Electrical Potentials -- Conceptual Solutions

1.) What does an absolute electrical potential actually tell you? That is:

a.) Is it a vector? If so, what does its direction signify?

Solution: Electrical potentials are modified potential energy functions. As such, they are scalar fields and have no direction associated with them.

b.) What does its magnitude tell you?

Solution: If you know the absolute electrical potential at a particular point in an electric field, you know how much *potential energy per unit charge* is available at that point due to the presence of the field. That is, electrical potentials are modified potential energy functions. (Note that *electrical potentials* are often referred to simply as *potentials*.)

c.) How are electrical potentials used in everyday life?

Solution: *Electrical potential differences* and *electric fields* go hand in hand. That is, if you create an electrical potential difference between two points, you will also have created an electric field between those two points (and vice versa). That means that when you switch a lamp on in your home, the electric field that motivates current to flow through the lamp's light bulb is actually being produced by an artificially created *electrical potential difference* between the two prongs of the lamp's electrical cord.

2.) An electrical potential field is oriented so that it becomes larger as you move to the right.

a.) What will a positive charge do if put in the field?

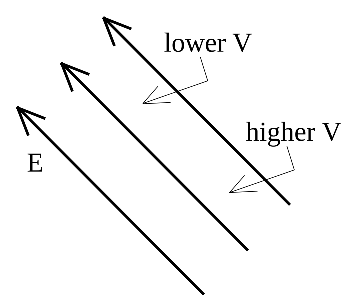
Solution: Positive charge "falls" through potential differences proceeding from *higher* to *lower* electrical potentials. In this case, that means they will accelerate to the left.

b.) What will a negative charge do if put in the field?

Solution: Negative charge does the opposite of positive charge, so they will accelerate from *lower* to *higher* electrical potential. In this case, that will be to the right.

c.) Is there an electric field associated with the potential and, if so, in what direction is it oriented?

Solution: By definition, the direction of an electric field is the direction a positive charge would accelerate if put in the field. As positive charge moves from higher to lower electrical potentials, electric field lines must be oriented from *higher* to *lower* electrical potentials.



3.) A point charge exists at the origin of a coordinate axis. A distance 2 meters down the *x axis*, the electric field is observed to be 12 nt/C.

a.) What is the electrical potential at that point?

Solution: The *electric field* function for a point mass is kQ/r^2, whereas the *electrical potential* function for a point charge is kQ/r. As the r terms are the same in both cases (i.e., the distance between the field producing charge and the point of interest), the electrical potential expression is evidently just r times E in this case. That means the electrical potential is $(12\ nt/C)(2\ meters) = 24\ volts$. (Note that a $nt \cdot m$ is an energy quantity, and energy divided by Coulombs is a *volt*.)

b.) You double the distance to 4 meters.

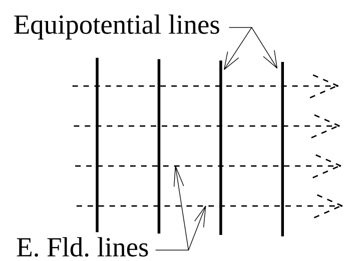
i.) What is the new electric field?

Solution: The electric field is a function of $1/r^2$. Doubling r will change the electric field by a factor of $1/(2)^2 = 1/4$.

ii.) What is the new electrical potential?

Solution: The electrical potential is a function of $1/r$. Doubling r will change the electrical potential by a factor of $1/2$.

4.) You have an electric field as shown. What will equipotential lines look like in the field?

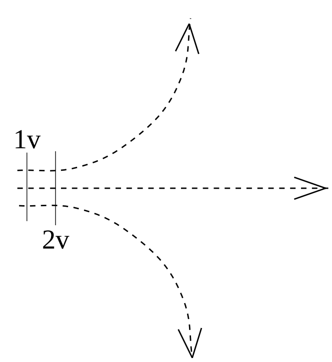


Solution: An equipotential line is a line upon which the electrical potential is the same at every point. It turns out that equipotential lines cut across electric field lines *at right angles*. So, for electric field lines to the right, the associated equipotential lines would be up and down (see sketch).

