Rotational Motion I -- Conceptual Solutions

1.) A ball and a hoop of equal mass and radius start side by side and proceed to roll down an incline. Which reaches the bottom first? Explain.

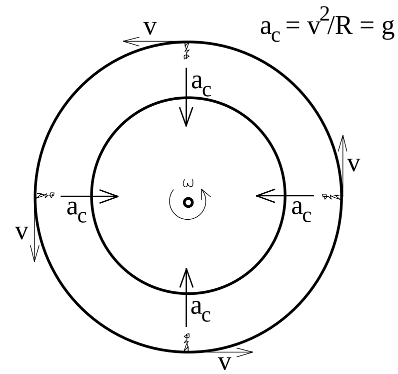
Solution: Which body has the greatest rotational inertia (i.e., moment of inertia)? It's the hoop. Why? Because on average, there is more mass farther out away from the central axis of the hoop than from the central axis of the ball. So which has the greatest angular acceleration? The body that has the least resistance to changing its rotational motion (i.e., the body with the least rotational inertia). As that is the ball, the ball will reach the bottom of the incline first.

2.) If you drive a car with oversized tires, how will your speedometer be affected?

Solution: A speedometer really has two parts to it. Attached to the wheel is a sensor that counts the number of wheel rotations the tire executes per unit time. That information is passed on to a meter located on the dashboard (i.e., what you and I would call *the speedometer*) which converts that data into a number that corresponds to the *translational distance traveled per unit time* (i.e., the velocity) in *miles per hour*. So let's assume that your car has stock tires on it, and a sensor reading of 7 turns per second corresponds to a distance traveled of approximately 44 feet per second. This would present itself on the speedometer as a velocity of 30 miles per hour (in fact, these numbers are approximately accurate--88 ft/sec corresponds to 60 mph; that means 44 ft/sec corresponds to 30 mph; a 1 ft radius tire has a circumference of $2\pi r = 2(3.14)(1\ ft) = 6.28\ ft$, so to cover 44 ft in a second, the wheel will have to rotate *44 ft/sec)/(6.28 ft/rotation)= 7 rotations per second* approximately). What happens when oversized tires are put on your car and the wheels are made to rotate, once again, 7 turns per second? The distance now traveled in a second will be greater than 44 ft because the tires now have a larger circumference, but the sensor will still register 7 rotations per second and the speedometer will still convert and present this as 30 mph. In short, the actual speed will be greater than the speed presented by the speedometer, which means the speedometer will register a speed that is slower than the actual speed of the car.

3.) Assume global warming is a reality. How will the earth's *moment of inertia* change as the Arctic ice caps melt?

Solution: If the Arctic ice caps melt, the released water will respond to the earth's rotation and flow outward away from the axis of rotation (i.e., toward the earth's equator). With more mass farther away from the axis of rotation, the earth's rotational inertia (i.e., moment of inertia) will increase.



4.) Artificial gravity in space can be produced by rotation.

a.) How so?

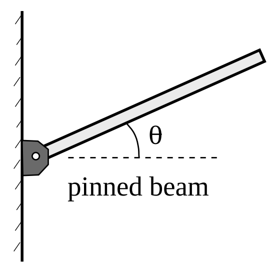
Solution: If you rotate a hoop-shaped space station about its central axis, the normal force provided by the station's floor will push objects on the floor (like a person) out of straight line motion and into circular motion. A person standing on the floor will, therefore, feel an *upward force* that will superficially appear to be no different than the force he or she might experience while standing on the earth. The difference is that because the apparent force is caused by the rotation of the station, we can relate that force and its associated acceleration with the velocity of the station. That is, centripetal force requires a centripetal acceleration of $a_c = v^2/r$, where v is the translational velocity of the station's floor and r is the floor's radius). Putting that acceleration equal to g yields $g = a_c = v^2/R$, or $v = (Rg)^{1/2}$. In other words, a floor rotating about a fixed point r units away with a translational velocity of $v = (Rg)^{1/2}$ will create an apparent acceleration equal to g, and anyone standing on such a floor will not be able to tell the difference between that situation and a situation in which the floor is stationary in the earth's gravitational field. In both cases, the individual will feel as though they weigh mg. Of additional interest here is the fact that the translational velocity v of the floor will be related to the angular velocity ω of the station by $v = r\omega$, so the angular velocity required to make the acceleration g will be $\omega = v/R = (Rg)^{1/2}/R = (g/R)^{1/2}$.

