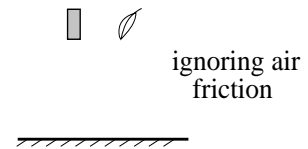


Newton's Laws -- Conceptual Solutions

- 1.) A car moves at a constant 30 m/s. Is it accelerating? If so, must there be an applied net force to maintain this motion? Explain.

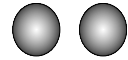
Solution: This is a tricky question. Most people see the words *constant 30 m/s* and think, *constant velocity implies no acceleration which means no force*. The problem is that velocity is a vector. There is more than one way to *change* a velocity vector. Indeed, the magnitude of the velocity vector isn't changing (it's constant), but its *direction* could be changing. Generally referred to as a *centripetal* acceleration, it is associated with a force that doesn't speed up or slow down a body but, rather, changes the *direction* of the body's motion. This requires an applied net force (N.F.L.--objects in motion tend to stay in motion in a straight line unless impinged upon by an external force). In short, *the car may or may not be accelerating. If it is moving along a curved path, there must be a net force motivating that motion.*

- 2.) Two objects of different mass will have different weights (that is, they will feel different gravitational forces), yet if air friction is ignored and you drop both from the same height, they will accelerate due to gravity at the same rate. How can this be?



Solution: Due to a quirk in the nature of things, if a block has twice the gravitational mass (i.e., twice the willingness to be attracted to another body . . . like the earth) as a feather, it will also have twice the inertial mass (i.e., twice the resistance to changing its motion) as does the feather. The greater weight (gravitational force) is counteracted by the body's greater resistance to changing its motion (inertia), and the net effect is that all objects in a frictionless situation will fall at the same rate.

- 3.) A solid copper ball and a hollow copper ball of the same radius are found in space. Both are weightless. Without cutting them open, how can you determine which is which?



Solution: Because the two objects are both made of copper, the hollow ball will be less massive. Being less massive, it will have less resistance to changing its motion. Therefore, if you are in space where the only force applied to either ball comes from you, the ball that is easier to motivate out of a stationary state will be the hollow, less massive ball. (This is like asking, *What will be harder to move in space, a 16 pound shot-put or a softball-size sphere made of styrofoam?*)

- 4.) A body is accelerated by some net force. If the force is halved, how will the velocity-change Δv alter? If the mass is halved instead, how will the velocity-change Δv alter?

Solution: *Velocity change* with time is acceleration, so *altering* the velocity change (as the question states) really suggests that there is a change of acceleration. According to Newton's Second Law, net force and acceleration are proportional ($F_{net} = ma$), so if the force is halved with the mass kept constant, the acceleration will also be halved. On the other hand, if the force is kept constant and the mass is halved, the acceleration must double (again, think about $F_{net} = ma$).

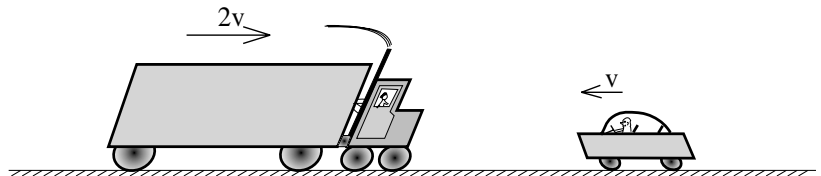
5.) It is easier to keep a crate moving across a frictional floor than it is to get it going in the first place. Why? That is, aside from the relative motion, what is the fundamental difference between the two cases?

Solution: You are looking at the difference between kinetic (sliding) friction and static friction. Static friction, due to the fact that there is no motion involved, is a situation in which the atomic structures of the two interacting bodies can meld more completely than would be the case with sliding friction. As that melding must be sheared to instigate motion, it is easier to keep a crate going (less melding, hence, less shearing needed) than to get it going in the first place.

6.) A heavy box attached to a parachute will reach the ground faster than a light box of equal size attached to an identical parachute. Why?