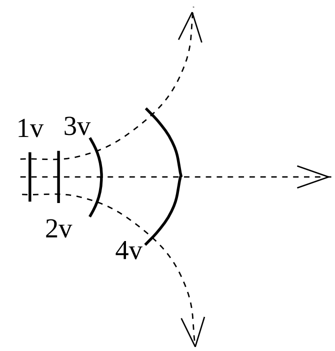
5.) How is the *electrical potential difference* between two points related to the amount of work required to move a charge q from one point to the other?

Solution: By definition, $W/q = -\Delta V$, so $W = -q\Delta V$.

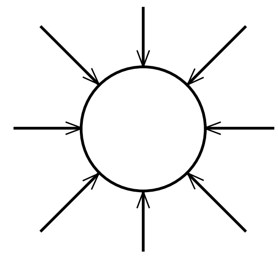
6.) The dotted lines in the sketch to the right are electric field lines. Shown also are the 1 volt and 2 volt equipotential lines. Draw in the 3 volt and 4 volt equipotential lines.



Solution: What's tricky here is the fact that as the electric field weakens, the equipotential lines get farther apart. This follows both mathematically and logically. Mathematically, the relationship $\boldsymbol{E} \cdot \boldsymbol{d} = -\Delta V$ suggests that if $\boldsymbol{E}$ gets smaller (this is what happens as the electric field lines get farther apart), the distance $\boldsymbol{d}$ between incremental equipotential lines must get larger if the voltage change is to stay the same. From a common sense perspective, if voltage changes are related to the amount of work the electric field does as a charge goes from one point to another, then a lessened electric field intensity will do less work per unit displacement, and the net displacement will have to increase to effect the same amount of work. In any case, the equipotential lines will be perpendicular to the electric field lines, and farther apart as the size of the electric field diminishes.



7.) To the right is a cut-away cross-section of a hollow ball of radius a. Given the electric field lines as shown:



a.) What do you know about the electrical potential on the surface of the shell?

Solution: There are a number of bits and pieces of information you need to know to answer this question. To begin with, the electric field outside the shell is generated by a negative charge on the shell (look at the direction of the electric field lines), and there is no field inside the shell. Electrical potentials generated by negative charges are *negative*. In addition, if we assume the total charge on the shell is $-Q$, the magnitude of the electric field from the shell outward is kQ/r^2, where k is a constant and r is the distance from the shell's center to the point of interest. That is, assuming you deal with $r \geq a$, there is no difference between this electric field and the electric field you'd end up with if you got rid of the shell and replaced it with a point charge equal to $-Q$ located at the shell's center. Putting it in still another way, as viewed from outside the shell, the field-producing charge *looks* like a point charge. If you can't tell the difference between the two situations, the solution to one will fit the solution to the other. We know the electrical potential for a negative point charge. It is $k(-Q)/r$. This function also defines the electrical potential outside the shell . . . and upon its surface as well. Evaluating at $r = a$, we get $V = -kQ/a$ at the shell's surface.

b.) How would *Part a* have been different if the electric field lines had been oriented outward?

Solution: The electric field outside the shell would have to be generated by a net positive charge on the shell. Electrical potentials generated by positive charges are *positive*. On the shell the electrical potential would be kQ/a.

c.) What do you know about the electrical potential inside the cavity?

Solution: The *electric field* between two points is directly proportional to the *rate of change* of electric potential between the two points (that is, E is related to the spatial derivative of the electrical potential function or, in one dimension, dV/dx). As the electric field is zero inside the shell (no electric field lines), the *change* of the electrical potential between any two points inside the hollow will be zero--the electrical potential will be the same everywhere in that region. As electrical potentials are continuous, that value will also have to be the same as the electrical potential at the shell's surface, or $k(-Q)/a$.

d.) What do you know about the electric field at the boundary between the *inside* and *outside* of the shell?

