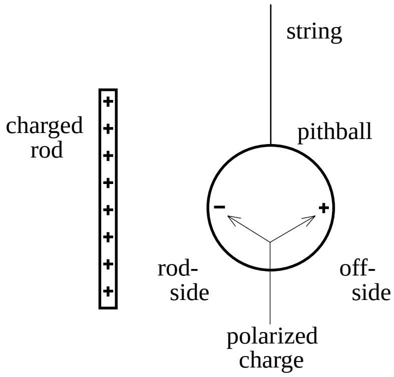
Electrostatics -- Conceptual Solutions

1.) The mass of an electron is $9.1x10^{-31}$ kg and its charge is $1.6x10^{-19}$ coulombs. If two electrons are separated by 1 meter, each will exert an electrical force and a gravitational force on one another. How do those forces compare?

Solution: The magnitude of the electrical force between any two *point charges* is called a *Coulomb force*. It is numerically equal to kq_1q_2/r^2, where k is a constant equal to $9x10^9\ nt\cdot m^2/C^2$ (it's normally written as $1/(4\pi\varepsilon_o)$, where ε_o is, itself, a constant called the permittivity of free space), the q terms are the magnitudes of the charges (in the MKS system, the units of charge are *coulombs*), and r is the distance in meters between the point charge). For two electrons, this force becomes $(9x10^9\ nt\cdot m^2/C^2)(1.6x10^{-19}\ C)^2/(1\ m)^2 = 2.3x10^{-28}\ nts$. The magnitude of the gravitational force between any two masses is Gm_1m_2/r^2, where G is the Universal Gravitational Constant equal to $6.67x10^{-11}\ nt\cdot m^2/kg^2$, the m*'s* are masses, and r is the distance between the centers of mass of the two bodies. For the two electrons, this equals $(6.67x10^{-11}\ nt\cdot m^2/kg^2)(9.1x10^{-31}\ kg)^2/(1\ m)^2 = 5.52x10^{-71}\ nts$. The ratio of these two forces is $F_e/F_g = 4.17x10^{42}$. That is, the electrical force is 10^{42} times larger.

2.) A light, small, styrofoam ball (this is called a *pithball*) is painted with a metallic paint and attached to a string that hangs freely in mid-air.

a.) What will the pithball do when a positively charged rod is brought close to it (the two don't touch)?



Solution: The bare bones information you need to understand this is as follows: 1.) Structures are deemed *metallic* if their atomic bonding allows their valence electrons (i.e., outer shell electrons) to roam freely throughout the material. Protons, on the other hand, are fixed within the nucleus of the atoms *and cannot move around*. 2.) Without the presence of an outside charge like that on the rod, the natural repulsion between the negative electrons motivates them to distribute evenly over a spherical, metallic surface. 3.) When a positive rod approaches such a surface, the electrons in the metal are attracted to and migrate toward the rod (negative charge is attracted to positive charge). 4.) The flow of electrons toward the rod-side of the sphere ceases when the natural repulsion between electrons makes it impossible for more electrons to join the crowd. 5.) At that point, the rod-side of the sphere is electrically negative. 6.) With positively charged protons fixed in the atomic lattice, and with electrons having moved to the rod-side of the sphere, the off-side is left with a net positive charge. 7.) In other words, the sphere has been electrically polarized.

With all of this in mind, what does the pithball do when the rod approaches? The electron shuffle (sounds like a line dance) produces a relatively large negative charge on the rod-side of the sphere which is *closer* to the rod than is an equally large *positive* charge left on the off-side of the sphere. The net effect is that the attraction between the rod and negative charge on the pithball will be greater than the repulsion between the rod and the positive charge on the off-side of the pithball, and the net force will motivate the pithball to move toward the rod. In short, the pithball will swing toward the rod.

b.) How would the results of *Part a* have changed if the rod had been negatively charged?

Solution: The only difference between the two cases is that the electrons on the rod-side of the pithball would migrate away from the negatively charged rod leaving the rod-side electrically positive and the off-side electrically negative. The positive side would be closer to the rod than the negative side, so once again there will be a net attraction and the pithball would swing up toward the rod.

c.) How would the results of *Part a* have changed if the pithball had not been coated with a metallic paint but, instead, was simply styrofoam?

Solution: The temptation is to assume that because there is no metallic bonding here, the electrons in the pithball will not be able to move and, hence, nothing will happen. It turns out that that isn't the case. Indeed, electrons in a covalently bonded material cannot migrate throughout the material as they can with metallic bonding, but they do move *within the atom*. Under normal circumstances (i.e., without a lot of outside charge around), the geometric center of "orbiting" electrons is at the center of the atom. That is, on average, the electrically negative part of an atom is centered at the same place where the electrically positive part of the atom (i.e., the proton) resides--at the center of the nucleus. When a positively charged rod comes close, the electrons in each atom of the structure spend more time on the rod-side of their respective atoms. In other words, the average position of the negative charge in the individual atoms no longer "covers" the positive charge--there is a very slight polarization. This is, indeed, VERY SLIGHT (after all, it is all happening inside atoms--structures that are only 10^{-10} meters across), nevertheless the shift creates a disparity between the electron's attraction to the rod and the proton's repulsion to the rod. The net effect is that the pithball will, again, feel a net force and will swing toward the rod.

d.) The rod and pithball in *Part a* touch. What are the consequences for the pithball?