b.) Assume a rotating space station produces an artificial acceleration equal to g. If the rotational speed is halved, how will that acceleration change?

Solution: We know that $v = r\omega$, $a_c = v^2/r$ and, as a consequence, $a_c = (r\omega)^2/r$. Putting $a = g$, we can write $g = (r\omega)^2/r$. From that relationship, halving ω evidently quarters the centripetal acceleration, and a_c goes to $g/4$.

5.) Make up a conceptual graph-based question for a friend. Make it a real stinker, but give enough information so the solution *can* be had (no fair giving an impossible problem).

6.) A beam of length L is pinned at one end. It is allowed to freefall around the pin, angularly accelerating at a rate of $\alpha = k\cos\theta$, where k is a constant. If you know the angle at which it started its freefall, is it kosher to use rotational kinematics to determine the angular position of the beam after $t = .2$ *seconds*? Explain.



Solution: Rotational kinematics are predicated on the assumption that the *angular acceleration* is a CONSTANT. This angular acceleration function changes with the angle, so it is not constant and rotational kinematics would not be applicable here.

7.) The angular velocity of an object is found to be $-4\,\mathbf{j}$ *radians per second*.

a.) What does the unit vector tell you?

Solution: The unit vector defines the *AXIS about which the body rotates*. That axis will always be perpendicular to the plane in which the motion occurs, so the unit vector also allows you to identify the plane of rotation.

b.) What does the negative sign tell you?

Solution: The negative sign tells you that the object's rotation is clockwise within the plane of motion as viewed from the positive side of the *y axis* (you *always* view from the positive side). This can be seen using the right hand rule. Position your RIGHT hand in hitchhiker position and orient your thumb so that it is in the *negative **j** direction* (i.e., down the page along the axis of rotation). In that position, your right hand will curl in the clockwise direction (again, as viewed from above on the $+y$ side of the origin). That is how you determine the clockwise/counterclockwise sense of the object's motion.

c.) What does the number tell you?

Solution: The number tells you how many *radians* the object rotates through *per unit time*.

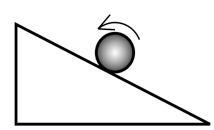
d.) How would questions *a* through *c* have changed if the $-4\,\boldsymbol{j}$ had been an angular position vector?

Solution: The unit vector would define the plane in which the angle is measured (how? the unit vector would be perpendicular to that plane), the negative sign would tell you that the angular position is measured *clockwise* from the *+x axis*, and the number would tell you how many *radians* you would have to rotate through, relative to the reference axis (i.e., the *+x axis*), to get to the object's position.

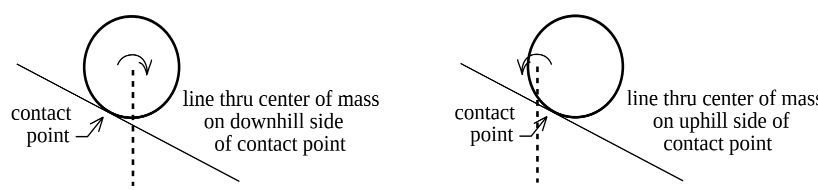
e.) How would questions *a* through *c* have changed if the $-4\,\boldsymbol{j}$ had been an angular acceleration vector?

Solution: The unit vector would define the plane in which the angular acceleration takes place, the negative sign would tell you that the angular acceleration is in the clockwise direction, and the number would tell you how many *radians/second* the motion changes *per second*.