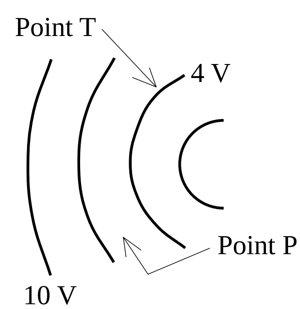
Solution: The electric field inside the shell is zero. Outside the shell, it is kQ/r^2. In other words, electric fields are *not* continuous functions.

e.) What do you know about the electrical potential at the boundary between the *inside* and *outside* of the shell?

Solution: As was said in *Part c*, the electrical potential at the boundary must be a continuous function.

f.) Where is the electrical potential zero?

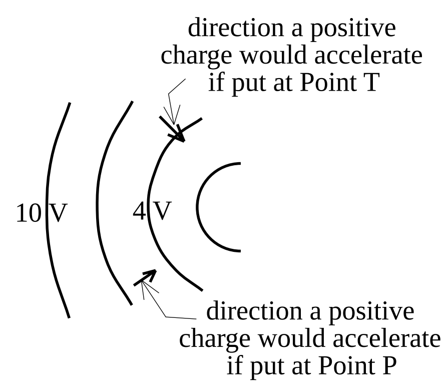
Solution: If you will remember back, in most cases a *potential energy* function is defined as zero where its associated force is zero. *Electrical potentials* are modified potential energy functions and have similar characteristics, relative to their associated *electric fields*. Although the electric field is zero for $r < a$, that is not due to a general dropping off of field intensity due to a moving away from the field producing charge. It is due to the fact that all the free charge carriers on the shell produce electric fields that add to zero inside the hollow. In short, for this configuration, we will take the electric field to be zero at infinity where the field has diminished due to distance, and we will take the electrical potential to be zero at that point also.



8.) An oddly shaped charge configuration produces the equipotentials shown to the right.

a.) In what direction will a positive charge accelerate if placed in the field at *Point P*? How about *Point T*?

Solution: Positive charges follow electric field lines, and electric field lines are always perpendicular to equipotential lines (remember, an equipotential line is a line upon which the electrical potential is always the same). Because electric fields are oriented "downstream," so to speak, with respect to equipotentials, a positive charge will travel along a path that is perpendicular to the equipotential lines in its vicinity, and in such a way as to travel from *higher* to *lower* electrical potential. For ease of viewing, I've reproduced the drawing and drawn the solution for this question on that sketch.



b.) What would be different if a negative charge had been placed at *Point P*?

Solution: Negative charges do exactly the opposite of positive charges, so an electron would travel perpendicularly to the equipotential lines and from *lower* to *higher* electrical potential.

c.) Is there any region in which:

i.) The magnitude of the electrical potential field is a constant? If so, identify it on the sketch.

Solution: That's what equipotential lines *are*--lines upon which the electrical potential is constant. There are *four* such lines depicted on our sketch, one for 10 volts, one for 7 volts, one for 4 volts, and one for 1 volt.

ii.) The magnitude of the electrical potential field is zero? If so, identify it on the sketch.

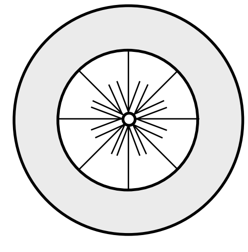
Solution: Given the equipotential lines, there must be a zero potential point just to the right of the far right line (i.e., the 1 volt line). (Note: voltages can be negative, so being zero doesn't mean you have necessarily hit the lowest potential possible.)

9.) What does the electrical potential V at a point tell you about the electric field $\boldsymbol{E}$ at that point?