Solution: As a parachute falls, air collides with the underside of the plummeting chute. This produces an upward, resistive force on the chute. With essentially no initial velocity at first, the dominating force in freefall is gravity. But as the velocity increases, this air-born resistive force increases (remember, it's opposite the direction of gravity) diminishing the net force on the chute and reducing its acceleration. When the force of gravity exactly counteracts this resistive force, the body stops accelerating and proceeds to the ground at what is called *terminal velocity*. To reach terminal velocity, a heavy box must be moving quite fast before air friction can counteract its relatively large weight. As such, a heavier box will have a higher terminal velocity associated with it and, as a consequence, will reach the ground faster than will a lighter box.

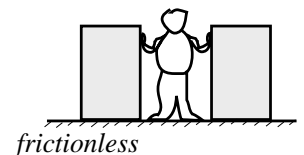
7.) A truck is ten times more massive and moves with twice the speed of a small car. The two collide. During the collision, which will experience the greater force? Which will experience the greater acceleration?



Solution: This is an old standby. According to Newton's Third Law, for every force in the universe there must be an equal and opposite *reaction* force. (Minor point: The word *reaction* is misleading as, in fact, the two forces happen at the same time . . . versus one happening and the other following in reaction). The idea is that one object can't exert a force on another object without the other object exerting the same force back on the first. It's just the way the universe works. What this means is that whatever force magnitude the truck experiences, the car must experience the same force magnitude. An object's acceleration is more complicated as it is related to both the magnitude of the net force (again, this is the same for both in this case) *and* the body's mass (remember, $F_{net} = ma$).

In this case, the car (with its lesser mass) will experience a greater acceleration (i.e., it will change velocity more radically) than will the more massive truck with its lesser acceleration . . . even though the product ma will be the same for both.

8.) A man stands wedged between two identical crates on a frictionless sheet of ice. Is there any way he can make the acceleration of one of the blocks greater than that of the other?



Solution: If the man pushes one block, his interaction with that block will force him toward the other block. That is, if he pushes the right

block to the right, it will push on him to the left with an equal force. Given the fact that he is jammed between the two boxes, that force will be transferred through him to the block on the left. As the two blocks have the same mass, the equal forces being applied to each will provide the same acceleration no matter what.

9.) Drop a rock from the mast of a moving boat. Will it hit the deck a.) in front of the mast, b.) next to the mast, or c.) behind the mast? Justify your response in terms of the force(s) acting on the rock.

Solution: The only force acting on the rock, ignoring frictional effects (we can effectively do this because the relative distance traveled is so small), is gravity. If the rock has an initial velocity in the horizontal (it does as it is initially traveling with the boat), it will continue to move with that horizontal velocity until it hits the deck. As the horizontal velocity of the rock and mast will be the same throughout, the rock should hit the deck next to the mast . . . and *b* is the answer.

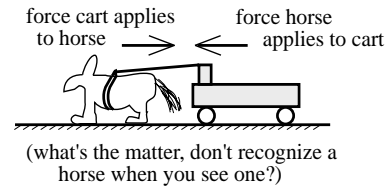
10.) A kid steps off a footstool and begins to freefall under the influence of gravity. That is, the earth applies a gravitational force to the kid. For this situation, what is the "reaction force" alluded to in Newton's Third Law?

Solution: In the first place, the term *reaction force* is a misnomer. The "action" and "reaction" forces really act at the same time. Ignoring this unfortunate play on words, if the earth exerts a gravitational force on the kid, the kid must exert an EQUAL AND OPPOSITE gravitational force on the earth. The reason the kid's response to the force is so noticeable (after all, the little devil *does* seem to pick up speed fairly quickly as he/she moves toward the earth) is because the child's mass is small in comparison to that of the earth. Remember, $F_{net} = ma$. For a given force, if *m* is small, *a* will be relatively large . . . and vice versa.

11.) A force F_1 stops a car. In terms of F_1 , how large must a new force be to stop the same car under the same circumstance but in half the distance? In half the time?