Solution: When the two touch, electrons on the pithball will transfer to the rod. This new negative charge density will be large where the contact occurs and non-existent at other places (the rod is covalently bonded so electrons can't roam about through it as would be the case if the rod had been metallic). Having lost electrons, the pithball now has a net charge that is positive. The resulting effect is that the positive charge on the rod and the newly positive pithball will repel, and the pithball will swing *away from the rod*.

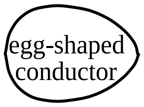
e.) You have a pithball that is covered with metallic paint. *Without allowing the pithball and rod to touch*, what clever thing could you do to make the pithball electrically negative?

Solution: Bring a positively charged rod close to the pithball. The valence electrons in the metallic covering will migrate toward the rod making the rod-side of the pithball negative and the off-side of the pithball electrically positive. If you touch the pithball on the off-side, you will "ground" that surface (i.e., electrons will flow from you to the exposed positive charge), neutralizing that side of the pithball. When the rod is then removed, the valence pithball's electrons will redistribute themselves over the pithball's metallic surface. But because there are now more electrons than before (remember, you transferred electrons from yourself to the pithball when you grounded the off-side), the surface will be electrically negative.

3.) If you put gas in a spherical shell, the gas will distribute itself pretty much evenly throughout the volume. If you put charge on a solid metal sphere, what will the charge do?

Solution: Like charges repel. They try to get as far away from charges of their own kind as possible. When you put charge on a sphere, the charge distributes itself over the surface of the sphere attempting to get as far away from others of its kind as possible. As such, no charge will move inside the sphere--it will all be held in dynamic tension on the sphere's surface.

4.) You have a charged, hollow, egg-shaped object made of copper. You put charge on the structure. How will the charge distribute itself over the surface? That is, will it distribute uniformly or what? If it doesn't distribute itself uniformly, how generally will it concentrate?



Solution: Due to the effect called *shielding*, charge densities go up on curved surfaces with the density getting larger and larger as the curvature gets more and more pointed. In other words, you will find more charge at the ends than in the middle, and more charge at the more pointed end than the less pointed end.

5.) Two point charges, one twice as large as the other, are placed a distance r units apart. How will the force on the smaller charge change if:

a.) The distance is doubled?

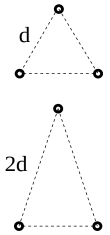
Solution: The Coulomb force is proportional to $1/r^2$. Double the distance and the force changes by $1/2^2$. In other words, it decreases by a factor of 4.

b.) The larger charge is doubled?

Solution: The Coulomb force is proportional to the size of each charge. Double one charge and the force will double.

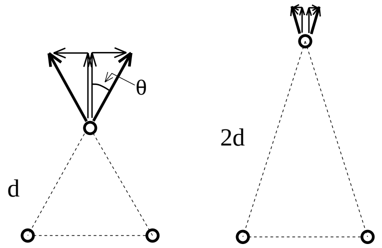
c.) How would the answers to *Parts a* and *b* have changed if you had been examining the larger charge instead of the smaller charge?

Solution: Coulomb force is a Newton's Third Law action/reaction pair (bad terminology, but you get the idea--for every force in the universe, there must exist an equal and opposite "reaction" force). In other words, the force the small charge experiences will be equal and opposite the force the large charge experiences.



6.) Three equal point charges are positioned at the corners of an equilateral triangle. The net force on the top charge is measured. The distance between the top charge and the other two charges is doubled. Decide which of the lettered responses below describes how the new net force on the top charge will change, then explain why that response is appropriate.

a.) Double.
b.) Halve.
c.) Quadruple.
d.) Quarter.
e.) None of the above.



Solution: It turns out that *response e* is the correct one. The temptation is to figure that if you double the distance between the charges, the force will decrease by a factor of 4 (the force *is* proportional to $1/r^2$). In fact, that is true of the *magnitude* of the force. The problem is that force is a *vector*--you have to take direction into account. The net force is the sum of the *vertical components* of the forces on the top charge due to the presence of the other two charges. This number, in turn, is dependent upon the *angle* shown in the sketch. When you move the top charge, you change the angle and, hence, the vertical component of the acting forces. So although the force magnitudes go down by a factor of four, the net force is greater than expected because more of the forces are now in the vertical.