Solution: The direction in which the *greatest spatial rate of electrical potential change* occurs is the direction of the electric field, and minus the magnitude of that rate of change equals the magnitude of the electric field at that point. That is, electric fields are oriented in the direction of greatest CHANGE of *potential energy per unit charge*, and their magnitudes are equal to the greatest *rate* at which that *change* takes place at the point. This is all wrapped up in the expression $\boldsymbol{E} = -\nabla V$.

10.) What does the electric field at a point tell you about the electrical potential at that point?

Solution: The electrical potential difference between two points tells you the amount of *work per unit charge* available due to the presence of the electric field between the two points. The relationship comes directly from the work/potential-energy relationship $U(b) - U(a) = -\int_a^b \boldsymbol{F} \bullet d\boldsymbol{r}$. By dividing that relationship to remove the q dependence, we get $(U/q)_b - (U/q)_a = -\int_a^b (\boldsymbol{F}/q) \bullet d\boldsymbol{r}$ which can be rewritten as $V(b) - V(a) = -\int_a^b \boldsymbol{E} \bullet d\boldsymbol{r}$. In other words, the area under the *electric field versus position* graph gives you the potential difference between the two points. Put a little differently, the anti-derivative of the electric field is the electrical potential function.



11.) To the right is a cut-away end-view of a thick-skinned conducting PIPE (I know, it looks like a ball--*it's the end of a pipe*) with a thin wire running down its axis. The wire's radius is a while the pipe's inside radius is b and its outside radius is c. Given the electric field lines as shown:

a.) Assume there is $-\lambda$*'s* worth of free *charge per unit length* associated with the wire. Looking at the electric field lines, is there any other free charge in the system and, if so, where is it found and how much is there?

Solution: If there was no more free charge in the system, positive charge would be induced on the inside of the pipe in the amount of $+\lambda$ (that would make for *zero charge enclosed* inside a Gaussian surface inside the conductor, thereby insuring zero electric field inside the conductor) leaving negative charge in the amount $-\lambda$ on the outside surface of the pipe. Using a Gaussian surface on the outside of the configuration would yield a net charge $-\lambda$ which would necessitate electric field lines in the sketch. There are no electric field lines outside the pipe, so there must be extra free charge in the system. In fact, the additional free charge is in the amount of $+\lambda$, and it is located on the inside surface of the pipe. Why inside surface? *Some* positive charge must reside there so that the *charge enclosed* inside a Gaussian surface inside the conductor is zero. If there is free charge available to do that job, it *will* do that job (actually, what really happens when the positive free charge is put on the pipe is that electrons are removed--with that, the electrons that are left rearrange themselves so as to cover over all the positive charge in the structure *except* positive charge on the inside surface of the pipe--that free positive charge will be in the amount $+\lambda$). In short, we have negative charge down the wire and an equal amount of positive charge on the pipe at $r = b$.

b.) What do you know about the electrical potential outside the pipe?

Solution: The temptation is to assume that the negatively charged wire is grounded and, hence, should be assumed to have zero electrical potential. The problem with doing this is that with no electric field outside the pipe, the electrical potential there must be a constant throughout the region *all the way to infinity*. Because the electrical potential in that region all the way to infinity must also be the same as the electrical potential at the outside surface of the pipe (remember, electrical potential functions must be continuous), which in turn must be equal to the electrical potential on the inside surface of the pipe (there is no electric field between $r = b$ and $r = c$, so the electrical potential difference between *any two points* in the pipe from $r = b$ outward must be zero, and the electrical potential must be equal throughout), it leaves us with the distasteful prospect of having the electrical potential of the inside surface of the pipe equaling the electrical potential at infinity. WE COULD DO THIS (again, electrical potential *differences* are all that are important), but let's not. Instead, let's assume that the electrical potential at infinity is zero and see where that takes us.

c.) What do you know about the electrical potential inside the dotted area?

Solution: Inside *area B*, the electric field is zero (no field lines), so the electrical potential is a constant. From *Parts a* and *b*, that constant is zero.

d.) What do you know about the electrical potential inside the hollow?