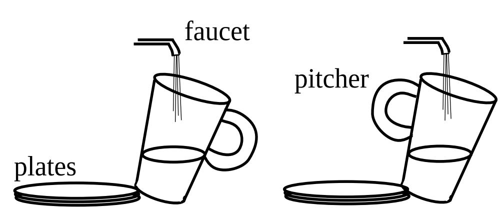
8.) A circular disk sits on an incline. When released, it freely rolls *up* hill. What must be true of the disk?



Solution: For a normal, homogeneous disk (i.e., a disk in which the mass is uniformly distributed), the *center of mass* of the disk will be to the right of a vertical line through the contact point between the disk and our incline (see the sketch). In that case, the disk will roll down the incline. If our disk had been inhomogeneous, on the other hand, it would have been possible to position the *center of mass* to the left of that line. In that case, the disk would have rolled the other way, or *up* the hill. In other words, what's true of the disk is that it's center of mass is not at its geometric center.



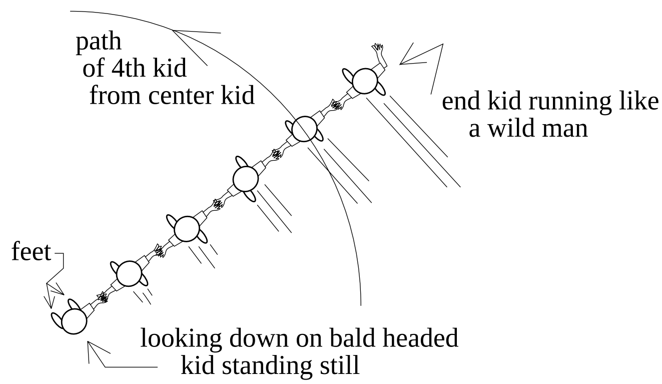
9.) Two people want to fill up their respective water pitchers. Both use a sink in which there are stacked plates. Neither is particularly fastidious, so each precariously perches his pitcher on the plates (notice I've made them guys?), then turns the faucet on. Which orientation is most likely to get the user into



trouble? Will the trouble surface immediately or will it take time? Explain.

Solution: For stability, the *center of mass* of an object must be over an area of support. The problem here is that as the water fills the tipped pitchers, the *center of mass* of each pitcher/water system will creep to the right. As the pitcher on the left has more mass to the right in the first place, it is most likely to have its *center of mass* ultimately (i.e., after some time) move beyond its support and, as a consequence, tip over.

10.) A group of kids hold hands. The kid at one end stays fixed while all the rest try to keep the line straight as they run in a circle (when I was a kid, we called this game *crack the whip*). As you can see in the sketch, the farther a kid is from the stationary center, the faster that kid has to move to keep up. If the speed of the kid one spot out from the center is v, what is the speed of the kid four spots out from the center (see sketch)? You can assume that each kid is the same size and takes up the same amount of room on the line.

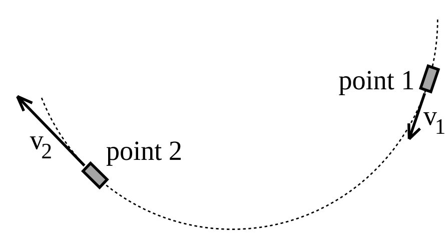


Solution: The angular velocity ω is the same for each kid (i.e., each kid is sweeping through the same number of *radians per second*), but the translational velocities get larger the farther one gets from the axis of rotation. The relationship between the angular velocity and the translational velocity of a kid r meters from the fixed axis is $v = r\omega$. That is, the translational velocity increases linearly with distance from the fixed axis. In short, a kid who is four times farther from the center than a second kid will have to travel four times as fast. For our problem, that velocity is equal to $4v$.