Solution: This is tricky. It seems to make sense that to stop the car in half the time, the car has to slow down twice as fast. In fact, the relationship between acceleration and time is more easily seen by considering a car at rest that accelerates through some distance. If you decide to cover the distance in half the time, how does the acceleration have to change? The relationship $\Delta x = .5at^2$ suggests that travel distance is dependent upon the square of the time (yes, this is for a somewhat different problem--a car picking up speed instead of one slowing down--but the concept is the same). In other words, halving the time requires the acceleration to increase by a factor of *four* if the distance is to stay the same. Given the fact that acceleration is proportional to force, the force in this case must be $F = 4F_1$. On the other hand, stopping the car in half the distance really does mean that the car has to slow down twice as fast as it originally did. The math justifies this claim. Specifically, $v_{stop}^2 = v_o^2 + 2a \Delta x$ with $v_{stop} = 0$ and *a* being negative means that $a = v_o^2 / 2 \Delta x$ (note that the sign of the acceleration term has been unembedded leaving *a* to denote a magnitude only). Therefore, if Δx halves, *a* doubles and the force we are looking for becomes $F = 2F_1$.

12.) A horse pulls on a cart. According to Newton's Third Law, the cart must exert an equal and opposite reaction force back on the horse. That is, if the horse pulls the cart with 50 newtons of force, the cart must pull back on the horse with 50 newtons of force. As this so-called action/reaction pair always adds to zero, it appears as though we are suggesting that the cart will never accelerate. This obviously can't be the case, so what's the problem here?



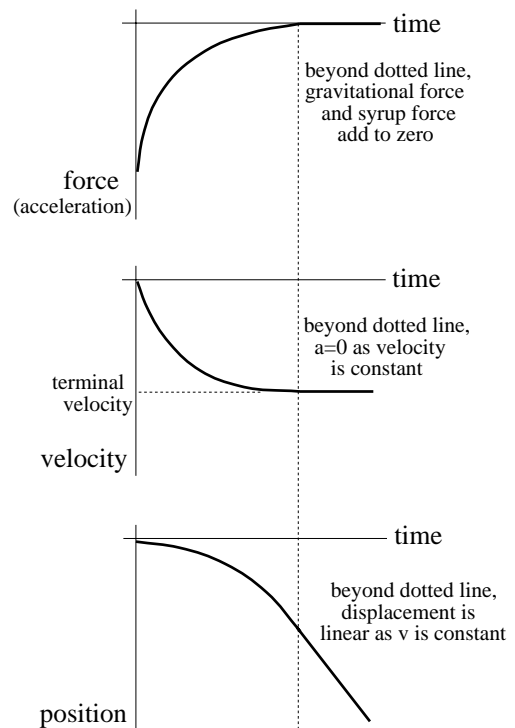
Solution: I first saw this problem in Hewitt's book *Conceptual Physics*. It is a great example of getting Newton's Laws confused. There is, indeed, an action/reaction pairing between the horse and the cart, but that isn't all there is. For motion, there must be a force between the horse's hooves and the street (in what direction is that force? . . . it had better be in the direction the horse is trying to go--if this isn't clear, ask your teacher about it). In other words, if you are going to look at the forces acting on a system, you have to include *all* the forces.

13.) Assuming friction is negligible, which will reach the bottom of an incline first, a large box or a small box? Explain.

Solution: In theory, if the incline is frictionless, both would hit the bottom at the same time. For a frictional situation, though, with the big box weighing more, kinetic friction will be greater on that box than on the smaller box so the smaller box should get to the bottom first.

14.) An object falls from rest into a syrupy fluid. What does its *net force versus time* graph look like? Its *velocity versus time* graph? Its *position versus time* graph?