Solution: We can use Gauss's Law to derive the electric field expression for the region inside the hollow. Doing so yields $-\lambda/(2\pi\varepsilon_o r)$, where $-\lambda$ is the linear density function for the charge on the wire, the negative sign signifies that the electric field is not outward but, instead, inward, and r is the perpendicular distance between the central axis and a point of interest inside the hollow. As this is a $1/r$ function, the electrical potential will be a $\ln r$ function (remember, the electrical potential is the anti-derivative of the electric field). In any case, the electrical potential turns out to be $-[-\lambda/(2\pi\varepsilon_o)][\ln(r) - \ln(b)]$. Notice that at $r = b$, the electrical potential is zero, and inside the hollow at $r < b$, the electrical potential is negative. This all makes sense, given our choice for *zero electrical* *potential*. That is, the electrical potential at $r = b$ is zero, but the electrical potential for $r < b$ is *less than zero*--it is *negative*. Consequence: If you let a positive charge go at $r = b$, it will accelerate toward the negative wire. In doing so, it will be accelerating from *higher* to *lower* electrical potential, just as expected.

e.) What do you know about the potential inside the wire?

Solution: Being a conductor, the net electric field inside the wire will be zero. From continuity, therefore, the region inside the wire will have the same electrical potential as the wire's surface. According to the expression we deduced in *Part c*, that value will be $-[-\lambda/(2\pi\varepsilon_o)][\ln(a) - \ln(b)]$.

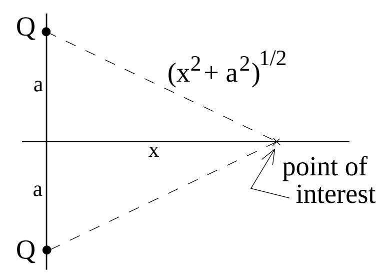
f.) When using Gauss's Law, we usually start with the innermost region and work our way outward. Why do we do that?

Solution: We work outward because Gauss's Law is predicated upon the notion that electric flux is governed by the charge enclosed within a Gaussian surface. Starting at the innermost part of the system allows us to easily identify that *charge enclosed* quantity with each successive section. As we move outward, we can build upon what has been done in the innermost regions.

g.) In asking about the electrical potential in this set-up, why did I start with the outermost area and work inward? (After all, that isn't the way we go when dealing with electric fields and Gauss's Law problems.)

Solution: An electrical potential, being a *1/r* type critter, is governed not only by the charge inside the surface upon which the point of interest resides, it is also affected by the charge *outside* that surface. As such, it is easiest to start with the outermost area where the electric field and *charge enclosed* are governed by what's *inside* that surface.

12.) Two equal, positive point charges are shown to the right. The net electrical potential a distance x units down the *x-axis* is $2kq/(x^2 + a^2)^{1/2}$, where k is a constant.



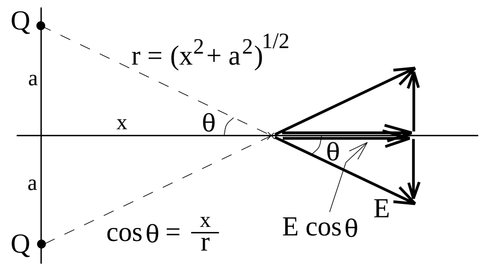
a.) What is the constant k?

Solution: The constant k is equal to $9x10^9\ nt{\cdot}m^2/C^2$.

b.) Justify the electrical potential expression. That is, use the electrical potential function for a point charge and do the problem.

Solution: The electrical potential for a negative point charge is $-kq/r$, where k is defined in *Part a*, $-q$ is the field producing charge, and r is the distance between the charge and the point of interest. In this case, $r = (x^2 + a^2)^{1/2}$. What is nice about electrical potentials is that they are scalars--they add like normal numbers (versus adding like vectors). With all this in mind, and given the symmetry, the sum of the electrical potentials will just be twice that of one.

c.) The electric field function for the situation we're talking about is $2kqx/(x^2 + a^2)^{3/2}(\mathbf{i})$. Using the *point charge* approach, justify this expression.