11.) A light, horizontal rod is pinned at one end. One of your stranger friends places a mass 10 centimeters from the pin and, while you are out of the room, takes a mysterious measurement. She then takes the same measurement when the mass is 20 centimeters, 30 centimeters, and 40 centimeters from the end. You get back into the room to find the graph shown to the right on the chalkboard. Your friend suggests that if you can determine what she has graphed, there might be something in it for you. What do you think she has graphed?

Solution: As the rotational inertia (i.e., the moment of inertia) of a point mass is proportional to the square of the distance between the mass and the axis of rotation ($I = mr^2$), the graph is most likely that of the moment of inertia of the mass about the pin.

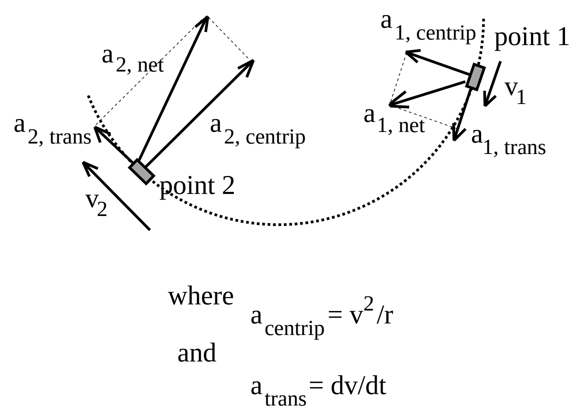
12.) A car rounds a corner. It goes into the curve with speed v_1 and exits the curve with greater speed v_2. Assume the magnitude of the velocity changes



uniformly over the motion and the motion is circular and in the x-y plane (see sketch).

a.) On the sketch, draw the direction of acceleration of the car at the two points shown.

Solution: There are two components to the acceleration of the car. The first component forces the car into a curved path. That component will be "center seeking" and, hence, will be toward the center of the curve upon which the car happens to be traveling at a particular instance. It will also be velocity dependent as $a_c = v^2/r$. To that we must vectorially add the acceleration that increases the *magnitude* of the velocity at a particular instance (dv/dt). That acceleration will be in the direction of motion (i.e., along the line of v) and will be constant (the velocity magnitude was assumed to be changing at a constant rate). Put together, the accelerations will be as shown in the sketch. Note that although the magnitude of the translational component dv/dt will be the same throughout the motion, the centripetal component v^2/r will be greater at the second spot because $v_2 > v_1$. Also, note that the magnitude of the net acceleration changes from point to point as does the relative angle of the acceleration--something to be aware of when driving a car that is executing such a maneuver.



b.) Identify the car's angular acceleration at the two points.

Solution: Because the magnitude of the angular acceleration is numerically equal to a_{trans}/r, it will be constant throughout the motion. And because the direction of an angular acceleration vector is always perpendicular to the plane in which the motion occurs, its direction will also be constant (in this case, because the rotational acceleration is clockwise in the *x-y plane*, that direction will be *into the page*, or in the *-**k** direction* . . . remember, any angular parameter that is associated with counterclockwise motion will be designated as being positive while any angular parameter associated with clockwise motion will be designated as being negative).

c.) Why are angular parameters preferred over translational parameters when it comes to rotational motion?

Solution: Assume you are looking at a body that is moving rotationally in the plane of this page. In comparison to the directional behavior of the body's translational parameters (i.e., its acceleration or velocity or whatever), the directional behavior of its rotational parameters is simple. That is, the direction of the vector associated with *all rotational parameters* (angular acceleration or angular velocity or whatever) will be along *one axis*--the axis perpendicular to the page. For rotational parameters, planar rotation is essentially a one-dimensional experience. What's more, if the object's translational velocity increases or decreases *uniformly* as it moves, the magnitude of a body's acceleration will change (see sketch in *Part a*) but the magnitude of the body's angular acceleration will be constant. So which would you prefer to deal with, an acceleration quantity whose magnitude and direction are constantly changing throughout the motion, or an angular acceleration quantity whose magnitude and direction are constant?