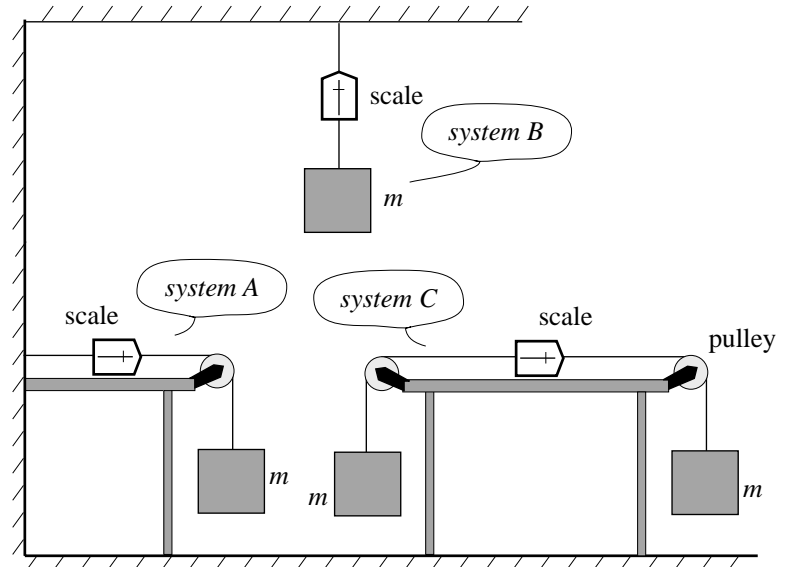
Solution: The *force versus time* graph will look the same as the *acceleration versus time* graph. Knowing that, we can determine the general form of the *velocity versus time* graph and *position versus time* graph using what we know about the relationship between those quantities. Note that if the object had fallen in air, it would have begun with an acceleration of 9.8 m/s^2 only to have its acceleration decrease as its velocity and, hence, frictional effects increased. At some point, the frictional force would exactly match gravity, the acceleration would cease, and the object would move at a constant *terminal velocity* for the rest of its trip. In a syrupy fluid, the same will happen except it will hit terminal velocity more quickly. The graphs are to the right. Note that the *force (acceleration) versus time* graph is just the derivative (i.e., slope) of the *velocity versus time* graph which is, in turn, the slope of the *position versus time* graph. Assuming POSITIVE is associated with UPWARD MOTION, everything is negative. Note: In doing this solution, I



started with the easiest motion to visualize, at least for me. That was the *velocity versus time* graph. I figured the velocity started from rest, increased at some non-linear rate until it leveled off at terminal velocity, then stayed there for the rest of the motion. From that graph, I deduced the other two graphs.

15.) Assuming all masses are the same size and the pulleys are ideal (i.e., massless and frictionless), which of the three scales in the figure will register the greatest force?

Solution: As bizarre as this might seem, they will all register the same force. The scale in the single pulley system (system A) on the left is essentially registering the weight of m (all an ideal pulley does is re-direct the line of the tension), so it and the ceiling hung scale (system B) will read the same number. The one that



usually confuses people is the double pulley situation (system C) on the right. Reconsider, though, system A. To keep the hanging mass from accelerating downward, the wall must apply a force that translates to the scale and mass via tension. That force pulls the scale to the left. Now consider system C. Its right side looks exactly like the system A (that is, there is a scale, a pulley, and a hanging mass). For the right side of system C to be in equilibrium, there must be a force applied to its scale *that is the same size* as the force applied to the scale in system A. Where does that force come from in system C? It comes from the hanging mass on the left side of the table. What's important to realize is that because the two situations both depict equilibrium, the force on the left side of the scale in both cases must be the same. And if that is so, both scales must measure the same value. Put a little differently, it doesn't matter how the force required for equilibrium is generated, if the hanging mass on the right side of each table is to sit in equilibrium, the same force must be applied to the left side of the scale *no matter where that force comes from*. In the one case, the wall provides the force. In the other case, a second hanging mass provides the force. In all cases, though, the scales will read the same value.

16.) A ball is dropped from rest a distance h units above a bathroom scale. When it hits, the scale measures an average force of 50 newtons. The ball is then dropped from a distance $2h$ units above the scale. Will the scale read 100 newtons, less than 100 newtons, or more than 100 newtons? Explain.