Solution: Yes, this is review, but it's review that you should be able to do quickly. The upper point charge is going to produce a field as shown in the sketch below. The lower point charge is going to produce a similar field, but upward and to the right. Due to both charge and geometric symmetry, the vector sum of the two fields finds the *y components* adding to zero leaving only the *x components*. In short, all we need to calculate is $E\cos\theta$. Noting from the sketch that $\cos\theta = x/(x^2 + a^2)^{1/2}$, letting $1/(4\pi\varepsilon_o) = k$, and using the electric field expression for a point charge, our electric field expression becomes $E = [kq/((x^2 + a^2)^{1/2})^2][x/(x^2 + a^2)^{1/2}]$. Doubling that (there are *two* charges involved) and adding in the unit vector yields $2kqx/(x^2 + a^2)^{3/2}(\mathbf{i})$.

d.) Using the del operator and the known electrical potential function, can you determine the electric field function presented in *Part c*? If so, do so.

Solution: This is conceptually tricky. Let's assume we can use the del operator on a point on the *x axis*. The math would yield: $\mathbf{E} = -\nabla V = -[\partial(2kq/(x^2 + a^2)^{1/2})/\partial x(\mathbf{i}) + \partial(2kq/(x^2 + a^2)^{1/2})/\partial y(\mathbf{j}) + \partial(2kq/(x^2 + a^2)^{1/2})/\partial z(\mathbf{k})]$. This set of partial derivatives yields a non-zero result only in the *x direction*, and that result is (lo, and behold) $\mathbf{E} = 2kqx/(x^2 + a^2)^{3/2}(\mathbf{i})$. It appears that using the del operator on the potential function we've derived yields a correct result. PLEASE NOTE: THIS IS VERY MISLEADING. In fact, you did get the right answer, but we have done something dirty--something that will become evident in *Part e*.

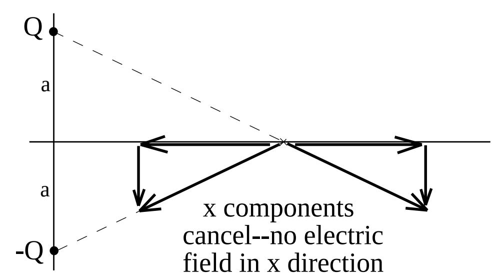
e.) If you try to do *Part d* but with a *negative charge* at the bottom and a *positive charge* at the top, you will run into a whopping big problem. Specifically, because the electrical potential of a positive charge a distance *r* units from a point of interest is kQ/r, and the electrical potential for a negative point charge is $-kQ/r$, the sum of the two electrical potentials at ANY point down the *x axis* will be ZERO.

i.) How are you going to use the del operator on *that*?

Solution: You aren't. There's obviously a problem.

ii.) In fact, if the electrical potential is *zero* down the axis, what does that tell you about the electric field at a point on the axis?

Solution: Remember how electrical potentials and electric fields are related. The electric field at a point is equal to the *rate* at which electrical potential changes at that point. In one dimension, this is dV/dx. In other words, the electrical potential can be zero at some point, but as long as the potential's values are something other than zero at points in the immediate vicinity, there



will be an electric field at the point. In this case, the electrical potential is zero down the *x axis*, so the change of electrical potential down the axis will be zero and the electric field down the axis (i.e., in the *x direction*) will be zero (this shouldn't be a surprise--due to the symmetry the *x components* of the electric field will add to zero). Move in the direction perpendicular to the axis, though, and you find non-zero electrical potential points just above and below the *x axis*. With a potential difference in that direction, you know there *is* an electric field in the *y axis*.

iii. It seems as though the del operator approach only allows you to determine the electric field when the system is symmetric and the charge is like (i.e., a situation like the one in *Part d*). Does this make sense? If not, what do you suppose is the problem?

