our graph--what kind of primary voltage change should go with that spike?). If the primary voltage changes linearly, we will see a constant induced current in the secondary (where do you find a constant current--what kind of a primary voltage goes with *that*?). If the primary voltage changes as a quadratic, we will see an induced secondary current that increases or decreases linearly with time (where do you find that on our graph?). And how big are the voltage changes? They are proportional to the areas under the current graphs over the time periods of interest. As long as you don't get messed up with the negative sign that is inherent in Faraday's Law, it's easy!




5.) The magnetic field down the axis of a coil varies with time as graphed (see next page). On the graph, sketch the induced EMF set up in the coil.

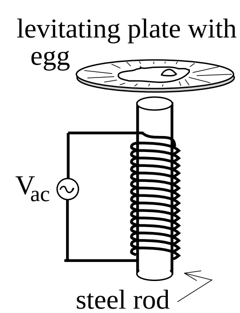
Solution: A change in the magnetic field will change the magnetic flux down the coil's axis which will, in turn, induce an EMF across the coil. If the magnetic field is constant, there is no change in the magnetic flux and the induced EMF will be zero. If the magnetic field changes linearly, the induced EMF will be a constant. In fact, the size of the EMF is determined using $\varepsilon = -N(d\Phi_m / dt) = -NA\cos\theta\,(dB/dt)$, where $N$ is the number of winds in the coil, $A$ is the constant cross sectional area of the coil, and $\theta$ is the angle between $\mathbf{B}$ and $\mathbf{A}$ (it is assumed to be zero degrees in this case). In other words, the induced EMF is related to the slope of the magnetic field function (actually, it's *minus* the slope . . .). In any case, the EMF's graph is superimposed on the original *B field* graph.




6.) If the graph in *Problem 5* had been of the EMF set up in the coil as a function of time, what could you say about the magnetic flux through the coil?

Solution: In general, if the EMF is related to the slope of the magnetic field function, the magnetic field function will be related to the area under the EMF graph.

7.) An AC source is attached to a coil that has a vertical, steel bar down its axis. When the power is turned on, an alternating magnetic field is set up along the axis of the bar. An aluminum plate is centered over the bar at its upper end. When power is provided to the coil, the plate levitates.




a.) Is aluminum a magnetizable material?

Solution: No! Aluminum is not like iron. Its atoms don't have more electrons spinning in one direction than the other, so aluminum atoms are not magnets unto themselves as is the case with iron atoms.

b.) Why does the plate levitate?

Solution: Although aluminum isn't magnetizable, it is a metal. As such, it has metallic bonding and it does have valence electrons that are free to move around within the structure. So what's going on within the plate? The changing magnetic field through the coil produces a changing magnetic flux through the plate. That changing flux induces an EMF that motivates free charge (electrons) in the plate to move about. The motion of those electrons sets up a magnetic field of its own (remember, charge in motion generates a magnetic field). This induced magnetic field will alternate, just as does the external magnetic field produced by the coil. The difference is that the two fields will be out of phase with one another. That is, when the field produced by the coil has the upper end of the steel acting like a *north pole*, the bottom surface of the plate will be acting like a *north pole*. The repulsion between the opposing magnetic fields provides the force that levitates the plate.

c.) An egg is broken onto the plate. What will happen to the egg . . . and why?
Solution: The motion of the electrons in the plate will cause the plate to heat. If you put an egg onto the plate, it will cook. As bizarre as this sounds, the demo actually works--you can cook an egg on the levitating aluminum plate.

8.) What is inductance? How is it comparable to resistance and capacitance?
Solution: The original presentation of Faraday's Law maintained that whenever there was a changing magnetic flux through a coil, that changing flux would be accompanied by an induced EMF. Mathematically, this was represented as $\varepsilon = -N(d\Phi_m/dt)$. When dealing with coils in electrical circuits, though, it was observed that the changing magnetic flux was really being generated by a change in the current through the coil. To reflect that fact in a mathematical sense, someone decided to write Faraday's Law in terms of $di/dt$ instead of $d\Phi_m/dt$. The proportionality constant required to make the relationship work was called *the inductance L* of the coil. With that constant, Faraday's Law became $\varepsilon = -L(di/dt)$. In short, inductance is the proportionality constant that relates the induced EMF (i.e., the voltage across the coil's leads) and the current change $di/dt$ that created the induced voltage in the first place. It is interesting to note that all of the circuit elements you have run into so far have had defining parameters that have been proportionality constants that related the voltage across the element to either the current through the element or the charge accumulated on the element. For resistors, *resistance* was defined such that $V = iR$; for capacitors, *capacitance* was defined as $q = CV$. *Inductance* follows the pattern nicely.

9.) How do transformers work?
Solution: A transformer is made up of two coils that are not electrically connected but that are magnetically coupled. Normally used in an AC setting, when the voltage changes across the coil in the primary coil, that creates a changing magnetic flux through the secondary coil (remember, the two coils are magnetically linked) which produces an EMF in the secondary coil. The EMF drives current in the secondary coil. Transformers are used primarily for transferring power from one part of an electrical circuit to another part without electrically connecting the two parts. In addition, if the number of winds $N_s$ in the secondary coil are greater than the number of winds $N_p$ in the primary coil, the secondary voltage will *step up* relative to the primary voltage (this is called a *step-up* transformer), with a proportional stepping down of current. (The opposite of this is the *step-down* transformer in which $N_s < N_p$.) Europe's electrical wall sockets, for instance, run at 220 volts. For you to use your electric shaver--a device that

runs on 110 volt AC--you have to use a step-down transformer to get the shaver to work properly.

10.) What's the difference between a generator and an electric motor?

Solution: In the case of a generator, you use mechanical energy to turn a shaft (you can do this turning manually, or by using a gas engine . . . or by using a waterfall over a turbine . . . or steam over a turbine . . . or whatever). Attached to the shaft is a coil bathed in a magnetic field. As the shaft, hence coil, rotates, a changing magnetic flux is produced through the coil and an alternative, induced EMF (i.e., a voltage) is generated across the coil's leads. In other words, a generator converts mechanical energy into electrical energy. An electric motor does the exact opposite. If you run current through a coil bathed in a magnetic field, the coil will feel a torque that will make it rotate. If the coil is attached to a shaft, the shaft will also rotate. In other words, an electric motor converts electrical energy into mechanical energy.