Solution: It turns out that it will read *less than 100 newtons*. The key here is in determining the velocity of the ball as it strikes the scale so that we can determine the acceleration that must exist if the ball is to come to rest (remember, force and acceleration are proportional). If we assume the time of collision is the same for both situations (a dubious assumption, but we have little else to go on here so we have to make it), and if we assume the acceleration is approximately constant throughout the slowdown (another

dubious assumption, but what the hell), we can use kinematics to write $F_{avg} = ma_{avg} = m(v_d - v_s)/t_{collision}$, where v_d is the velocity of the ball when the scale is fully depressed (this value will be zero at that point) and v_s is the velocity of the ball just as it hits the scale. With $v_d = 0$, this becomes $F_{avg} = -mv_s/t$ (in this case, we can ignore the negative sign--we are only interested in the magnitude of the force). Also, we know that this number will numerically equal 50 newtons when the ball falls from a height h , so we can re-write our expression (ignoring the negative sign) as $50 = mv_s/t$. If we can determine a relationship between the *before collision* velocity v_s for both cases (after all, according to our expression, v_s is proportional to the average force), we can determine the relationship between the average force generated by the two falls. For the fall of height h , this expression is $v_s^2 = v_o^2 + 2(-g)(-h)$, with v_o being the initial fall velocity (this is zero). That relationship yields $v_s = (2gh)^{1/2}$. Using this same expression for a height of $2h$ yields $v_{2h} = (2g(2h))^{1/2} = (2)^{1/2}v_s$. In other words, $v_{2h} = (2)^{1/2}v_s$. As the forces are proportional to these velocities, we can write $F_{avg,2h} = (2)^{1/2}F_{avg,h} = (2)^{1/2}(50 \text{ nts}) = 70 \text{ nts}$. The new force will be less than 100 newtons.

17.) A massive object is placed on a frictionless table. It takes 2 newtons of force to accelerate it at $.5 \text{ m/s}^2$. The object is taken into space where it is weightless. The force required to accelerate the object at $.5 \text{ m/s}^2$ will be (a) less than, (b) equal to, or (c) more than 2 newtons.

Solution: On a frictionless table, all your force is doing as it accelerates the object is overcoming the object's inertia. In gravitationless space, the same is the case. The two forces should, therefore, be the same.

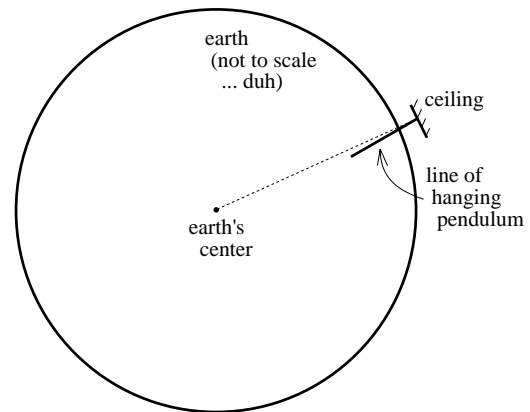
18.) Magic Mountain is an amusement park in Southern California that is known for its giant roller coaster rides. One of the rides, Superman, consists of a cart that is accelerated along a horizontal stretch of track (via magnetic induction, no less) to somewhere around 100 mph in less than four seconds. The cart's path then curves into a vertical climb up an enormously high tower. At the top it comes to rest whereupon it proceeds to freefall several hundred feet back down the vertical section of the track and out the curve onto the horizontal freefall section where it finally comes to rest. The drop from the top is billed as *pure freefall*. In theory, if you took this ride and released a dime at the top (i.e., just as you began to freefall back down the track), the dime should sit motionless in front of you as both you and it gravitationally accelerated back toward Earth. For two reasons (one obvious and one not so obvious), **DOING THIS WOULD BE A BAD IDEA--A REALLY, REALLY BAD IDEA**. What is the obvious problem and, for the hotshots, what is the not so obvious problem?