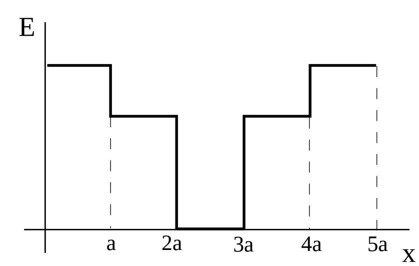
Solution: If the del operator approach is to work, it has to work in all cases. The key to unscrambling this problem is to remember that rarely can you get away with using a specific, selective set of points for an evaluation that is supposed to be general. That is what we have done here--we've only allowed for points whose *y coordinate* were zero (that is, we did the evaluation *only* for points along the *x axis*). What we should have done, to be technically correct, was to derive a *general* electrical potential function--one that is good *for the whole field* (versus one that is good just for points along the *x axis*). If we had used the del operator on *that* kind of function, then evaluated its results at $x = a, y = 0$, the correct electric field expression for that point would have appeared, direction and all. Figuring out a general electrical potential function--one that is good for any arbitrary point *x, y*--is messy, but once you've done it the del operator approach works just fine.

iv.) What would you have to do to make the del operator approach work in an *unlike-charge* situation? Explain.

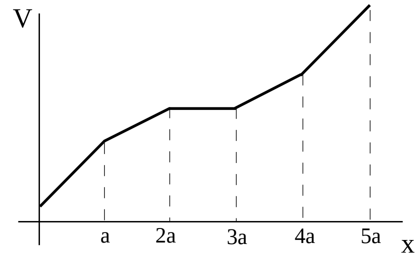
Solution: See *Part 12e-iii*.

v.) Why *did* the approach work as presented in the *like-charge* situation?

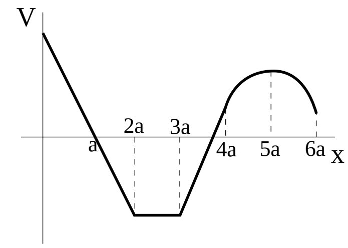
Solution: Notice that in the sketch I defined the *y coordinate* of the point of interest to be $y = 0$. Selectively omitting the *y* variability in the electrical potential function was our downfall. It meant that there would be no *y variable*, hence, no *y derivative*, in the use of the del operator. Because symmetry mandates the field be in the *x* direction, and because there is an *x* variable in the electrical potential function, we ended up with an electric field expression that worked even though it is the consequence of our setting the problem up incorrectly. Sadly, though it is tainted, this *mickey mouse* approach *does* yield the correct result when symmetry exists and all of the charges in the system are of the same kind. For that restricted situation, you could use the tainted technique if you were careful with your variable selection (though you'd have to shower after doing so).

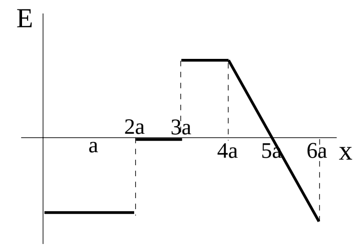


13.) The graph of an electric field in one dimension is shown to the right. What would the corresponding *electrical potential* graph look like?



Solution: The electrical potential difference between any two points is the area under the *electric field versus position* graph ($\Delta V = \int \mathbf{E} \cdot d\mathbf{r}$). Assuming the electrical potential is some positive value, it will get bigger between *zero* and *a*, will get still bigger but not as fast between *a* and *2a*, will not change at all between *2a* and *3a*, then will get bigger again, etc. The graph is shown to the right.





14.) The graph of an electrical potential field in one dimension is shown to the right. What would the corresponding *electric field* graph look like?

Solution: The slope of the electrical potential function at any point is equal to the electric field function ($E = -dV/dx$). That slope is constant and negative in the section from *zero* to *2a*, then is zero, then constant and positive from *3a* to *4a*, then varies from positive to zero to negative in a relatively linear way. The graph is shown to the right.