Solution: If you go along with the idea that you don't want to kill someone, then you don't want to do anything that might allow the freely falling dime to come into contact with someone's skull. If you and the dime were accelerating at the same rate, it would still be in front of you when you got to the bottom and you could grab it before pulling out onto the horizontal stretch. The "obvious problem" is that that won't happen. The dime, being

relatively light, will hit its terminal velocity fairly quickly and, as a consequence, will not keep up with you as you accelerate downward (it will appear to accelerate up and away from you). If people happen to be walking under the ride, they could get clobbered by the falling object (amusement parks get around this problem by closing off the area under such rides). The "not so obvious problem" is associated with what is called Bernoulli's effect. As the dime falls, the air velocity on one side of the dime will be greater than on the other side due to its spin. As a consequence, the variation will cause a pressure difference between the two sides. The high pressure side will push the dime motivating it to veer off away from you to the left or the right. If conditions are just right (or just wrong, depending upon how you look at it), the dime could conceivably sail out beyond the restricted area under the ride and actually hit someone. In short, if you must observe this freefall phenomenon, use something relatively massive (maybe a baseball?) whose terminal velocity won't be reached so quickly and whose mass won't allow Bernoulli's effect to push it around so easily.

19.) A pendulum in Los Angeles (22° latitude) does not hang directly toward the center of the earth. Explain why not.

Solution: A simple pendulum is a string with a mass attached to one end. Think about holding such a pendulum while standing on the edge of a rotating *merry go round*. From your perspective, what does the pendulum bob appear to do? It seems to push out away from the center (actually, it's really trying to follow straight-line motion--your holding it, via the string, applies a force that pulls it into circular motion . . . hence the feeling that it pushes you outward). If the earth were stationary, a pendulum would be gravitationally attracted to the center of the earth and the string would orient itself between the pendulum's contact point (i.e., where the pendulum is attached to, say, the ceiling) and the earth's center. The problem is that the earth is rotating. This means that along with the tension force required to counteract gravity, there must be a tension *component* that pulls the pendulum bob into circular motion. The consequence is that the line of the pendulum will not be toward the earth's center but will be off a bit toward the equator (see sketch).



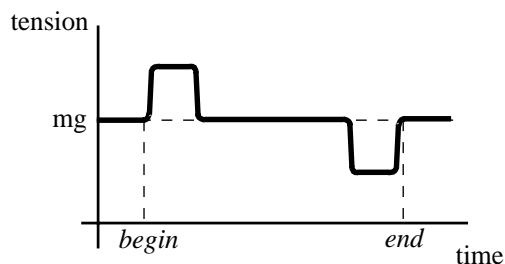
20.) A block on a frictional incline plane compresses an ideal spring by a distance d . The spring is released firing the block up the incline. Old George maintains that the block will go up the incline and, upon returning, will recompress the spring by some distance *less than* d . Why, in the real world, might he think that, and what additional, exotic thing might happen that could prove him wrong?

Solution: If the incline had been frictionless, the magnitude of the block's velocity as it left the spring would, due to the symmetry of the situation, have been the same as the magnitude of the velocity when it returned. If that had been the case, one would expect that the spring would again be depressed the same distance d . With friction in the picture, the block would not rise as high on the incline as otherwise would have been the case, and would not be moving as fast when it got back down to the spring. In that situation, you would expect the spring to be depressed a distance *less than* d . The exotic twist comes in the fact that *kinetic frictional force* is not as strong as *static frictional force*,

so it is possible that when the block gets to the top of its motion, the gravitational component will not be large enough to overcome the *static frictional force* being applied and the block will simply stay there without returning down the incline at all. Tricky, eh?

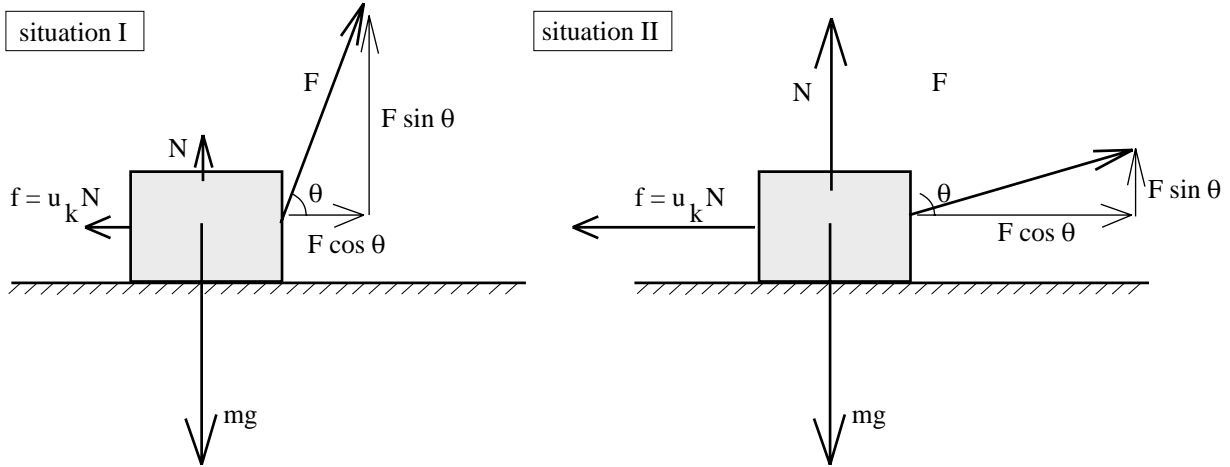
21.) A mass m is attached to one end of a string. The other end of the string is attached to the ceiling of an elevator. The elevator proceeds from the first floor to the sixtieth floor. What might you expect the graph of the string tension to look like, relative to the force mg , as the motion proceeds? Being the big-hearted guy that I am, I'll give you a hint: the tension IS mg before the elevator begins to move. Justify each part of your graph.

Solution: The tension in a string hanging from a ceiling will equal the weight of the mass attached to it as long as the system isn't accelerating. If there is acceleration, the tension will be other than mg . So, the elevator accelerates upward at a constant rate (I'm assuming it's constant) to start. The tension must not only support the weight of the pendulum bob, it must also increase the bob's velocity. In other words, the tension associated with that initial acceleration will be greater than mg . When the elevator reaches cruising speed, assuming it doesn't accelerate throughout the ascent, the velocity *change* will be zero and, hence, so will the acceleration. In that case (i.e., for constant velocity), the tension in the line will simply be mg (note that this is a consequence of N.F.L.: objects in motion stay in motion with a constant velocity *unless impinged upon by a net external force*--in this case, gravity and tension are external forces that add to zero so the net force on the bob is zero and the bob proceeds with constant velocity). When the elevator approaches the end, it must accelerate negatively, slowing the body down (if you've ever been in an elevator that is doing this, you know that you end up feeling light on your feet--the same happens here). The tension doesn't have to support the full weight of the pendulum bob because the bob's velocity is decreasing (it's the opposite of having to increase the tension to counter both gravity *and an increase* in velocity). The graph depicts all of this.



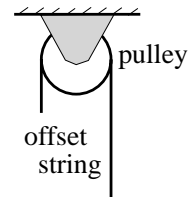
22.) A block on a horizontal frictional surface is pulled by a rope oriented at some non-zero angle, relative to the horizontal. Is there an optimal angle at which the block's acceleration will be a maximum and, if so, how would you go about theoretically determining that angle?

Solution: There are two forces to deal with here. The easier one to see is the force generated by the rope (I'll call this F). It will have two components, one in the vertical and one in the horizontal. The second is friction. The acceleration of the block will depend upon the vector sum (actually, because they are in opposite directions, it'll be a subtraction) of the frictional force and the horizontal component of F . The frictional force depends upon the *normal force* which, in turn, depends upon the *vertical component* of F . The relationships can be seen in the sketches labeled *situation I* and *situation II*. It should be obvious from looking at the sketches that with a big angle comes a little normal force (the large vertical component of F means that the surface providing the normal force will not have to muster much force to counteract mg), a small frictional force



(N will be small, so the coefficient of kinetic friction times N will be small), and a small horizontal component of F . The *vector* sum of friction and the horizontal component of F (this will be $f - F \cos \theta$) will be small as the two are about the same size. As a consequence, the acceleration will be relatively small. Likewise, with a small angle comes a large normal force, a large frictional force, and a large horizontal component of F . The *vector* sum of friction and the horizontal component of F will, again, be small as the two will be about the same size. As a consequence, the acceleration will be relatively small. There is, indeed, an angle somewhere between the two situations in which the normal force isn't too big so the frictional force isn't too big, but the horizontal component of F is largish. In that case, the vector sum of the horizontal vectors will be relatively large as will the acceleration. How do you get that angle? Use N.S.L. to generate an expression for F as a function of θ , then maximize that function (i.e., note that the slope of the F vs θ graph will be zero at a maximum, take the derivative of F to determine its slope function, set that function equal to zero to hone in on its maximum, then solve for the angle that satisfies that situation).