15.) A charge q in a constant electric field sits at a point where the electrical potential is V_1, where V_1 is positive. It accelerates from rest to a position at which the electrical potential is $2V_1$. At that point, the particle's velocity is v.

a.) Is the charge positive or negative?

Solution: The charge is negative. How so? It accelerates from a lower potential (i.e. V_1) to higher potential (i.e. $2V_1$). *Negative charges* do that--positive charges accelerate from *higher* to *lower* electrical potentials.

b.) If q had gone to a position where the electrical potential was $3V_1$, how fast would it have been going (do this in terms of the known velocity v)?

Solution: This is a conservation of energy problem. In the original problem, the electrical potential difference was $2V_1 - V_1 = V_1$. That means the work done was $W = -\Delta V = -(-q)V_1$. Also, given that the initial velocity was zero, the kinetic energy change was $.5mv^2$. We can write out the work/energy theorem as $.5mv_1^{\,2} = qV_1$, where v_1 is the velocity of the charge after the initial acceleration. The electrical potential difference in the second situation is $3V_1 - V_1 = 2V_1$. Using the work/energy theorem with that information yields $.5mv_2^{\,2} = 2qV_1$. Taking a ratio of the two expressions, the $.5m$ terms cancel as do the qV_1 terms leaving $v_1^{\,2}/v_2^{\,2} = (1)/(2)$. Evidently, $v_2 = (2)^{1/2}v_1$. Again, not a very tidy result, but not surprising given the *velocity squared* characteristic of kinetic energy quantities.

16.) A positive charge q in a constant electric field is released from rest and travels 10 meters. At the end of the 10 meter run, its velocity is v. How far would it have had to travel to attain a velocity of $2v$?

Solution: This is not a problem whose *outcome* is terribly important. What is important is the thinking you did in coming to conclusions over it. That is, it was designed to make you think about the relationship between energy conservation, electrical potential differences, electric fields, and distances traveled in electrical potential fields. Hopefully, your thinking went as follows: The charge's initial potential energy is qV_1, where V_1 is the absolute electrical potential at the starting point. The conservation of energy maintains that $KE_1 + PE_1 + W_{ext} = KE_2 + PE_2$. Substituting in for the non-zero parts, this becomes $qV_1 = .5mv^2 + qV_2$, where V_2 is the absolute electrical potential at the same point the mass's velocity is v. Putting the *kinetic energy* term on the left side and the qV_1 term on the right side of the equation, and noting that $qV_2 - qV_1 = \Delta V$, we can write this as $-.5mv^2 = \Delta V$, or $.5mv^2 = -\Delta V$. In other words, $-\Delta V \ \alpha \ v^2$ (note that "α" symbolizes the words *is proportional to*). From out of our bag of relationships, we know that $-\Delta V = \boldsymbol{E}\cdot\boldsymbol{d}$. From that we can deduce that for a constant electric field $\boldsymbol{E}$, the magnitude of the displacement d is proportional to $-\Delta V$ . . . which is proportional to v^2. So, for the initial situation, we can write that $d_1 \ \alpha \ v^2$, and for the secondary situation we can write $d_2 \ \alpha \ (2v)^2$ (note that $d_1 = 10$ *meters* and d_2 is what we are looking for). Taking the ratio of the two yields $10/d_2 = v^2/(4v^2)$. Canceling appropriately and solving yields $d_2 = 40$ *meters*. This surprising solution shouldn't be so surprising. Remember back in the energy chapter when you were asked if accelerating from zero to 20 mph required the same amount of energy as accelerating from 20 mph to 40 mph? The answer was *NO*. It takes a lot more energy to motivate an object moving fast to still higher speeds than it does to motivate an object moving slower to higher speeds. This is the same idea. A whole lot more work is required to double the velocity, hence the requirement of a whole lot more distance.