23.) You want to set up the following device: Place a string over a pulley, then attach unequal masses to each end (this device is called *an Atwood Machine*--typically, the question asked for such a device is *what will the acceleration of the system be if allowed to freefall?*).



Unfortunately, you are symmetrically challenged. Every time you thread the string over the pulley (i.e., before you get the masses attached), you put more string on one side of the pulley than on the other side. That means that when you release the string to pick up the masses, the string free-wheels over the pulley and ends up on the ground (really irritating).

You may be symmetrically challenged, but you aren't stupid. Realizing that you aren't going to be able to make the miserable thing work, you change the problem to *what is the string's freefall acceleration as it free-wheels over the pulley?* Without answering the question itself, answer the following: Will the acceleration be constant (i.e., could you use kinematics on this if you were clever?) and, if not, what parameters (i.e., height above the ground, initial velocity, what?) will determine what the string's acceleration is at a given instant?

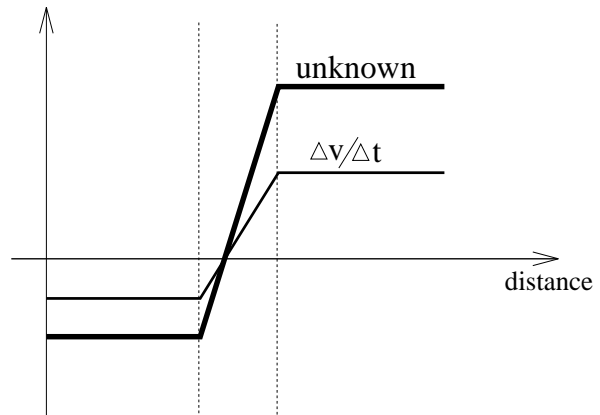
Solution: What determines the acceleration will be the disparity in weight between the string on one side of the pulley and that on the other side. As that disparity will change with time (i.e., as more string slides over the pulley, more weight will be on the down side), the acceleration would not be constant and kinematics would not be an acceptable option.

24.) A net force of 2 newtons is applied to a mass. The speed of the object doesn't change. How can this be?

Solution: If the force is perpendicular to the direction of motion, the magnitude of the velocity won't change but the direction will. Evidently, that is what is happening here.

25.) The graph shown depicts an object's change of velocity with time. What might the unknown function be?

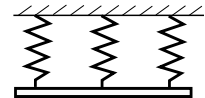
Solution: *Change of velocity with time* is acceleration, so the known function mimics that of acceleration. According to N.S.L., $F_{net} = ma$. As the known and unknown functions appear to be proportional to one another, I suspect the unknown function is that of the net force on the object.



26.) John was big, but he wasn't too bright. He needed to transport several 60 pound cubical microwave cookers (each was enameled plastic with no feet) across town in his car, but his beat up, rusty hulk of a vehicle was just a little too small. To accommodate the last two, he put them side by side on the roof. Once there, he urged the car forward only to find that they both broke loose when stopping. To remedy the problem, he put one on top of the other thereby doubling the normal force thereby doubling the frictional force. What problem is he likely to run into?

Solution: He is only doubling the normal force on the bottom microwave. Assuming the car's surface isn't slick (it *was* rusty), the top one will not enjoy the extra friction.

27.) Three identical springs are attached at the ceiling. A bar of mass m is hooked to the group. If the new system's equilibrium position is d units below the springs' unstretched lengths, what must the spring constant be for each spring?



Solution: Summing the forces at the new equilibrium position yields $3kd - mg = ma = 0$ (i.e., $a = 0$ at equilibrium). Solving yields a spring constant of $k = mg/(3d)$.